Chapter 8

Chapter 8 Opener

Try It Yourself (p. 333)

1. Area = Area of rectangle + Area of semicircle

$= \ell w + \frac{1}{2}\pi r^2$

$\approx (15)(8) + \frac{1}{2}(3.14)(4)^2$

$= 120 + 25.12$

$= 145.12 \text{ m}^2$

The area of the figure is about 145.12 square meters.

2. Area = Area of horizontal rectangle + Area of vertical rectangle

$= \ell w + \ell w$

$= (9)(4) + (5)(10)$

$= 36 + 50$

$= 86 \text{ cm}^2$

The area of the figure is 86 square centimeters.

3. $A = \pi r^2 \approx 3.14(5)^2 = 3.14 \bullet 25 = 78.5 \text{ ft}^2$

The area of the circle is about 78.5 square feet.

4. $r = \frac{1}{2}d = \frac{1}{2}(26) = 13 \text{ in.}$

$A = \pi r^2 \approx 3.14(13)^2 = 3.14 \bullet 169 = 530.66 \text{ in.}^2$

The area of the circle is about 530.66 square inches.

5. $r = \frac{1}{2}d = \frac{1}{2}(7) = 3.5 \text{ cm}$

$A = \pi r^2 \approx 3.14(3.5)^2 = 3.14 \bullet 12.25 = 38.465 \text{ cm}^2$

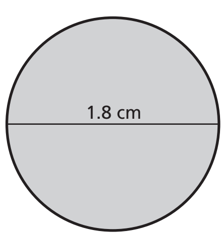
The area of the circle is about 38.465 square centimeters.

Section 8.1

8.1 Activity (pp. 334–335)

1. a. *Sample answer:*

Area of the face of a dime:

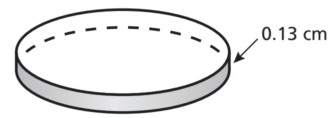


$A = \pi r^2 = \pi(0.9)^2 = 0.81\pi \approx 2.54 \text{ cm}^2$

The area of the face of a dime is about 2.54 square centimeters.

b. *Sample answer:*

Height of 12 dimes:



Because the height of 1 dime is about 0.13 centimeter, the height of 12 dimes is about $12 \bullet 0.13 = 1.56$ centimeters.

Volume of 12 dimes:

$V = (\text{area of face}) \bullet (\text{height of 12 dimes})$

$= 0.81\pi \bullet 1.56$

$= 1.2636\pi \text{ cm}^3$

The volume of 12 dimes is about 3.97 cubic centimeters.

c. To find the volume of a cylinder, multiply the area of the base by the height.

$V = Bh = \pi r^2 h$

2. a. *Sample answer:*

small candle: 2 inch radius, 3 inch height
medium candle: 2 inch radius, 5 inch height
large candle: 2 inch radius, 8 inch height

b. *Sample answer:*

small candle: $2
medium candle: $5
large candle: $8

c. *Sample answer:* No, but they should be because the person is paying for the amount of wax to make each candle, which is the volume of the wax.

3. Pour water into the beaker until it flows out the side tube. Place an empty cylinder at the end of this side tube. Gently lower the object into the beaker. The volume of the object is equal to the amount of water that flows into the cylinder.

4. a. *Sample answer:* The taller cylinder looks like it has the greater volume.

b. Volume of short cylinder:

$V = Bh = \pi(3)^2 \bullet 4 = 9\pi \bullet 4 = 36\pi \approx 113.1 \text{ units}^3$

Volume of tall cylinder:

$V = Bh$

$= \pi(2)^2 \bullet 9$

$= 4\pi \bullet 9$

$= 36\pi$

$\approx 113.1 \text{ units}^3$

The cylinders have the same volume.

5. To find the volume of a cylinder, calculate the area of the base and then multiply by the height.

6. Both formulas are $V = Bh$, where B is the area of the base and h is the height.

8.1 On Your Own (pp. 336–337)

1. $V = Bh = \pi(4)^2(15) = 240\pi \approx 754.0 \text{ ft}^3$

   The volume is about 754.0 cubic feet.

2. The diameter is 8 centimeters. So, the radius is 4 centimeters.

   $$V = Bh$$
   $$176 = \pi(4)^2(h)$$
   $$176 = 16\pi h$$
   $$3.5 \approx h$$

   The height is about 3.5 centimeters.

3. The missing salsa fills a cylinder with a height of $10 - 5 = 5$ centimeters and a radius of 5 centimeters.

   $V = Bh = \pi(5)^2(5) = 125\pi \approx 392.7 \text{ cm}^3$

   About 392.7 cubic centimeters of salsa are missing from the jar.

4. Find the volume of the tower. The diameter is 15 meters. So, the radius is 7.5 meters.

   $V = Bh = \pi(7.5)^2(5) = 281.25\pi \approx 883.573 \text{ m}^3$

   Calculate the number of gallons:

   $$883.573 \cancel{\text{m}^3} \times \frac{264 \text{ gal}}{1 \cancel{\text{m}^3}} \approx 233{,}263 \text{ gal}$$

   The water tower contains about 233,263 gallons of water.

8.1 Exercises (pp. 338–339)

Vocabulary and Concept Check

1. The question "How much does it take to cover the cylinder?" is different because it deals with surface area and the other three deal with volume.

   Surface area:

   $$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(5)^2 + 2\pi(5)(12) \\ &= 50\pi + 120\pi \\ &= 170\pi \\ &\approx 534.1 \text{ cm}^2 \end{aligned}$$

   Volume:

   $V = Bh = \pi(5)^2(12) = 300\pi \approx 942.5 \text{ cm}^3$

   It will take about 534.1 square centimeters to cover the cylinder. It will take about 942.5 cubic centimeters to fill the cylinder, which is also the capacity of the cylinder and how much the cylinder contains.

2. The volume of the cube is greater because the cylinder can be placed inside the cube with room to spare in the corners.

Practice and Problem Solving

3. $V = Bh = \pi(9)^2(6) = 486\pi \approx 1526.8 \text{ ft}^3$

   The volume is about 1526.8 cubic feet.

4. The diameter is 3 meters, so the radius is 1.5 meters.

   $V = Bh = \pi(1.5)^2(3) = 6.75\pi \approx 21.2 \text{ m}^3$

   The volume is about 21.2 cubic meters.

5. $V = Bh = \pi(7)^2(5) = 245\pi \approx 769.7 \text{ ft}^3$

   The volume is about 769.7 cubic feet.

6. $V = Bh = \pi(5)^2(10) = 250\pi \approx 785.4 \text{ ft}^3$

   The volume is about 785.4 cubic feet.

7. $V = Bh = \pi(3)^2(10) = 90\pi \approx 282.7 \text{ mm}^3$

   The volume is about 282.7 cubic millimeters.

8. $V = Bh = \pi(2)^2(1) = 4\pi \approx 12.6 \text{ ft}^3$

   The volume is about 12.6 cubic feet.

9. $V = Bh = \pi(6)^2(7) = 252\pi \approx 791.7 \text{ in.}^3$

   The volume is about 791.7 cubic inches.

10. The diameter is 15 meters, so the radius is 7.5 meters.

    $V = Bh = \pi(7.5)^2(5) = 281.25\pi \approx 883.6 \text{ m}^3$

    The volume is about 883.6 cubic meters.

11. The diameter is 8 centimeters, so the radius is 4 centimeters.

    $V = Bh = \pi(4)^2(16) = 256\pi \approx 804.2 \text{ cm}^3$

    The volume is about 804.2 cubic centimeters.

12. The diameter is 16 feet, so the radius is 8 feet.

    $V = Bh = \pi(8)^2(4) = 256\pi \approx 804.2 \text{ ft}^3$

    Calculate the number of gallons:

    $$804.2 \cancel{\text{ft}^3} \times \frac{7.5 \text{ gal}}{1 \cancel{\text{ft}^3}} = 6031.5 \text{ gal}$$

    The pool could contain about 6032 gallons of water.

13. The diameter is 8 feet, so the radius is 4 feet.

    $$V = Bh$$
    $$250 = \pi(4)^2(h)$$
    $$250 = 16\pi h$$
    $$5 \approx h$$

    The height is about 5 feet.

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14. The diameter is 32 inches, so the radius is 16 inches.

$$\begin{aligned} V &= Bh \\ 10{,}000\pi &= \pi(16)^2(h) \\ 10{,}000\pi &= 256\pi h \\ 39 &\approx h \end{aligned}$$

The height is about 39 inches.

15.

$$\begin{aligned} V &= Bh \\ 600{,}000 &= \pi r^2(76) \\ 600{,}000 &= 76\pi r^2 \\ 2512.97 &\approx r^2 \\ 50 &\approx r \end{aligned}$$

The radius is about 50 centimeters.

16. When the diameter of a cylinder is halved, the radius is also halved. So, the volume of a cylinder with one-half the radius is

$$V = Bh = \pi\left(\frac{1}{2} \bullet r\right)^2 h = \pi\left(\frac{1}{4}r^2\right)h = \frac{1}{4}\pi r^2 h.$$

So, when the diameter is halved, the volume is one-fourth the original volume.

17. Volume of a "round" bale:

The diameter is 4 feet, so the radius is 2 feet.

$$V = Bh = \pi(2)^2(5) = 20\pi \approx 62.8 \text{ ft}^3$$

Volume of a "square" bale:

$$V = \ell wh = (2)(2)(4) = 16 \text{ ft}^3$$

Because $62.8 \div 16 = 3.925$, about 4 "square" bales contain the same amount of hay as one "round" bale.

18. Volume of the tank:

$$V = Bh = \pi(2)^2(6) = 24\pi \approx 75.398 \text{ ft}^3$$

Calculate the weight:

$$75.398 \cancel{\text{ft}^3} \times \frac{62.5 \text{ lb}}{1 \cancel{\text{ft}^3}} \approx 4712 \text{ lb}$$

The water in the tank weighs about 4712 pounds.

19. Height of the cylinder:

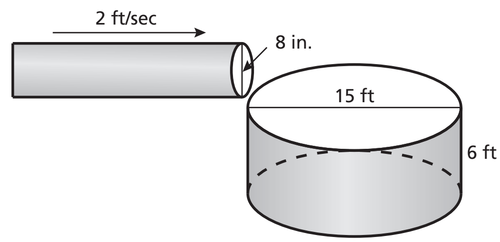
$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ 1850 &= 2\pi(9)^2 + 2\pi(9)(h) \\ 1850 &= 162\pi + 18\pi h \\ h &= \frac{1850 - 162\pi}{18\pi} \end{aligned}$$

Volume of cylinder:

$$\begin{aligned} V = Bh &= \pi(9)^2\left(\frac{1850 - 162\pi}{18\pi}\right) \\ &= 9\left(\frac{1850 - 162\pi}{2}\right) \\ &= 9(925 - 81\pi) \\ &\approx 6035 \text{ m}^3 \end{aligned}$$

The volume of the cylinder is about 6035 cubic meters.

20.



a. The diameter of the pipe is 8 inches, so the radius is 4 inches. The water flows at a rate of 2 feet per second, which is 24 inches per second.

$$V = Bh = \pi(4)^2(24) = 384\pi \text{ in.}^3$$

The amount of water that flows out of the pipe each second is 384π, or 1206.37 cubic inches.

b. Calculate the time in seconds:

$$5 \cancel{\text{min}} \times \frac{60 \text{ sec}}{1 \cancel{\text{min}}} = 300 \text{ sec}$$

Volume in the tank:

$$300 \cancel{\text{sec}} \times \frac{384\pi \text{ in.}^3}{\cancel{\text{sec}}} = 115{,}200\pi \text{ in.}^3$$

Find the height of water in the tank:

The diameter of the tank is 15 feet, so the radius is 7.5 feet.

$$7.5 \cancel{\text{ft}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} = 90 \text{ in.}$$

$$\begin{aligned} V &= Bh \\ 115{,}200\pi &= \pi(90)^2 h \\ 115{,}200\pi &= 8100\pi h \\ 14.22 &\approx h \end{aligned}$$

After 5 minutes, the height of the water is about 14.22 inches.

c. Volume of the tank:

The height of the tank is $6\ \cancel{\text{ft}} \times \frac{12\text{ in.}}{1\ \cancel{\text{ft}}} = 72$ inches.

$$V = Bh = \pi(90)^2(72) = 583{,}200\pi \approx 1{,}832{,}177\text{ in.}^3$$

The tank can hold 1,832,177 cubic inches of water. So, the amount of water in 75% of the tank is $0.75 \bullet 1{,}832{,}177 \approx 1{,}374{,}133$ cubic inches.

Time:

$$1{,}374{,}133\ \cancel{\text{in.}^3} \times \frac{1\ \cancel{\text{sec}}}{384\pi\ \cancel{\text{in.}^3}} \times \frac{1\text{ min}}{60\ \cancel{\text{sec}}} \approx 18.98\text{ min}$$

It will take about 19 minutes to fill 75% of the tank.

Fair Game Review

21.
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 20^2 + 21^2 &\stackrel{?}{=} 29^2 \\ 400 + 441 &\stackrel{?}{=} 841 \\ 841 &= 841 \ \checkmark \end{aligned}$$

It *is* a right triangle.

22.
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 1^2 + 2.4^2 &\stackrel{?}{=} 2.6^2 \\ 1 + 5.76 &\stackrel{?}{=} 6.76 \\ 6.76 &= 6.76 \ \checkmark \end{aligned}$$

It *is* a right triangle.

23.
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5.6^2 + 8^2 &\stackrel{?}{=} 10.6^2 \\ 31.36 + 64 &\stackrel{?}{=} 112.36 \\ 95.36 &\neq 112.36 \ \times \end{aligned}$$

It is *not* a right triangle.

24. C;

$$\begin{aligned} 3x + 4y &= -10 \\ 2x - 4y &= 0 \end{aligned}$$

Add the two equations.

$$\begin{aligned} 3x + 4y &= -10 \\ +\ 2x - 4y &= 0 \\ \hline 5x &= -10 \\ \frac{5x}{5} &= \frac{-10}{5} \\ x &= -2 \end{aligned}$$

$$\begin{aligned} 2x - 4y &= 0 \\ 2(-2) - 4y &= 0 \\ -4 - 4y &= 0 \\ -4y &= 4 \\ \frac{-4y}{-4} &= \frac{4}{-4} \\ y &= -1 \end{aligned}$$

So, the solution is $(-2, -1)$.

Section 8.2

8.2 Activity (pp. 340–341)

1. 3; $3 \times \text{Volume of a Cone} = \text{Volume of a Cylinder}$

$$\begin{aligned} 3 \times \text{Volume of a Cone} &= Bh \\ \text{Volume of a Cone} &= \frac{1}{3}Bh \end{aligned}$$

2. *Volumes of Prisms and Cylinders*

Volume = Area of base $\times$ height

Volumes of Pyramids and Cones

$$\text{Volume} = \frac{1}{3}\begin{pmatrix}\text{Volume of prism or cylinder} \\ \text{with same base and height}\end{pmatrix}$$

Formulas for the area of a base:

Square and rectangle: $A = \ell w$

Triangle: $A = \frac{1}{2}bh$

Circle: πr^2

3. The volume of the stack remains the same because the paper takes up the same amount of space.

4. *Sample answer*: Find the volume of a cylinder with the same base and height as the cone and multiply by $\frac{1}{3}$, or use the formula $V = \frac{1}{3}Bh$.

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5. a. triangle; *Sample answer*: Find the volume of the entire cone and divide it by 2.

   b. circle; *Sample answer*: Find the volume of the entire cone and subtract the volume of one of the sections to find the volume of the other section.

8.2 On Your Own (p. 343)

1. $V = \frac{1}{3}Bh = \frac{1}{3}\pi(6)^2(15) = 180\pi \approx 565.5 \text{ cm}^3$

   The volume is about 565.5 cubic centimeters.

2. $$V = \frac{1}{3}Bh$$
   $$7200 = \frac{1}{3}\pi(15)^2(h)$$
   $$7200 = 75\pi h$$
   $$30.6 \approx h$$

   The height is about 30.6 yards.

3. Time:

   $$2513 \cancel{\text{mm}^3} \times \frac{1 \text{ sec}}{60 \cancel{\text{mm}^3}} \approx 41.88 \text{ sec}$$

   You have about 42 seconds to answer the question.

4. Volume:

   $V = \frac{1}{3}Bh = \frac{1}{3}\pi(5)^2(12) = 100\pi \approx 314 \text{ mm}^3$

   The volume of the sand is about 314 cubic millimeters.

   Time:

   $$314 \cancel{\text{mm}^3} \times \frac{1 \text{ sec}}{50 \cancel{\text{mm}^3}} \approx 6.28 \text{ sec}$$

   You have about 6 seconds to answer the question.

8.2 Exercises (pp. 344–345)

Vocabulary and Concept Check

1. The height of a cone is the perpendicular distance from the base to the vertex.

2. The formulas for the volume of a cone and a pyramid multiply one-third by the product of the area of the base and the height. However, the base of a cone is a circle, so the area of the base is πr^2, and the base of a pyramid is not.

3. The volume of a cone is one-third the volume of a cylinder, so you would divide by 3.

Practice and Problem Solving

4. $V = \frac{1}{3}Bh = \frac{1}{3}\pi(2)^2(4) = \frac{16\pi}{3} \approx 16.8 \text{ in.}^3$

   The volume of the cone is about 16.8 cubic inches.

5. $V = \frac{1}{3}Bh = \frac{1}{3}\pi(3)^2(3) = 9\pi \approx 28.3 \text{ m}^3$

   The volume is about 28.3 cubic meters.

6. $V = \frac{1}{3}Bh = \frac{1}{3}\pi(5)^2(10) = \frac{250\pi}{3} \approx 261.8 \text{ mm}^3$

   The volume is about 261.8 cubic millimeters.

7. $V = \frac{1}{3}Bh = \frac{1}{3}\pi(1)^2(2) = \frac{2\pi}{3} \approx 2.1 \text{ ft}^3$

   The volume is about 2.1 cubic feet.

8. $V = \frac{1}{3}Bh = \frac{1}{3}\pi(5)^2(8) = \frac{200\pi}{3} \approx 209.4 \text{ cm}^3$

   The volume is about 209.4 cubic centimeters.

9. $V = \frac{1}{3}Bh = \frac{1}{3}\pi\left(\frac{7}{2}\right)^2(9) = \frac{147\pi}{4} \approx 115.5 \text{ yd}^3$

   The volume is about 115.5 cubic yards.

10. $V = \frac{1}{3}Bh = \frac{1}{3}\pi(4)^2(7) = \frac{112\pi}{3} \approx 117.3 \text{ ft}^3$

    The volume is about 117.3 cubic feet.

11. $V = \frac{1}{3}Bh = \frac{1}{3}\pi\left(2\frac{1}{2}\right)^2(10) = \frac{125\pi}{6} \approx 65.4 \text{ in.}^3$

    The volume is about 65.4 cubic inches.

12. $V = \frac{1}{3}Bh = \frac{1}{3}\pi(2)^2(8) = \frac{32\pi}{3} \approx 33.5 \text{ cm}^3$

    The volume is about 33.5 cubic centimeters.

13. The diameter was used instead of the radius. The diameter of the cone is 2 meters, so the radius is 1 meter.

    $V = \frac{1}{3}Bh = \frac{1}{3}\pi(1)^2(3) = \pi \text{ m}^3$

14. Volume of Glass A:

    $V = \frac{1}{3}Bh = \frac{1}{3}\pi(4)^2(8) = \frac{128\pi}{3} \approx 134.0 \text{ cm}^3$

    Volume of Glass B:

    $V = \frac{1}{3}Bh = \frac{1}{3}\pi(3)^2(10) = 30\pi \approx 94.2 \text{ cm}^3$

    Because Glass A has a greater volume, it can hold more liquid. Glass A can hold about $134.0 - 94.2 = 39.8$ cubic centimeters more than Glass B.

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15. The diameter is $\frac{2}{3}$ foot, so the radius is $\frac{1}{2} \bullet \frac{2}{3} = \frac{1}{3}$ foot.

$$V = \frac{1}{3}Bh$$
$$\frac{1}{18}\pi = \frac{1}{3}\pi\left(\frac{1}{3}\right)^2(h)$$
$$\frac{1}{18}\pi = \frac{1}{27}\pi h$$
$$1.5 = h$$

The height is 1.5 feet.

16. The diameter is 10 centimeters, so the radius is 5 centimeters.

$$V = \frac{1}{3}Bh$$
$$225 = \frac{1}{3}\pi(5)^2(h)$$
$$225 = \frac{25}{3}\pi h$$
$$8.6 \approx h$$

The height is about 8.6 centimeters.

17.
$$V = \frac{1}{3}Bh$$
$$3.6 = \frac{1}{3}\pi(r)^2(4.2)$$
$$3.6 = 1.4\pi r^2$$
$$0.81 \approx r^2$$
$$0.9 \approx r$$

The radius is about 0.9 inch, so the diameter is about 1.8 inches.

18. The volume of a cone is $\frac{1}{3}Bh$ and the volume of a cylinder is Bh. So, if the volume of a cone is 20π cubic meters, the volume of a cylinder is $3(20\pi) = 60\pi$ cubic meters.

19. Volume of the cone:

The diameter of the cone is 4.8 inches, so the radius is 2.4 inches.

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi(2.4)^2(10) = 19.2\pi \approx 60.32 \text{ in.}^3$$

The volume of the vase is about 60.32 cubic inches, which is the amount of water in the vase. So, 20% of the water is $0.2(60.32) = 12.064$ cubic inches of water.

Time: $12.064 \cancel{\text{in.}^3} \times \frac{1 \text{ min}}{0.5 \cancel{\text{in.}^3}} \approx 24.1 \text{ min}$

It will take about 24.1 minutes for 20% of the water to leak from the vase.

20. a. Volume of one paper cup:

The diameter is 8 centimeters, so the radius is 4 centimeters.

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi(4)^2(11) = \frac{176\pi}{3} \approx 184.3 \text{ cm}^3$$

Amount of lemonade in cubic centimeters:

$$10 \cancel{\text{gal}} \times \frac{3785 \text{ cm}^3}{1 \cancel{\text{gal}}} = 37{,}850 \text{ cm}^3$$

Because $37{,}850 \div 184.3 \approx 205.37$, the number of cups needed is 206 cups.

b. Because $206 \div 50 = 4.12$, you should buy 5 packages of paper cups. This will give you $5(50) = 250$ cups.

c. Because there are 37,850 cubic centimeters of lemonade to sell, 80% of the lemonade is $0.8(37{,}850) = 30{,}280$ cubic centimeters. Because $30{,}280 \div 184.3 \approx 164.30$, the number of cups used is 165 cups. So, the number of cups left over is $250 - 165 = 85$ cups.

21. Because the diameter of the cone is $2x$, the radius is x. Because the radius of the cone and the cylinder are the same, the area of the base is the same. The volume of the cone is one-third the volume of the cylinder, so the height of the cone must be three times the height of the cylinder. The height of the cone is $3y$.

22. The triangle in Example 3 is similar to the triangle formed by the new timer.

$$\frac{r}{30} = \frac{10}{24}$$
$$r = \frac{300}{24}$$
$$r = 12.5$$

So, the radius of the new triangle is 12.5 millimeters.

$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}\pi(12.5)^2(30)$$
$$= 1562.5\pi$$
$$\approx 4909$$

The volume of the sand is about 4909 cubic millimeters.

$$4909 \cancel{\text{mm}^3} \times \frac{1 \text{ sec}}{50 \cancel{\text{mm}^3}} = 98.18$$

So, you have about 98 seconds to answer the question.

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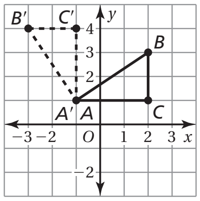
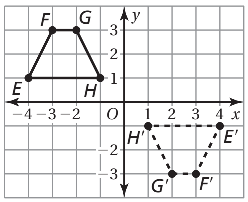
Fair Game Review

23.



The coordinates of the image are $A'(-1, 1)$, $B'(-3, 4)$, and $C'(-1, 4)$.

24.



The coordinates of the image are $E'(4, -1)$, $F'(3, -3)$, $G'(2, -3)$, and $H'(1, -1)$.

25. D;

Area of $\Delta ABC = \frac{1}{2}bh$;

Area of $\Delta XYZ = \frac{1}{2}(3b)(3h) = 9\left[\frac{1}{2}bh\right]$

So, the area of ΔXYZ is 9 times greater than the area of ΔABC.

Study Help

Available at *BigIdeasMath.com*.

Quiz 8.1–8.2

1. The diameter is 4 yards, so the radius is 2 yards.

$V = Bh = \pi(2)^2(3.5) = 14\pi \approx 44.0 \text{ yd}^3$

The volume is about 44.0 cubic yards.

2. $V = Bh = \pi(3)^2(4) = 36\pi \approx 113.1 \text{ ft}^3$

The volume is about 113.1 cubic feet.

3. $V = \frac{1}{3}Bh = \frac{1}{3}\pi(5)^2(6) = 50\pi \approx 157.1 \text{ cm}^3$

The volume is about 157.1 cubic centimeters.

4. The diameter is 12 inches, so the radius is 6 inches.

$V = \frac{1}{3}Bh = \frac{1}{3}\pi(6)^2(11) = 132\pi \approx 414.7 \text{ in.}^3$

The volume is 414.7 cubic inches.

5.

$$V = Bh$$
$$340 = \pi(3)^2 h$$
$$340 = 9\pi h$$
$$12.0 \approx h$$

The height is about 12.0 feet.

6.

$$V = \frac{1}{3}Bh$$
$$938 = \frac{1}{3}\pi r^2(4.7)$$
$$938 = \frac{4.7}{3}\pi r^2$$
$$190.6 \approx r^2$$
$$13.8 \approx r$$

The radius is about 13.8 centimeters.

7. The diameter of the cone is 6 centimeters, so the radius is 3 centimeters.

$$V = \frac{1}{3}Bh$$
$$84.78 = \frac{1}{3}\pi(3)^2(h)$$
$$84.78 = 3\pi h$$
$$9 \approx h$$

The height is about 9 centimeters.

8. *Original cylinder*

$V = Bh = \pi(5)^2(1) = 25\pi \text{ m}^3$

Dimensions tripled

$V = Bh = \pi(15)^2(3) = 675\pi \text{ m}^3$

The ratio of the volumes is $\frac{675\pi \cancel{\text{m}^3}}{25\pi \cancel{\text{m}^3}} = 27.$ So, the volume of the new cylinder is 27 times the volume of the original cylinder.

9. *Total volume*

$V = Bh = \pi(1.5)^2(16) = 36\pi \approx 113.10$

The total volume of the container is about 113.10 cubic inches, so the volume of the white sand is about $113.10 - 42.39 - 28.26 = 42.45$ cubic inches.

10. Volume of Can A:

The diameter is 6 inches, so the radius is 3 inches.

$V = Bh = \pi(3)^2(6) = 54\pi \text{ in.}^3$

Height of Can B:

The diameter is 4 inches, so the radius is 2 inches.

$$\begin{aligned} V &= Bh \\ 54\pi &= \pi(2)^2(h) \\ 54\pi &= 4\pi h \\ 13.5 &= h \end{aligned}$$

The height of Can B is 13.5 inches.

Section 8.3

8.3 Activity (pp. 348–349)

1. The height of the cylinder is equal to the diameter of the ball, which is twice the radius, $2r$. $\frac{2}{3}$ of the cylinder is filled with rice.

2.

$V = \pi r^2 h$ Write formula for volume of a cylinder.

$= \frac{2}{3}\pi r^2 h$ Multiply by $\frac{2}{3}$ because the volume of a sphere is $\frac{2}{3}$ of the volume of the cylinder.

$= \frac{2}{3}\pi r^2(2r)$ Substitute $2r$ for h.

$= \frac{4}{3}\pi r^3$ Simplify.

3.

$V = \frac{1}{3}Bh$ Write formula for volume of a pyramid.

$= n\frac{1}{3}Br$ Multiply by the number of small pyramids n and substitute r for h.

$= \frac{1}{3}(4\pi r^2)r$ $4\pi r^2 \approx n \bullet B$

This result is equal to the result in Activity 2.

$$\begin{aligned} \frac{1}{3}(4\pi r^2)r &= \frac{1}{3} \bullet 4 \bullet \pi \bullet r \bullet r \bullet r \\ &= \left(\frac{1}{3} \bullet 4\right) \bullet \pi \bullet (r \bullet r \bullet r) \\ &= \frac{4}{3}\pi r^3 \end{aligned}$$

4. *Sample answer:* Use the formula $V = \frac{4}{3}\pi r^3$.

5. circle; *Sample answer:* Find the volume of the entire sphere and divide it by 2.

8.3 On Your Own (p. 351)

1. The diameter is 16 feet, so the radius is 8 feet.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(8)^3 \\ &= \frac{2048}{3}\pi \\ &\approx 2144.7 \end{aligned}$$

The volume is about 2144.7 cubic feet.

2.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ 36\pi &= \frac{4}{3}\pi r^3 \\ 27 &= r^3 \\ 3 &= r \end{aligned}$$

The radius is 3 meters.

3. The two hemispheres combine to one sphere.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2)^3 = \frac{4}{3}\pi(8) = \frac{32}{3}\pi$

The height of the cylinder is 4 inches and the radius is 2 inches.

$V = Bh = \pi(2)^2(4) = 16\pi$

So, the volume of the composite solid is $\frac{32}{3}\pi + 16\pi = \frac{80}{3}\pi \approx 83.8$ cubic inches.

4. The height of the cone is 5 meters and the radius is 3 meters.

$V = \frac{1}{3}Bh = \frac{1}{3}\pi(3)^2(5) = 15\pi$

The height of the cylinder is 9 meters and the radius is 3 meters.

$V = Bh = \pi(3)^2(9) = 81\pi$

So, the volume of the composite solid is $15\pi + 81\pi = 96\pi \approx 301.6$ cubic meters.

8.3 Exercises (pp. 352–353)

Vocabulary and Concept Check

1. A hemisphere is one-half of a sphere.

2. sphere; It does not have a base.

Practice and Problem Solving

3. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5)^3 = \frac{500\pi}{3} \approx 523.6$

The volume is about 523.6 cubic inches.

4. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(7)^3 = \frac{1372\pi}{3} \approx 1436.8$

The volume is about 1436.8 cubic feet.

5. The diameter is 18 millimeters, so the radius is 9 millimeters. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(9)^3 = 972\pi \approx 3053.6$

The volume is about 3053.6 cubic millimeters.

6. The diameter is 12 yards, so the radius is 6 yards.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = 288\pi \approx 904.8$

The volume is about 904.8 cubic yards.

7. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = 36\pi \approx 113.1$

The volume is about 113.1 cubic centimeters.

8. The diameter is 28 meters, so the radius is 14 meters.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(14)^3 = \frac{10,976\pi}{3} \approx 11,494.0$

The volume is about 11,494.0 cubic meters.

9.
$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ 972\pi &= \frac{4}{3}\pi r^3 \\ 729 &= r^3 \\ 9 &= r \end{aligned}$$

The radius is 9 millimeters.

10.
$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ 4.5\pi &= \frac{4}{3}\pi r^3 \\ 3.375 &= r^3 \\ 1.5 &= r \end{aligned}$$

The radius is 1.5 centimeters.

11.
$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ 121.5\pi &= \frac{4}{3}\pi r^3 \\ 91.125 &= r^3 \\ 4.5 &= r \end{aligned}$$

The radius is 4.5 feet.

12. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(10)^3 = \frac{4000\pi}{3} \approx 4189$

The volume of the globe is about 4189 cubic inches.

13.
$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ \frac{125}{6}\pi &= \frac{4}{3}\pi r^3 \\ \frac{125}{8} &= r^3 \\ \frac{5}{2} &= r \\ 2.5 &= r \end{aligned}$$

The radius of the softball is 2.5 inches.

14. The radius of the hemisphere is 4 centimeters.

$V = \frac{1}{2} \bullet \frac{4}{3}\pi r^3 = \frac{2}{3}\pi(4)^3 = \frac{128\pi}{3}$

The sides of the cube are 8 centimeters.

$V = s^3 = (8)^3 = 512$

So, the volume of the composite solid is $\frac{128\pi}{3} + 512 \approx 646.0$ cubic centimeters.

15. The radius of the cone is 8 feet.

$V = \frac{1}{3}Bh = \frac{1}{3}\pi(8)^2(6) = 128\pi$

The radius of the cylinder is 8 feet.

$V = Bh = \pi(8)^2(4) = 256\pi$

So, the volume of the composite solid is $128\pi + 256\pi = 384\pi \approx 1206.4$ cubic feet.

16. The radius of the hemisphere is 3 inches.

$V = \frac{1}{2} \bullet \frac{4}{3}\pi r^3 = \frac{2}{3}\pi(3)^3 = 18\pi$

The radius of the cylinder is 3 inches.

$V = Bh = \pi(3)^2(11) = 99\pi$

So, the volume of the composite solid is $99\pi - 18\pi = 81\pi \approx 254.5$ cubic inches.

17. The volume of the sphere is $V = \frac{4}{3}\pi r^3$.

The volume of the cylinder is $V = Bh = \pi r^2 h$.

The solids have the same volume and radius.

$$\begin{aligned} \frac{4}{3}\pi r^3 &= \pi r^2 h \\ \frac{4}{3}r &= h \\ r &= \frac{3}{4}h \end{aligned}$$

So, the radius is $r = \frac{3}{4}h$.

Chapter 8

18. Volume of container = Volume not occupied by rubber balls + 3 • Volume of a rubber ball

Volume of container: $V = Bh$

$= \pi(3)^2(18)$

$= 162\pi$

Volume of a rubber ball: $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(3)^3$

$= 36\pi$

Let x represent the volume not occupied by rubber balls.

$$\begin{aligned} 162\pi &= x + 3(36\pi) \\ 162\pi &= x + 108\pi \\ -108\pi & \quad\quad -108\pi \\ 54\pi &= x \\ 170 &\approx x \end{aligned}$$

So, the amount of space in the container that is not occupied by rubber balls is about 170 cubic centimeters.

19. $$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ 4500\pi &= \frac{4}{3}\pi r^3 \\ 3375 &= r^3 \\ 15 &= r \end{aligned}$$

The radius is 15 inches, so the diameter is 30 inches.

The side length of the box is 30 inches.

Surface Area $= 6s^2 = 6(30)^2 = 5400$ in.^{2}

Volume $= s^3 = (30)^3 = 27{,}000$ in.^{3}

So, the surface area of the box is 5400 square inches, and the volume is 27,000 cubic inches.

20. Volume of Sphere = 4 • Volume of Cone

$$\begin{aligned} \frac{4}{3}\pi r^3 &= 4\left[\frac{1}{3}\pi r^2 h\right] \\ \frac{4}{3}\pi r^3 &= \frac{4}{3}\pi r^2 h \\ r &= h \end{aligned}$$

So, the volume of a sphere with radius r is four times the volume of a cone with radius r when the height of the cone is equal to the radius.

Fair Game Review

21. The coordinates of the red figure are $A(1, 1)$, $B(4, 1)$, and $C(2, 3)$. The coordinates of the blue figure are $A'(2, 2)$, $B'(8, 2)$, and $C'(4, 6)$. Each x-and y-coordinate of ABC is multiplied by 2 to produce the vertices of $A'B'C'$. So, the dilation is an enlargement with a scale factor of 2.

22. The coordinates of the red figure are $A(-3, -3)$, $B(0, 3)$, $C(6, 3)$, and $D(6, -3)$. The coordinates of the blue figure are $A'(-1, -1)$, $B'(0, 1)$, $C'(2, 1)$, and $D'(2, -1)$. Each x- and y-coordinate of $ABCD$ is multiplied by $\frac{1}{3}$ to produce the vertices of $A'B'C'D'$.

So, the dilation is a reduction with a scale factor of $\frac{1}{3}$.

23. A;

Find the missing dimension using indirect measurement.

$$\frac{\text{height of person}}{\text{length of person's shadow}} = \frac{\text{height of flagpole}}{\text{length of flagpole's shadow}}$$

$$\begin{aligned} \frac{5}{6} &= \frac{h}{30} \\ \frac{150}{6} &= h \\ 25 &= h \end{aligned}$$

The height of the flagpole is 25 feet.

Chapter 8

Section 8.4

8.4 Activity (pp. 354–355)

1. a.

Radius	1	1	1	1	1
Height	1	2	3	4	5
Surface Area	4π	6π	8π	10π	12π
Volume	π	2π	3π	4π	5π

Solid with a height of 1:

$$S = 2\pi r^2 + 2\pi rh = 2\pi(1)^2 + 2\pi(1)(1) = 2\pi + 2\pi = 4\pi$$

$$V = Bh = \pi(1)^2(1) = \pi$$

Solid with a height of 2:

$$S = 2\pi r^2 + 2\pi rh = 2\pi(1)^2 + 2\pi(1)(2) = 2\pi + 4\pi = 6\pi$$

$$V = Bh = \pi(1)^2(2) = 2\pi$$

Solid with a height of 3:

$$S = 2\pi r^2 + 2\pi rh = 2\pi(1)^2 + 2\pi(1)(3) = 2\pi + 6\pi = 8\pi$$

$$V = Bh = \pi(1)^2(3) = 3\pi$$

Solid with a height of 4:

$$S = 2\pi r^2 + 2\pi rh = 2\pi(1)^2 + 2\pi(1)(4) = 2\pi + 8\pi = 10\pi$$

$$V = Bh = \pi(1)^2(4) = 4\pi$$

Solid with a height of 5:

$$S = 2\pi r^2 + 2\pi rh = 2\pi(1)^2 + 2\pi(1)(5) = 2\pi + 10\pi = 12\pi$$

$$V = Bh = \pi(1)^2(5) = 5\pi$$

As the height increases by 1, the surface area increases by 2π and the volume increases by π. The ratios of the radius to the height are $1:1$, $1:2$, $1:3$, $1:4$, and $1:5$ respectively. The dimensions are not proportional because the ratios of the radius to the height are not equivalent.

b.

Radius	1	2	3	4	5
Height	1	2	3	4	5
Surface Area	4π	16π	36π	64π	100π
Volume	π	8π	27π	64π	125π

Solid with a height of 1:

$$S = 2\pi r^2 + 2\pi rh = 2\pi(1)^2 + 2\pi(1)(1) = 2\pi + 2\pi = 4\pi$$

$$V = Bh = \pi(1)^2(1) = \pi$$

Solid with a height of 2:

$$S = 2\pi r^2 + 2\pi rh = 2\pi(2)^2 + 2\pi(2)(2) = 8\pi + 8\pi = 16\pi$$

$$V = Bh = \pi(2)^2(2) = 8\pi$$

Solid with a height of 3:

$$S = 2\pi r^2 + 2\pi rh = 2\pi(3)^2 + 2\pi(3)(3) = 18\pi + 18\pi = 36\pi$$

$$V = Bh = \pi(3)^2(3) = 27\pi$$

Solid with a height of 4:

$$S = 2\pi r^2 + 2\pi rh = 2\pi(4)^2 + 2\pi(4)(4) = 32\pi + 32\pi = 64\pi$$

$$V = Bh = \pi(4)^2(4) = 64\pi$$

Solid with a height of 5:

$$S = 2\pi r^2 + 2\pi rh = 2\pi(5)^2 + 2\pi(5)(5) = 50\pi + 50\pi = 100\pi$$

$$V = Bh = \pi(5)^2(5) = 125\pi$$

A solid with radius n and height n has a surface area of $(2n)^2\pi$ and a volume of $n^3\pi$. The ratios of the radius to the height are $1:1$, $2:2$, $3:3$, $4:4$, and $5:5$ respectively. The dimensions are proportional because the ratios of the radius to the height are equivalent.

Chapter 8

2.

Base Side	6	12	18	24	30
Height	4	8	12	16	20
Slant Height	5	10	15	20	25
Surface Area	96	384	864	1536	2400
Volume	48	384	1296	3072	6000

Solid with a base side length of 6:

$$S = \text{area of base} + 4(\text{area of one lateral face})$$
$$= (6)(6) + 4\left(\frac{1}{2} \bullet 6 \bullet 5\right)$$
$$= 36 + 60$$
$$= 96$$
$$V = \frac{1}{3}Bh = \frac{1}{3}(6)(6)(4) = 48$$

Solid with a base side length of 12:

$$S = \text{area of base} + 4(\text{area of one lateral face})$$
$$= (12)(12) + 4\left(\frac{1}{2} \bullet 12 \bullet 10\right)$$
$$= 144 + 240$$
$$= 384$$
$$V = \frac{1}{3}Bh = \frac{1}{3}(12)(12)(8) = 384$$

Solid with a base side length of 18:

$$S = \text{area of base} + 4(\text{area of one lateral face})$$
$$= (18)(18) + 4\left(\frac{1}{2} \bullet 18 \bullet 15\right)$$
$$= 324 + 540$$
$$= 864$$
$$V = \frac{1}{3}Bh = \frac{1}{3}(18)(18)(12) = 1296$$

Solid with a base side length of 24:

$$S = \text{area of base} + 4(\text{area of one lateral face})$$
$$= (24)(24) + 4\left(\frac{1}{2} \bullet 24 \bullet 20\right)$$
$$= 576 + 960$$
$$= 1536$$
$$V = Bh = \frac{1}{3}(24)(24)(16) = 3072$$

Solid with a base side length of 30:

$$S = \text{area of base} + 4(\text{area of one lateral face})$$
$$= (30)(30) + 4\left(\frac{1}{2} \bullet 30 \bullet 25\right)$$
$$= 900 + 1500$$
$$= 2400$$
$$V = \frac{1}{3}Bh = \frac{1}{3}(30)(30)(20) = 6000$$

As the dimensions of the solid increase by a factor of n, the surface area increases by a factor of n^2 and the volume increases by a factor of n^3. The ratios of the base side length to the height to the slant height can be simplified to $6 : 4 : 5$. The dimensions are proportional because the ratios of the corresponding dimensions are equivalent.

3. When the dimensions of a solid increase by a factor of k, the surface area increases by a factor of k^2.

4. When the dimensions of a solid increase by a factor of k, the volume increases by a factor of k^3.

5. a. 25; When the dimensions of a prism increase by a factor of 5, the surface area increases by a factor of $5^2 = 25$.

   b. 125; When the dimensions of a solid increase by a factor of 5, the volume increases by a factor of $5^3 = 125$.

8.4 On Your Own (pp. 356–358)

1. Check to see if corresponding linear measures are proportional.

   Cylinder A and Cylinder D:

$$\frac{\text{Height of A}}{\text{Height of D}} = \frac{4}{4.5} = \frac{8}{9}$$
$$\frac{\text{Radius of A}}{\text{Radius of D}} = \frac{6}{7.5} = \frac{4}{5}$$

   Because the ratios are not proportional, Cylinder A is not similar to Cylinder D.

   Cylinder B and Cylinder D:

$$\frac{\text{Height of B}}{\text{Height of D}} = \frac{3}{4.5} = \frac{2}{3}$$
$$\frac{\text{Radius of B}}{\text{Radius of D}} = \frac{5}{7.5} = \frac{2}{3}$$

   Because the ratios are proportional, Cylinder B is similar to Cylinder D.

   Cylinder C and Cylinder D:

   Because Cylinder C is proportional to Cylinder A, and Cylinder A is not proportional to Cylinder D, Cylinder C is not proportional to Cylinder D. So, Cylinder C is not similar to Cylinder D.

Chapter 8

2. Missing width:

$$\frac{\text{Width of A}}{\text{Width of B}} = \frac{\text{Height of A}}{\text{Height of B}}$$

$$\frac{8 \text{ in.}}{w} = \frac{20 \text{ in.}}{8 \text{ in.}}$$

$$8 \bullet 8 = w \bullet 20$$

$$64 = 20w$$

$$3.2 = w$$

The width of Prism B is 3.2 inches.

Missing length:

$$\frac{\text{Length of A}}{\text{Length of B}} = \frac{\text{Height of A}}{\text{Height of B}}$$

$$\frac{11 \text{ in.}}{\ell} = \frac{20 \text{ in.}}{8 \text{ in.}}$$

$$11 \bullet 8 = \ell \bullet 20$$

$$88 = 20\ell$$

$$4.4 = \ell$$

The length of Prism B is 4.4 inches.

3. $$\frac{\text{Surface area of Blue}}{\text{Surface area of Red}} = \left(\frac{\text{Width of Blue}}{\text{Width of Red}}\right)^2$$

$$\frac{608}{S} = \left(\frac{8}{5}\right)^2$$

$$\frac{608}{S} = \frac{64}{25}$$

$$608 \bullet 25 = S \bullet 64$$

$$15{,}200 = 64S$$

$$237.5 = S$$

The surface area of the red solid is 237.5 square meters.

4. $$\frac{\text{Surface area of Red}}{\text{Surface area of Blue}} = \left(\frac{\text{Diameter of Red}}{\text{Diameter of Blue}}\right)^2$$

$$\frac{S}{110} = \left(\frac{5}{4}\right)^2$$

$$\frac{S}{110} = \frac{25}{16}$$

$$S \bullet 16 = 110 \bullet 25$$

$$16S = 2750$$

$$S \approx 171.9$$

The surface area of the red solid is about 171.9 square centimeters.

5. $$\frac{\text{Volume of Red}}{\text{Volume of Blue}} = \left(\frac{\text{Height of Red}}{\text{Height of Blue}}\right)^3$$

$$\frac{V}{288} = \left(\frac{5}{12}\right)^3$$

$$\frac{V}{288} = \frac{125}{1728}$$

$$V \bullet 1728 = 288 \bullet 125$$

$$1728V = 36{,}000$$

$$V \approx 20.8 \text{ cm}^3$$

The volume of the red solid is about 20.8 cubic centimeters.

6. $$\frac{\text{Volume of Blue}}{\text{Volume of Red}} = \left(\frac{\text{Side length of Blue}}{\text{Side length of Red}}\right)^3$$

$$\frac{9}{V} = \left(\frac{3}{4}\right)^3$$

$$\frac{9}{V} = \frac{27}{64}$$

$$9 \bullet 64 = V \bullet 27$$

$$576 = 27V$$

$$21.3 \approx V$$

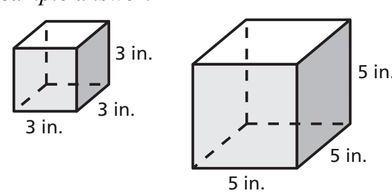
The volume of the red solid is about 21.3 cubic inches.

8.4 Exercises (pp. 359–361)

Vocabulary and Concept Check

1. Similar solids are solids of the same type that have proportional corresponding linear measures.

2. *Sample answer:*



Practice and Problem Solving

3. a. When the dimensions of a cube increase by a factor of $\frac{3}{2}$, the surface area increases by a factor of $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$.

b. When the dimensions of a cube increase by a factor of $\frac{3}{2}$, the volume increases by a factor of $\left(\frac{3}{2}\right)^3 = \frac{27}{8}$.

Chapter 8

4. $$\frac{\text{Length of smaller prism}}{\text{Length of larger prism}} = \frac{2}{6} = \frac{1}{3}$$

$$\frac{\text{Width of smaller prism}}{\text{Width of larger prism}} = \frac{1}{3}$$

$$\frac{\text{Height of smaller prism}}{\text{Height of larger prism}} = \frac{3}{9} = \frac{1}{3}$$

Because the corresponding linear measures are proportional, the solids are similar.

5. $$\frac{\text{Length of larger prism}}{\text{Length of smaller prism}} = \frac{4}{2} = \frac{2}{1}$$

$$\frac{\text{Width of larger prism}}{\text{Width of smaller prism}} = \frac{2}{1}$$

$$\frac{\text{Height of larger prism}}{\text{Height of smaller prism}} = \frac{4}{4} = \frac{1}{1}$$

Because the corresponding linear measures are not proportional, the solids are not similar.

6. $$\frac{\text{Base side length of smaller pyramid}}{\text{Base side length of larger pyramid}} = \frac{5}{10} = \frac{1}{2}$$

$$\frac{\text{Height of smaller pyramid}}{\text{Height of larger pyramid}} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{\text{Slant height of smaller pyramid}}{\text{Slant height of larger pyramid}} = \frac{6.5}{13} = \frac{1}{2}$$

Because the corresponding linear measures are proportional, the solids are similar.

7. $$\frac{\text{Radius of smaller cone}}{\text{Radius of larger cone}} = \frac{9}{20}$$

$$\frac{\text{Height of smaller cone}}{\text{Height of larger cone}} = \frac{12}{21} = \frac{4}{7}$$

$$\frac{\text{Slant height of smaller cone}}{\text{Slant height of larger cone}} = \frac{15}{29}$$

Because the corresponding linear measures are not proportional, the solids are not similar.

8. $$\frac{\text{Diameter of larger cylinder}}{\text{Diameter of smaller cylinder}} = \frac{\text{Height of larger cylinder}}{\text{Height of smaller cylinder}}$$

$$\frac{10}{d} = \frac{4}{10}$$

$$100 = 4d$$

$$25 = d$$

The diameter of the smaller cylinder is 25 inches.

9. $$\frac{\text{Length of smaller prism}}{\text{Length of larger prism}} = \frac{\text{Height of smaller prism}}{\text{Height of larger prism}}$$

$$\frac{12}{b} = \frac{5}{7.5}$$

$$90 = 5b$$

$$18 = b$$

The length of the larger triangular prism is 18 meters.

$$\frac{\text{Width of smaller prism}}{\text{Width of larger prism}} = \frac{\text{Height of smaller prism}}{\text{Height of larger prism}}$$

$$\frac{6}{h} = \frac{5}{7.5}$$

$$45 = 5h$$

$$9 = h$$

The width of the larger prism is 9 meters.

$$\frac{\text{Hypotenuse of smaller prism}}{\text{Hypotenuse of larger prism}} = \frac{\text{Height of smaller prism}}{\text{Height of larger prism}}$$

$$\frac{13}{c} = \frac{5}{7.5}$$

$$97.5 = 5c$$

$$19.5 = c$$

The hypotenuse of the larger triangle is 19.5 meters.

10. $$\frac{\text{Surface area of Blue}}{\text{Surface area of Red}} = \left(\frac{\text{Length of Blue}}{\text{Length of Red}}\right)^2$$

$$\frac{336}{S} = \left(\frac{4}{6}\right)^2$$

$$\frac{336}{S} = \frac{16}{36}$$

$$12{,}096 = 16S$$

$$756 = S$$

The surface area of the red solid is 756 square meters.

11. $$\frac{\text{Surface area of Blue}}{\text{Surface area of Red}} = \left(\frac{\text{Radius of Blue}}{\text{Radius of Red}}\right)^2$$

$$\frac{1800}{S} = \left(\frac{20}{15}\right)^2$$

$$\frac{1800}{S} = \left(\frac{4}{3}\right)^2$$

$$\frac{1800}{S} = \frac{16}{9}$$

$$16{,}200 = 16S$$

$$1012.5 = S$$

The surface area of the red solid is 1012.5 square inches.

Chapter 8

12. $$\frac{\text{Volume of Blue}}{\text{Volume of Red}} = \left(\frac{\text{Side length of Blue}}{\text{Side length of Red}}\right)^3$$

$$\frac{5292}{V} = \left(\frac{21}{7}\right)^3$$

$$\frac{5292}{V} = (3)^3$$

$$\frac{5292}{V} = 27$$

$$5292 = 27V$$

$$196 = V$$

The volume of the red solid is 196 cubic millimeters.

13. $$\frac{\text{Volume of Blue}}{\text{Volume of Red}} = \left(\frac{\text{Radius of Blue}}{\text{Radius of Red}}\right)^3$$

$$\frac{7850}{V} = \left(\frac{10}{12}\right)^3$$

$$\frac{7850}{V} = \left(\frac{5}{6}\right)^3$$

$$\frac{7850}{V} = \frac{125}{216}$$

$$1{,}695{,}600 = 125V$$

$$13{,}564.8 = V$$

The volume of the red solid is 13,564.8 cubic feet.

14. The ratio of the volumes is equal to the cube of the ratio of the corresponding linear measures.

$$\frac{108}{V} = \left(\frac{3}{5}\right)^3$$

$$\frac{108}{V} = \frac{27}{125}$$

$$13{,}500 = 27V$$

$$500 = V$$

The volume of the larger solid is 500 cubic inches.

15. $$\frac{220}{S} = \left(\frac{4}{7}\right)^2$$

$$\frac{220}{S} = \frac{16}{49}$$

$$10{,}780 = 16S$$

$$673.75 = S$$

The surface area of the larger can is 673.75 square centimeters.

16. Volume of the 1 : 18 scale model:

$$\frac{V}{390} = \left(\frac{1}{18}\right)^3$$

$$\frac{V}{390} = \frac{1}{5832}$$

$$5832V = 390$$

$$V \approx 0.067 \text{ in.}^3$$

Volume of the 1 : 24 scale model:

$$\frac{V}{390} = \left(\frac{1}{24}\right)^3$$

$$\frac{V}{390} = \frac{1}{13{,}824}$$

$$13{,}824V = 390$$

$$V \approx 0.028 \text{ in.}^3$$

$$0.067 - 0.028 = 0.039 \text{ in.}^3$$

The 1 : 18 scale model has a greater engine volume by about 0.04 cubic inch.

17. a. The height of the original statue is

$$7 \cancel{\text{ft}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} = 84 \text{ in.}$$

$$\frac{\text{Height of original statue}}{\text{Height of model statue}} = \frac{84}{10} = \frac{8.4}{1}$$

$$\frac{\text{Volume of original statue}}{\text{Volume of model statue}} = \left(\frac{8.4}{1}\right)^3 \approx \frac{592.7}{1}$$

Because the original statue and the model statue are each made of marble, the ratio of their volumes is proportional to the ratio of their weights.

$$\frac{\text{Weight of original statue}}{\text{Weight of model statue}} = \frac{\text{Volume of original statue}}{\text{Volume of model statue}}$$

$$\frac{x}{16} = \frac{592.7}{1}$$

$$x = 9483.2$$

The weight of the original statue is about 9483 pounds.

b. The height of the original statue is

$$20 \text{ ft} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 240 \text{ inches.}$$

$$\frac{\text{Height of original statue}}{\text{Height of model statue}} = \frac{240}{10} = \frac{24}{1}$$

$$\frac{\text{Volume of original statue}}{\text{Volume of model statue}} = \left(\frac{24}{1}\right)^3 = \frac{13{,}824}{1}$$

$$\frac{\begin{array}{c}\text{Weight of}\\ \text{original statue}\end{array}}{\begin{array}{c}\text{Weight of}\\ \text{model statue}\end{array}} = \frac{\begin{array}{c}\text{Volume of}\\ \text{original statue}\end{array}}{\begin{array}{c}\text{Volume of}\\ \text{model statue}\end{array}}$$

$$\frac{x}{16} = \frac{13{,}824}{1}$$

$$x = 221{,}184$$

The weight of the original statue would be 221,184 pounds.

18.

Height (in.)	1	2	3	4	5	6	7
Surface area (in.^2)	S	$4S$	$9S$	$16S$	$25S$	$36S$	$49S$
Volume (in.^3)	V	$8V$	$27V$	$64V$	$125V$	$216V$	$343V$

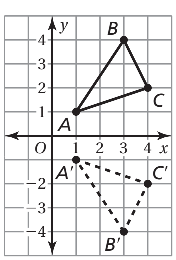
As the dimensions of the doll increase by a factor of n, the surface area increases by a factor of n^2 and the volume increases by a factor of n^3. When the doll's height increases from 1 inch to 2 inches, it is increased by a factor of 2. So, the surface area will be $2^2 = 4$ times greater and the volume will be $2^3 = 8$ times greater.

19. a. yes; Because all circles are similar, the slant height and the circumference of the base of the cones are proportional.

b. no; The ratio of the volumes of similar solids is equal to the cube of the ratio of their corresponding linear measures.

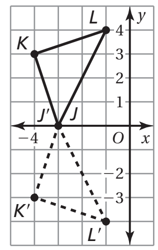
Fair Game Review

20.



The coordinates of the image are $A'(1, -1)$, $B'(3, -4)$, and $C'(4, -2)$.

21.



The coordinates of the image are $J'(-3, 0)$, $K'(-4, -3)$, and $L'(-1, -4)$.

22. B; The lines are parallel because the equations have the same slope and different y-intercepts. So, the lines will not intersect, and the system of linear equations has no solution.

Quiz 8.3–8.4

1. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(8)^3 = \frac{2048\pi}{3} \approx 2144.7$

The volume is about 2144.7 cubic inches.

2. The diameter is 32 centimeters, so the radius is 16 centimeters.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(16)^3 = \frac{16{,}384\pi}{3} \approx 17{,}157.3$

The volume is about 17,157.3 cubic centimeters.

3.

$$V = \frac{4}{3}\pi r^3$$
$$4500\pi = \frac{4}{3}\pi r^3$$
$$3375 = r^3$$
$$15 = r$$

The radius is 15 yards.

4.

$$V = \frac{4}{3}\pi r^3$$
$$\frac{32}{3}\pi = \frac{4}{3}\pi r^3$$
$$8 = r^3$$
$$2 = r$$

The radius is 2 feet.

5. The solid is made up of a cylinder and a cone.

Cylinder:

$V = Bh = \pi r^2 h = \pi(8)^2(12) = 768\pi \text{ ft}^3$

Cone:

$V = \frac{1}{3}Bh = \frac{1}{3}\pi(8)^2(9) = 192\pi \text{ ft}^3$

$768\pi + 192\pi = 960\pi \approx 3015.9 \text{ ft}^3$

The volume of the composite solid is about 3015.9 cubic feet.

Chapter 8

6. $\dfrac{\text{Diameter of larger cylinder}}{\text{Diameter of smaller cylinder}} = \dfrac{6}{4} = \dfrac{3}{2}$

$\dfrac{\text{Height of larger cylinder}}{\text{Height of smaller cylinder}} = \dfrac{7.5}{5} = \dfrac{3}{2}$

Because the corresponding linear measures are proportional, the solids are similar.

7. Width:

$$\frac{\text{Length of larger prism}}{\text{Length of smaller prism}} = \frac{\text{Width of larger prism}}{\text{Width of smaller prism}}$$

$$\frac{10}{4} = \frac{w}{1}$$

$$10 = 4w$$

$$2.5 = w$$

The width of the larger prism is 2.5 inches.

Height:

$$\frac{\text{Length of larger prism}}{\text{Length of smaller prism}} = \frac{\text{Height of larger prism}}{\text{Height of smaller prism}}$$

$$\frac{10}{4} = \frac{h}{2}$$

$$20 = 4h$$

$$5 = h$$

The height of the larger prism is 5 inches.

8. $\dfrac{\text{Surface area of Blue}}{\text{Surface area of Red}} = \left(\dfrac{\text{Height of Blue}}{\text{Height of Red}}\right)^2$

$$\frac{18.84}{S} = \left(\frac{2}{4}\right)^2$$

$$\frac{18.84}{S} = \left(\frac{1}{2}\right)^2$$

$$\frac{18.84}{S} = \frac{1}{4}$$

$$75.36 = S$$

The surface area of the red cone is 75.36 square meters.

9. $V = \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(2)^3 = \dfrac{32\pi}{3} \approx 33.5$

The volume is about 34 cubic centimeters.

10. $\dfrac{\text{Volume of smaller box}}{\text{Volume of larger box}} = \left(\dfrac{2}{3}\right)^3$

$$\frac{V}{162} = \frac{8}{27}$$

$$27V = 1296$$

$$V = 48$$

The volume of the smaller box is 48 cubic inches.

11. The token is a cylinder. The gold ring is the portion remaining when the inner cylinder is removed.

Entire cylinder:

$$V = Bh = \pi(10)^2(2) = 200\pi$$

Inner cylinder:

$$V = Bh = \pi(9)^2(2) = 162\pi$$

$$200\pi - 162\pi = 38\pi \approx 119.4 \text{ mm}^3$$

The volume of the gold ring is about 119.4 cubic millimeters.

Chapter 8 Review

1. The diameter is 15 feet, so the radius is 7.5 feet.

$$V = Bh = \pi(7.5)^2(7) = 393.75\pi \approx 1237.0 \text{ ft}^3$$

The volume is about 1237.0 cubic feet.

2. $V = Bh = \pi(2)^2(10) = 40\pi \approx 125.7 \text{ in.}^3$

The volume is about 125.7 cubic inches.

3. $V = Bh = \pi(3)^2(12) = 108\pi \approx 339.3$

The volume is about 339.3 cubic yards.

4. $V = Bh = \pi(9)^2(18) = 1458\pi \approx 4580.4$

The volume is about 4580.4 cubic inches.

5. The diameter is 3 inches, so the radius is 1.5 inches.

$$V = Bh$$

$$25 = \pi(1.5)^2 h$$

$$25 = 2.25\pi h$$

$$3.5 \approx h$$

The height is about 4 inches.

6. $V = \pi r^2 h$

$$7599 = \pi r^2(20)$$

$$120.9 = r^2$$

$$11 \approx r$$

The radius is about 11 meters.

7. $V = \dfrac{1}{3}Bh = \dfrac{1}{3}\pi(8)^2(12) = 256\pi \approx 804.2 \text{ m}^3$

The volume is about 804.2 cubic meters.

8. The diameter is 4 centimeters, so the radius is 2 centimeters.

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi(2)^2(10) = \frac{40\pi}{3} \approx 41.9 \text{ cm}^3$$

The volume is about 41.9 cubic centimeters.

9.
$$\begin{aligned} V &= \frac{1}{3}Bh \\ 3052 &= \frac{1}{3}\pi(9)^2(h) \\ 3052 &= 27\pi h \\ 36.0 &\approx h \end{aligned}$$

The height is about 36.0 inches.

10. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12)^3 = 2304\pi \approx 7238.2$

The volume is about 7238.2 cubic feet.

11.
$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ 12{,}348\pi &= \frac{4}{3}\pi r^3 \\ 9261 &= r^3 \\ 21 &= r \end{aligned}$$

The radius is 21 inches.

12. The solid is made up of two cones.

Smaller cone:

$V = \frac{1}{3}Bh = \frac{1}{3}\pi(6)^2(12) = 144\pi \text{ m}^3$

Larger cone:

$V = \frac{1}{3}Bh = \frac{1}{3}\pi(6)^2(18) = 216\pi \text{ m}^3$

$144\pi + 216\pi = 360\pi \approx 1131.0 \text{ m}^3$

The volume of the composite solid is about 1131.0 cubic meters.

13. The solid is made up of a square prism and a square pyramid.

Square prism:

$V = Bh = (6)(6)(2) = 72 \text{ ft}^3$

Square pyramid:

$V = \frac{1}{3}Bh = \frac{1}{3}(6)(6)(5) = 60 \text{ ft}^3$

$72 + 60 = 132 \text{ ft}^3$

The volume of the composite solid is 132 cubic feet.

14. The solid is made up of a cylinder and a hemisphere.

Cylinder:

$V = Bh = \pi(2)^2(4) = 16\pi$

Hemisphere:

$V = \frac{1}{2} \bullet \frac{4}{3}\pi r^3 = \frac{2}{3}\pi(2)^3 = \frac{16\pi}{3} = 5\frac{1}{3}\pi$

$16\pi + \frac{16\pi}{3} = \frac{64\pi}{3} \approx 67.0 \text{ cm}^3$

The volume of the composite solid is about 67.0 cubic centimeters.

15.
$$\begin{aligned} \frac{\text{Volume of Red}}{\text{Volume of Blue}} &= \left(\frac{\text{Height of Red}}{\text{Height of Blue}}\right)^3 \\ \frac{V}{4608} &= \left(\frac{12}{24}\right)^3 \\ \frac{V}{4608} &= \left(\frac{1}{2}\right)^3 \\ \frac{V}{4608} &= \frac{1}{8} \\ 8V &= 4608 \\ V &= 576 \text{ m}^3 \end{aligned}$$

The volume of the red pyramid is 576 cubic meters.

16.
$$\begin{aligned} \frac{\text{Surface area of Red}}{\text{Surface area of Blue}} &= \left(\frac{\text{Height of Red}}{\text{Height of Blue}}\right)^2 \\ \frac{S}{154} &= \left(\frac{6}{8}\right)^2 \\ \frac{S}{154} &= \left(\frac{3}{4}\right)^2 \\ \frac{S}{154} &= \frac{9}{16} \\ 16S &= 1386 \\ S &\approx 86.6 \end{aligned}$$

The surface area of the red prism is about 86.6 square yards.

Chapter 8 Test

1. $V = Bh = \pi(20)^2(30) = 12{,}000\pi \approx 37{,}699.1 \text{ mm}^3$

The volume is about 37,699.1 cubic millimeters.

2. The diameter is 3 centimeters, so the radius is 1.5 centimeters.

$V = \frac{1}{3}Bh = \frac{1}{3}\pi(1.5)^2(6) = 4.5\pi \approx 14.1 \text{ cm}^3$

The volume is 14.1 cubic centimeters.

3. The diameter is 26 feet, so the radius is 13 feet.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(13)^3 = \frac{8788\pi}{3} \approx 9202.8 \text{ ft}^3$

The volume is about 9202.8 cubic feet.

4. The solid is formed from a cylinder and a cone.

Cylinder:

$V = Bh = \pi(6)^2(12) = 432\pi \text{ m}^3$

Cone:

$V = \frac{1}{3}Bh = \frac{1}{3}\pi(6)^2(10) = 120\pi \text{ m}^3$

$432\pi + 120\pi = 552\pi \approx 1734.2 \text{ m}^3$

The volume of the composite solid is about 1734.2 cubic meters.

Chapter 8

5. a. $$\frac{\text{Height of Blue}}{\text{Height of Red}} = \frac{\text{Slant height of Blue}}{\text{Slant height of Red}}$$

$$\frac{4}{6} = \frac{5}{\ell}$$

$$4\ell = 30$$

$$\ell = 7.5$$

The slant height of the red pyramid is 7.5 centimeters.

b. $$\frac{\text{Surface area of Blue}}{\text{Surface area of Red}} = \left(\frac{\text{Height of Blue}}{\text{Height of Red}}\right)^2$$

$$\frac{96}{S} = \left(\frac{4}{6}\right)^2$$

$$\frac{96}{S} = \left(\frac{2}{3}\right)^2$$

$$\frac{96}{S} = \frac{4}{9}$$

$$864 = 4S$$

$$216 = S$$

The surface area of the red pyramid is 216 square centimeters.

6. Volume of the cone:

The diameter is 5 inches, so the radius is 2.5 inches.

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi(2.5)^2(5) = \frac{31.25\pi}{3} \approx 32.7 \text{ in.}^3$$

Volume of the cylinder:

The diameter is 3 inches, so the radius is 1.5 inches.

$$V = Bh = \pi(1.5)^2(5.5) = 12.375\pi \approx 38.9 \text{ in.}^3$$

$$38.9 - 32.7 = 6.2 \text{ in.}^3$$

The cylindrical glass holds about 6.2 cubic inches more liquid than the cone-shaped glass.

7. $$\frac{\text{Volume of smaller cone}}{\text{Volume of larger cone}} = \left(\frac{3}{4}\right)^3$$

$$\frac{18}{V} = \frac{27}{64}$$

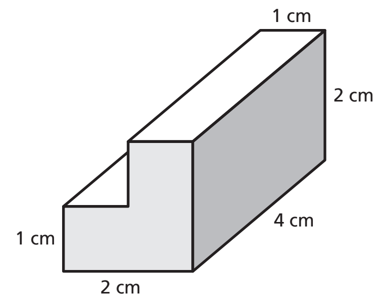
$$1152 = 27V$$

$$42.7 \approx V$$

The volume of the larger cone is about 42.7 cubic inches.

8. *Sample answer:*

Composite Solid 1:



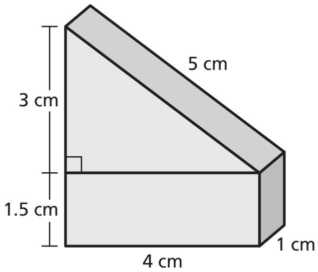
Volume of solid:

$$\begin{aligned} V &= \text{Volume of smaller prism} \\ &\quad + \text{Volume of larger prism} \\ &= Bh + Bh \\ &= (1)(4)(1) + (2)(4)(1) \\ &= 4 + 8 \\ &= 12 \text{ cm}^3 \end{aligned}$$

Surface area of solid:

$$\begin{aligned} S &= \text{Surface area of smaller prism} \\ &\quad + \text{Surface area of larger prism} \\ &= \ell w + 2\ell h + 2wh + \ell w + 2\ell h \\ &\quad + 2wh + \text{area of top} \\ &= (1)(4) + 2(1)(1) + 2(4)(1) \\ &\quad + (2)(4) + 2(2)(1) + 2(4)(1) + (1)(4) \\ &= 4 + 2 + 8 + 8 + 4 + 8 + 4 \\ &= 38 \text{ cm}^2 \end{aligned}$$

Composite Solid 2:



Volume of solid:

$$\begin{aligned} V &= \text{Volume of rectangular prism} \\ &\quad + \text{Volume of triangular prism} \\ &= Bh + Bh \\ &= (4)(1)(1.5) + \frac{1}{2}(3)(4)(1) \\ &= 6 + 6 \\ &= 12 \text{ cm}^3 \end{aligned}$$

Surface area of solid:

$$\begin{aligned} S &= \text{Surface area of rectangular prism} \\ &\quad + \text{Surface area of triangular prism} \\ &= \ell w + 2\ell h + 2wh + 2(\text{area of base}) \\ &\quad + \text{area of lateral faces} \\ &= (4)(1) + 2(4)(1.5) + 2(1)(1.5) \\ &\quad + 2\left(\frac{1}{2}\right)(3)(4) + (3)(1) + (5)(1) \\ &= 4 + 12 + 3 + 12 + 3 + 5 \\ &= 39 \text{ cm}^2 \end{aligned}$$

The two composite solids have the same volume, but different surface areas.

9. *Glass A:*

The diameter is 3.5 inches, so the radius is 1.75 inches.

$$\begin{aligned} V &= Bh \\ &= \pi(1.75)^2(4) \\ &= 12.25\pi \\ &\approx 38.5 \text{ in.}^3 \end{aligned}$$

Glass B:

$$\begin{aligned} V &= Bh \\ &= \pi(1.5)^2(5) \\ &= 11.25\pi \\ &\approx 35.3 \text{ in.}^3 \end{aligned}$$

The volume of Glass A is greater, so Glass A can hold more milk.

10. The cube has a greater volume because the sphere can be placed inside the cube but does not completely fill it.

Chapter 8 Standards Assessment

1. D;

$$\begin{aligned} \frac{w}{3} &= 3(w - 1) - 1 \\ \frac{w}{3} &= 3w - 3 - 1 \\ \frac{w}{3} &= 3w - 4 \\ (3)\frac{w}{3} &= 3(3w - 4) \\ w &= 9w - 12 \\ -9w &\quad -9w \\ -8w &= -12 \\ \frac{-8w}{-8} &= \frac{-12}{-8} \\ w &= \frac{3}{2} \end{aligned}$$

2. H;

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \left(\frac{1}{3}\right)\left(\frac{22}{7}\right)(14)^2(20) \\ &= \left(\frac{1}{3}\right)\left(\frac{22}{7}\right)(196)(20) \\ &= \frac{86{,}240}{21} \\ &= 4106\frac{2}{3} \text{ cm}^3 \end{aligned}$$

The volume of the cone is $4106\frac{2}{3}$ cubic centimeters.

3. C;

$$\begin{aligned} -\frac{3}{2}(8x - 10) &= -20 \\ \left(-\frac{2}{3}\right)\left[-\frac{3}{2}(8x - 10)\right] &= -20\left(-\frac{2}{3}\right) \\ 8x - 10 &= \frac{40}{3} \\ 8x &= \frac{70}{3} \\ x &= \frac{35}{12} \end{aligned}$$

4. G;

The x-coordinates are unchanged and the y-coordinates are opposites for the image.

5. A;

The ordered pairs are $(2, 5)$, $(4, -2)$, $(6, -7)$, and $(8, 1)$.

Chapter 8

6. 3°F; $m = \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{36 - 54}{6 - 0} = \dfrac{-18}{6} = -3$

So, the temperature fell 3°F each hour.

7. I;

$$\begin{aligned} A &= P + PI \\ A - P &= PI \\ \frac{A - P}{P} &= I \end{aligned}$$

8. 9 in.^{3};

$$\begin{aligned} \frac{1296}{V} &= \frac{\pi r^2 h}{\pi\left(\frac{r}{12}\right)^2 h} \\ \frac{1296}{V} &= \frac{r^2}{\left(\frac{r}{12}\right)^2} \\ \frac{1296}{V} &= \frac{r^2}{\frac{r^2}{144}} \\ \frac{1296}{V} &= 144 \\ 1296 &= 144V \\ 9 &= V \end{aligned}$$

The volume of the new (smaller) cylinder is 9 cubic inches.

9. B; Graph B is a straight line, so it represents a linear function.

10. 113.04 in.^{3}; *Answer should include, but is not limited to:* The lantern is a composite solid. The lantern is made of a cone and a cylinder.

$$\begin{aligned} \text{Volume of cone} &= \frac{1}{3}\pi r^2 h \\ &\approx \frac{1}{3}(3.14)(2)^2(3) \\ &= \frac{1}{3}(3.14)(4)(3) \\ &= 12.56 \text{ in.}^3 \end{aligned}$$

The volume of the cone is about 12.56 cubic inches.

$$\begin{aligned} \text{Volume of cylinder} &= \pi r^2 h \\ &\approx (3.14)(2)^2(8) \\ &= (3.14)(4)(8) \\ &= 100.48 \text{ in.}^3 \end{aligned}$$

The volume of the cylinder is about 100.48 cubic inches.

To find the volume of a composite figure, add the volumes of the smaller figures that are in the composite figure. To find the volume of the lantern, add the volume of the cone to the volume of the cylinder.

$$\begin{aligned} \text{Volume of lantern} &= \text{Volume of cone} \\ &\quad + \text{Volume of cylinder} \\ &\approx 12.56 \text{ in.}^3 + 100.48 \text{ in.}^3 \\ &= 113.04 \text{ in.}^3 \end{aligned}$$

So, the volume of the lantern is about 113.04 cubic inches.