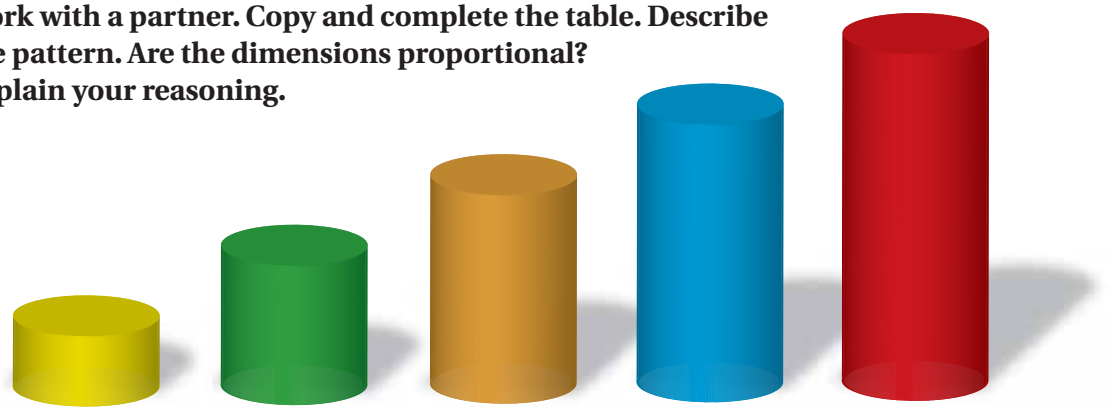
8.4 Surface Areas and Volumes of Similar Solids

Essential Question When the dimensions of a solid increase by a factor of k, how does the surface area change? How does the volume change?

1 ACTIVITY: Comparing Surface Areas and Volumes

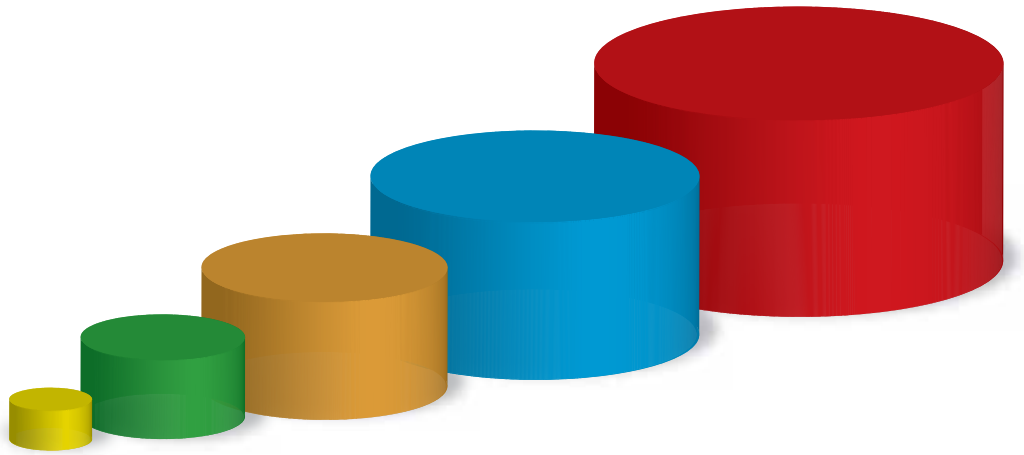
Work with a partner. Copy and complete the table. Describe the pattern. Are the dimensions proportional? Explain your reasoning.

a.



Radius	1	1	1	1	1
Height	1	2	3	4	5
Surface Area					
Volume					

b.



Radius	1	2	3	4	5
Height	1	2	3	4	5
Surface Area					
Volume					

Common Core

Geometry

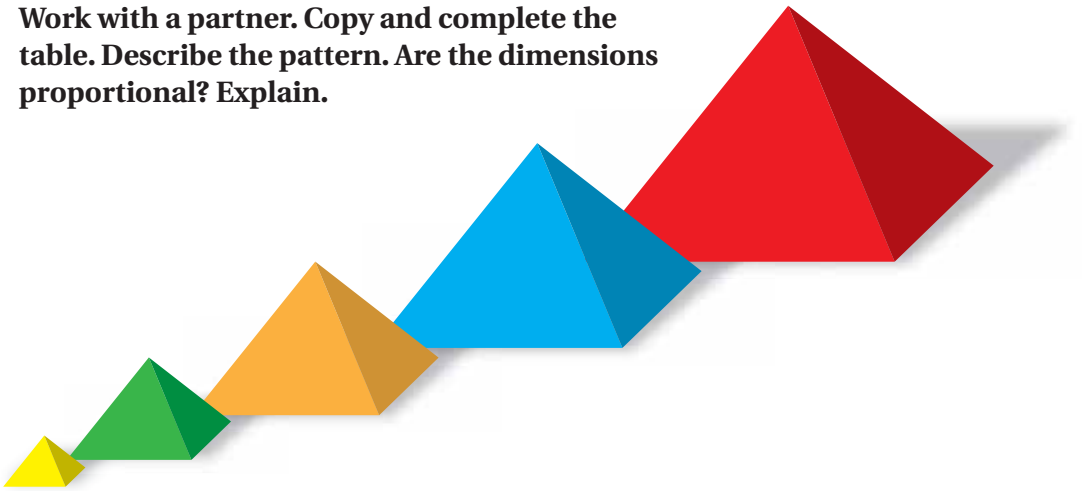
In this lesson, you will

- identify similar solids.
- use properties of similar solids to find missing measures.
- understand the relationship between surface areas of similar solids.
- understand the relationship between volumes of similar solids.
- solve real-life problems.

Applying Standard 8.G.9

2 ACTIVITY: Comparing Surface Areas and Volumes

Work with a partner. Copy and complete the table. Describe the pattern. Are the dimensions proportional? Explain.



Math Practice 8

Repeat Calculations

Which calculations are repeated? How does this help you describe the pattern?

Base Side	6	12	18	24	30
Height	4	8	12	16	20
Slant Height	5	10	15	20	25
Surface Area					
Volume					

What Is Your Answer?

3. **IN YOUR OWN WORDS** When the dimensions of a solid increase by a factor of k, how does the surface area change?

4. **IN YOUR OWN WORDS** When the dimensions of a solid increase by a factor of k, how does the volume change?

5. **REPEATED REASONING** All the dimensions of a prism increase by a factor of 5.

   a. How many times greater is the surface area? Explain.

   5 10 25 125

   b. How many times greater is the volume? Explain.

   5 10 25 125

Practice Use what you learned about surface areas and volumes of similar solids to complete Exercise 3 on page 359.

8.4 Lesson

Check It Out
Lesson Tutorials
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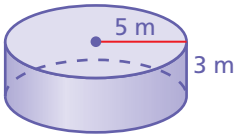
Key Vocabulary
similar solids, *p. 356*

Similar solids are solids that have the same shape and proportional corresponding dimensions.

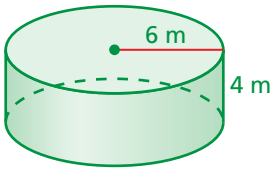
EXAMPLE 1 Identifying Similar Solids

Which cylinder is similar to Cylinder A?

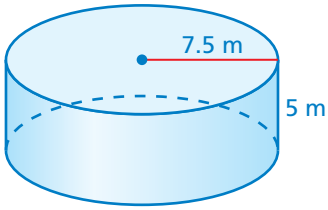
Cylinder B



Cylinder A



Cylinder C



Check to see if corresponding dimensions are proportional.

Cylinder A and Cylinder B

$\frac{\text{Height of A}}{\text{Height of B}} = \frac{4}{3}$ $\quad$ $\frac{\text{Radius of A}}{\text{Radius of B}} = \frac{6}{5}$

Not proportional

Cylinder A and Cylinder C

$\frac{\text{Height of A}}{\text{Height of C}} = \frac{4}{5}$ $\quad$ $\frac{\text{Radius of A}}{\text{Radius of C}} = \frac{6}{7.5} = \frac{4}{5}$

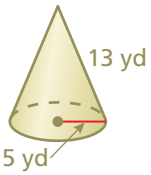
Proportional

∴ So, Cylinder C is similar to Cylinder A.

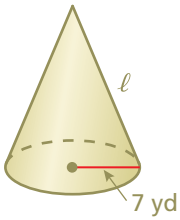
EXAMPLE 2 Finding Missing Measures in Similar Solids

The cones are similar. Find the missing slant height ℓ.

Cone X



Cone Y



$\frac{\text{Radius of X}}{\text{Radius of Y}} = \frac{\text{Slant height of X}}{\text{Slant height of Y}}$

$\frac{5}{7} = \frac{13}{\ell}$ Substitute.

$5\ell = 91$ Cross Products Property

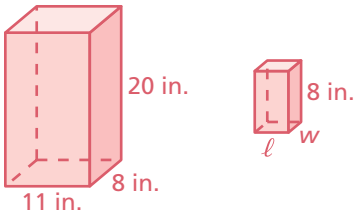
$\ell = 18.2$ Divide each side by 5.

∴ The slant height is 18.2 yards.

On Your Own

Now You're Ready
Exercises 4–9

1. Cylinder D has a radius of 7.5 meters and a height of 4.5 meters. Which cylinder in Example 1 is similar to Cylinder D?
2. The prisms at the right are similar. Find the missing width and length.



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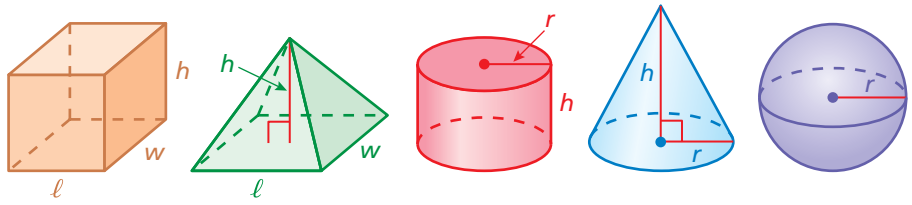
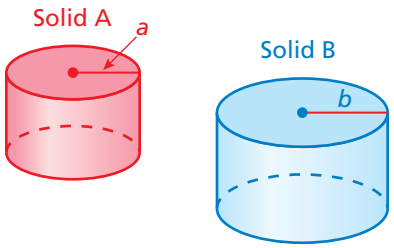
Key Ideas

Linear Measures



Surface Areas of Similar Solids

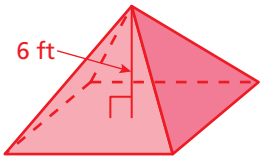
When two solids are similar, the ratio of their surface areas is equal to the square of the ratio of their corresponding linear measures.



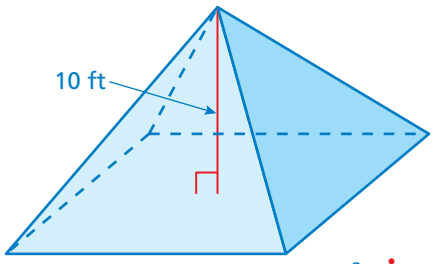
$$\frac{\text{Surface Area of A}}{\text{Surface Area of B}} = \left(\frac{a}{b}\right)^2$$

EXAMPLE 3 Finding Surface Area

Pyramid A



Pyramid B



Surface Area = 600 ft²

The pyramids are similar. What is the surface area of Pyramid A?

$$\frac{\text{Surface Area of A}}{\text{Surface Area of B}} = \left(\frac{\text{Height of A}}{\text{Height of B}}\right)^2$$

$$\frac{S}{600} = \left(\frac{6}{10}\right)^2 \quad \text{Substitute.}$$

$$\frac{S}{600} = \frac{36}{100} \quad \text{Evaluate.}$$

$$\frac{S}{600} \cdot 600 = \frac{36}{100} \cdot 600 \quad \text{Multiplication Property of Equality}$$

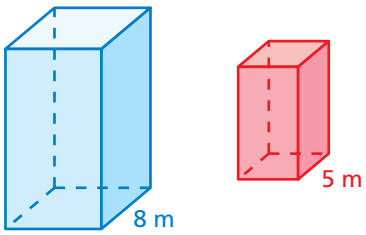
$$S = 216 \quad \text{Simplify.}$$

∴ The surface area of Pyramid A is 216 square feet.

On Your Own

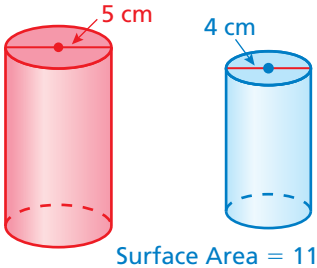
The solids are similar. Find the surface area of the red solid. Round your answer to the nearest tenth.

3.



Surface Area = 608 m²

4.



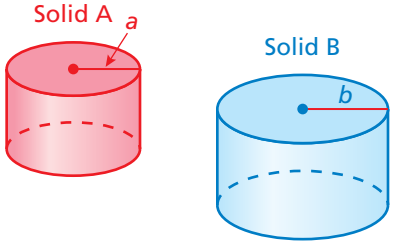
Surface Area = 110 cm²

Key Idea

Volumes of Similar Solids

When two solids are similar, the ratio of their volumes is equal to the cube of the ratio of their corresponding linear measures.

$$\frac{\text{Volume of A}}{\text{Volume of B}} = \left(\frac{a}{b}\right)^3$$



EXAMPLE 4 Finding Volume

Original Tank

Volume = 2000 ft^3

The dimensions of the touch tank at an aquarium are doubled. What is the volume of the new touch tank?

(A) 150 ft^3 (B) 4000 ft^3

(C) 8000 ft^3 (D) 16,000 ft^3

The dimensions are doubled, so the ratio of the dimensions of the original tank to the dimensions of the new tank is 1 : 2.

$$\frac{\text{Original volume}}{\text{New volume}} = \left(\frac{\text{Original dimension}}{\text{New dimension}}\right)^3$$

$$\frac{2000}{V} = \left(\frac{1}{2}\right)^3 \quad \text{Substitute.}$$

$$\frac{2000}{V} = \frac{1}{8} \quad \text{Evaluate.}$$

$$16{,}000 = V \quad \text{Cross Products Property}$$

∴ The volume of the new tank is 16,000 cubic feet. So, the correct answer is (D).

Study Tip

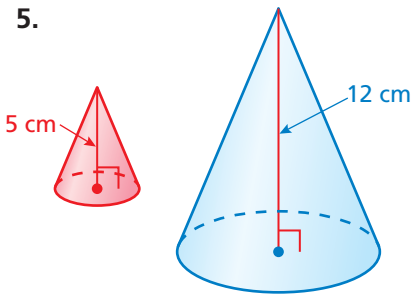
When the dimensions of a solid are multiplied by k, the surface area is multiplied by k^2 and the volume is multiplied by k^3.

On Your Own

Now You're Ready
Exercises 10–13

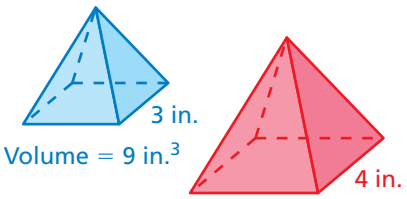
The solids are similar. Find the volume of the red solid. Round your answer to the nearest tenth.

5.



Volume = 288 cm^3

6.



Volume = 9 in.^{3}

8.4 Exercises

Check It Out
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Vocabulary and Concept Check

1. **VOCABULARY** What are similar solids?
2. **OPEN-ENDED** Draw two similar solids and label their corresponding linear measures.

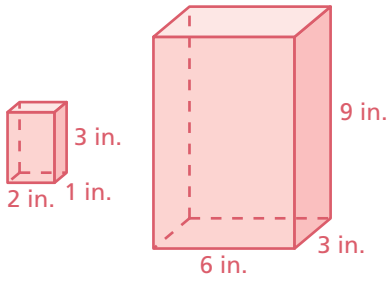
Practice and Problem Solving

3. **NUMBER SENSE** All the dimensions of a cube increase by a factor of $\frac{3}{2}$.
   a. How many times greater is the surface area? Explain.
   b. How many times greater is the volume? Explain.

Determine whether the solids are similar.

1

4.



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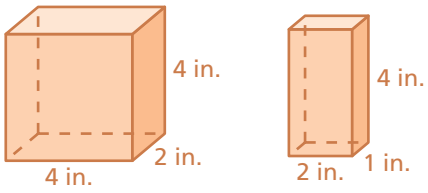

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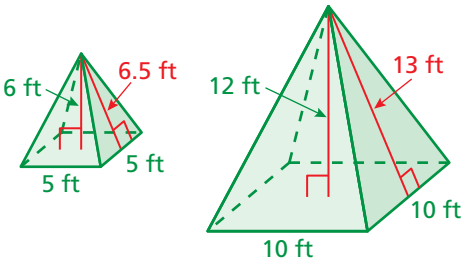

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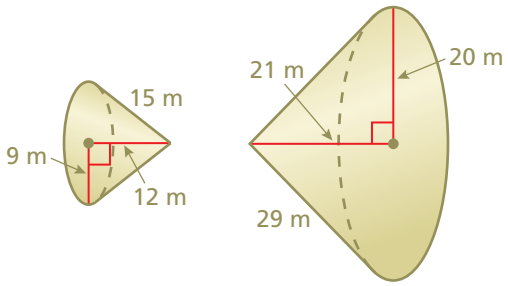
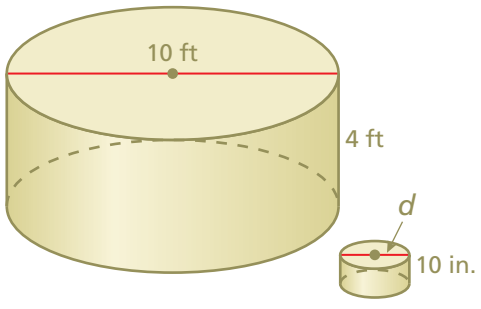


The solids are similar. Find the missing dimension(s).

2

8.



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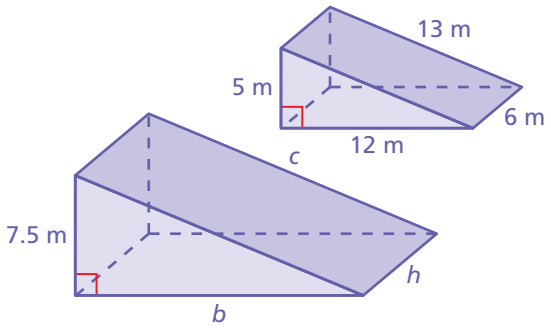
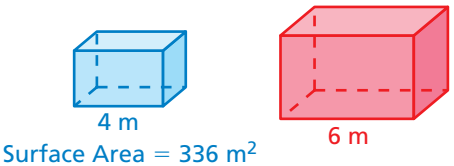


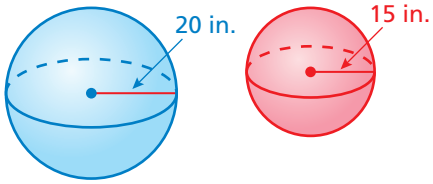
The solids are similar. Find the surface area S or volume V of the red solid. Round your answer to the nearest tenth.

3 4 **10.**



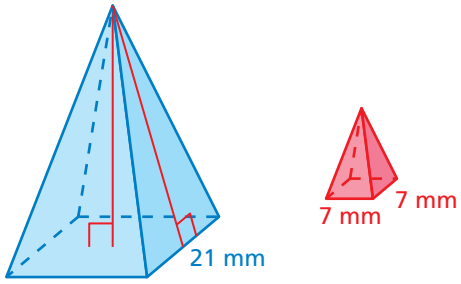
Surface Area = 336 m^2

11.



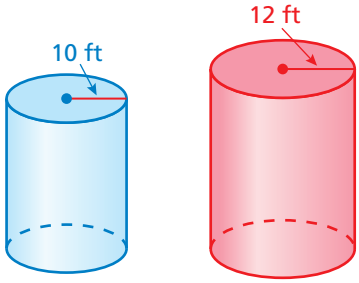
Surface Area = 1800 $in.^2$

12.



Volume = 5292 mm^3

13.



Volume = 7850 ft^3

14. ERROR ANALYSIS The ratio of the corresponding linear measures of two similar solids is 3 : 5. The volume of the smaller solid is 108 cubic inches. Describe and correct the error in finding the volume of the larger solid.

✗

$$\frac{108}{V} = \left(\frac{3}{5}\right)^2$$

$$\frac{108}{V} = \frac{9}{25}$$

$$300 = V$$

The volume of the larger solid is 300 cubic inches.

15. MIXED FRUIT The ratio of the corresponding linear measures of two similar cans of fruit is 4 to 7. The smaller can has a surface area of 220 square centimeters. Find the surface area of the larger can.

16. ENGINE The volume of a car engine is 390 cubic inches. Which scale model of the car has the greater engine volume, a 1 : 18 scale model or a 1 : 24 scale model? How much greater is it?

17. MARBLE STATUE You have a small marble statue of Wolfgang Mozart. It is 10 inches tall and weighs 16 pounds. The original statue is 7 feet tall.

a. Estimate the weight of the original statue. Explain your reasoning.

b. If the original statue were 20 feet tall, how much would it weigh?

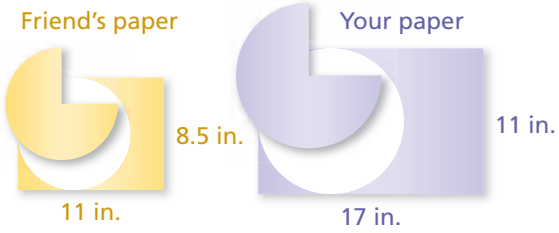
Wolfgang Mozart

18. REPEATED REASONING The largest doll is 7 inches tall. Each of the other dolls is 1 inch shorter than the next larger doll. Make a table that compares the surface areas and the volumes of the seven dolls.

19. Precision You and a friend make paper cones to collect beach glass. You cut out the largest possible three-fourths circle from each piece of paper.

a. Are the cones similar? Explain your reasoning.

b. Your friend says that because your sheet of paper is twice as large, your cone will hold exactly twice the volume of beach glass. Is this true? Explain your reasoning.



Fair Game Review What you learned in previous grades & lessons

Draw the figure and its reflection in the x-axis. Identify the coordinates of the image. *(Section 2.3)*

20. $A(1, 1), B(3, 4), C(4, 2)$

21. $J(-3, 0), K(-4, 3), L(-1, 4)$

22. MULTIPLE CHOICE Which system of linear equations has no solution? *(Section 5.4)*

(A) $y = 4x + 1$, $y = -4x + 1$

(B) $y = 2x - 7$, $y = 2x + 7$

(C) $3x + y = 1$, $6x + 2y = 2$

(D) $5x + y = 3$, $x + 5y = 15$