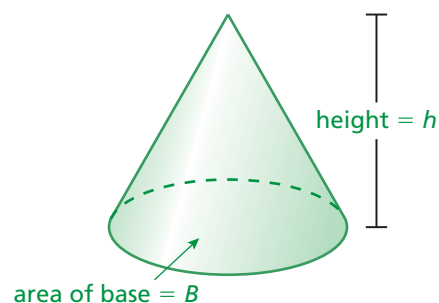# 8.2 Volumes of Cones

## Essential Question How can you find the volume of a cone?

You already know how the volume of a pyramid relates to the volume of a prism. In this activity, you will discover how the volume of a cone relates to the volume of a cylinder.

height = h

area of base = B

1 ACTIVITY: Finding a Formula Experimentally

Work with a partner. Use a paper cup that is shaped like a cone.

- Estimate the height of the cup.
- Trace the top of the cup on a piece of paper. Find the diameter of the circle.
- Use these measurements to draw a net for a cylinder with the same base and height as the paper cup.
- Cut out the net. Then fold and tape it to form an open cylinder.
- Fill the paper cup with rice. Then pour the rice into the cylinder. Repeat this until the cylinder is full. How many cones does it take to fill the cylinder?
- Use your result to write a formula for the volume of a cone.
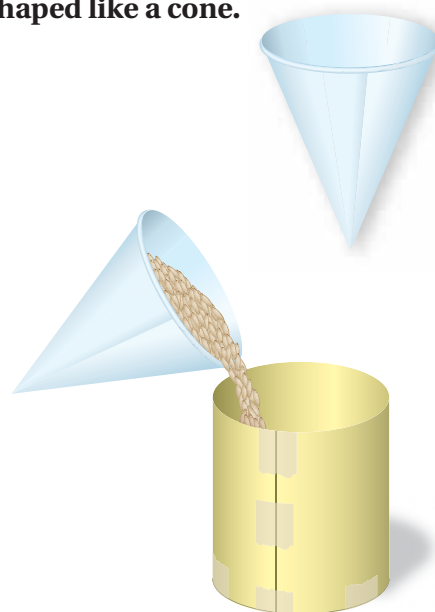
2 ACTIVITY: Summarizing Volume Formulas

Work with a partner. You can remember the volume formulas for prisms, cylinders, pyramids, and cones with just two concepts.

Volumes of Prisms and Cylinders

Volume = ⬜ Area of base ⬜ × ⬜

Volumes of Pyramids and Cones

Volume = ⬜ Volume of prism or cylinder with same base and height

Make a list of all the formulas you need to remember to find the area of a base. Talk about strategies for remembering these formulas.
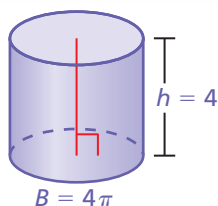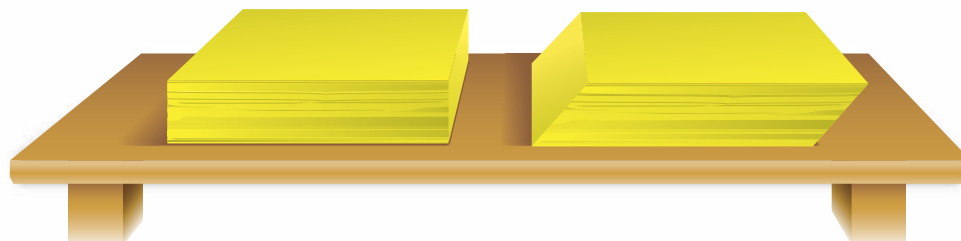
3   ACTIVITY: Volumes of Oblique Solids

Work with a partner. Think of a stack of paper. When you adjust the stack so that the sides are oblique (slanted), do you change the volume of the stack? If the volume of the stack does not change, then the formulas for volumes of right solids also apply to oblique solids.
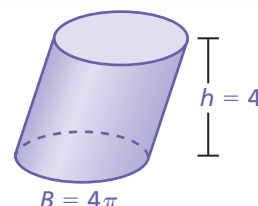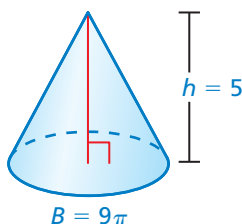
Math Practice 2

Use Equations

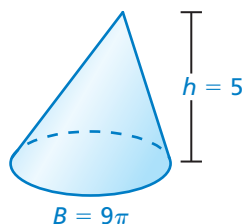What equation would you use to find the volume of the oblique solid? Explain.

$h = 4$

$B = 4\pi$

Right cylinder

$h = 4$

$B = 4\pi$

Oblique cylinder

$h = 5$

$B = 9\pi$
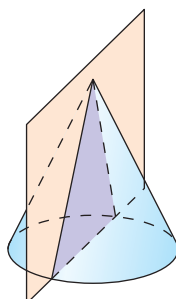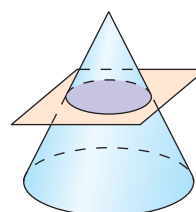
Right cone

$h = 5$

$B = 9\pi$

Oblique cone

What Is Your Answer?

4. IN YOUR OWN WORDS How can you find the volume of a cone?

5. Describe the intersection of the plane and the cone. Then explain how to find the volume of each section of the solid.

a.

b.

Practice

Use what you learned about the volumes of cones to complete Exercises 4–6 on page 344.

8.2 Lesson

Key Idea

Volume of a Cone

Study Tip

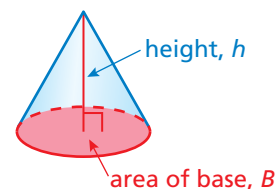The *height* of a cone is the perpendicular distance from the base to the vertex.

Words The volume V of a cone is one-third the product of the area of the base and the height of the cone.

height, h

area of base, B

Area of base

Algebra $V = \dfrac{1}{3}Bh$

Height of cone

EXAMPLE 1 Finding the Volume of a Cone

Study Tip

Because $B = \pi r^2$, you can use $V = \dfrac{1}{3}\pi r^2 h$ to find the volume of a cone.

Find the volume of the cone. Round your answer to the nearest tenth.

The diameter is 4 meters. So, the radius is 2 meters.

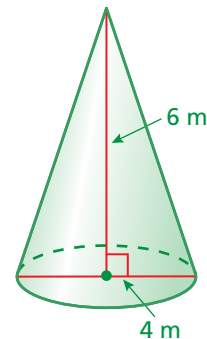$V = \dfrac{1}{3}Bh$ Write formula for volume.

$= \dfrac{1}{3}\pi(2)^2(6)$ Substitute.

$= 8\pi \approx 25.1$ Use a calculator.

6 m

4 m

The volume is about 25.1 cubic meters.

EXAMPLE 2 Finding the Height of a Cone

Find the height of the cone. Round your answer to the nearest tenth.

$V = \dfrac{1}{3}Bh$ Write formula for volume.

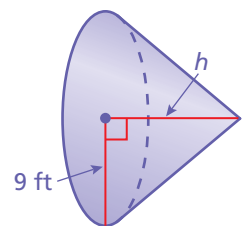$956 = \dfrac{1}{3}\pi(9)^2(h)$ Substitute.

$956 = 27\pi h$ Simplify.

$11.3 \approx h$ Divide each side by 27π.

h

9 ft

Volume = 956 ft³

The height is about 11.3 feet.

On Your Own

Find the volume V or height h of the cone. Round your answer to the nearest tenth.

1.

6 cm

15 cm

$V \approx$

2.

$h \approx$

15 yd

Volume = 7200 yd³
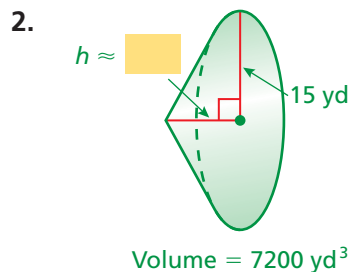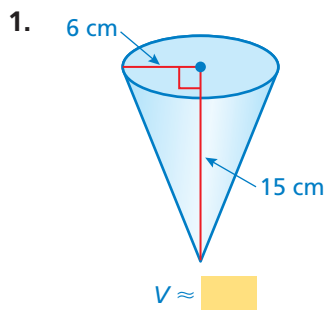
EXAMPLE 3 Real-Life Application

├── 30 mm ──┤

10 mm

24 mm
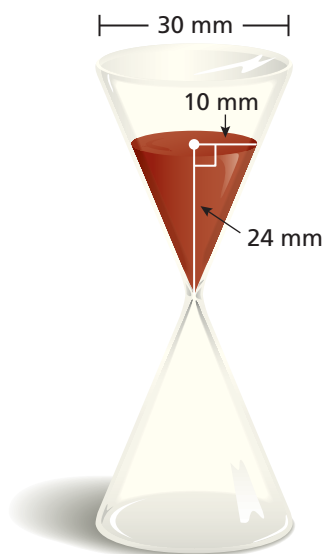
You must answer a trivia question before the sand in the timer falls to the bottom. The sand falls at a rate of 50 cubic millimeters per second. How much time do you have to answer the question?

Use the formula for the volume of a cone to find the volume of the sand in the timer.

$$V = \frac{1}{3}Bh \qquad \text{Write formula for volume.}$$

$$= \frac{1}{3}\pi(10)^2(24) \qquad \text{Substitute.}$$

$$= 800\pi \approx 2513 \qquad \text{Use a calculator.}$$

The volume of the sand is about 2513 cubic millimeters. To find the amount of time you have to answer the question, multiply the volume by the rate at which the sand falls.

$$2513 \ \text{mm}^3 \times \frac{1 \ \text{sec}}{50 \ \text{mm}^3} = 50.26 \ \text{sec}$$

So, you have about 50 seconds to answer the question.

On Your Own

3. WHAT IF? The sand falls at a rate of 60 cubic millimeters per second. How much time do you have to answer the question?

4. WHAT IF? The height of the sand in the timer is 12 millimeters, and the radius is 5 millimeters. How much time do you have to answer the question?

Vocabulary and Concept Check

1. **VOCABULARY** Describe the height of a cone.

2. **WRITING** Compare and contrast the formulas for the volume of a pyramid and the volume of a cone.

3. **REASONING** You know the volume of a cylinder. How can you find the volume of a cone with the same base and height?
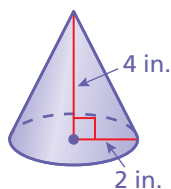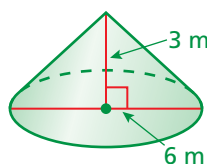
Practice and Problem Solving

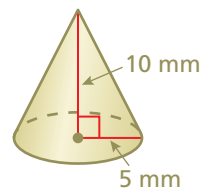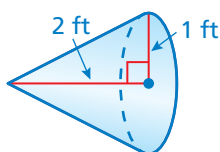**Find the volume of the cone. Round your answer to the nearest tenth.**

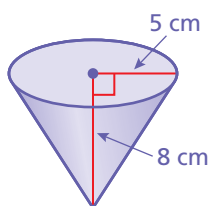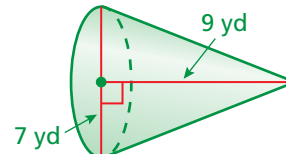**4.**

4 in.

2 in.

5.

3 m

6 m

6.

10 mm

5 mm

7.

2 ft   1 ft

8.

5 cm

8 cm

9.

9 yd
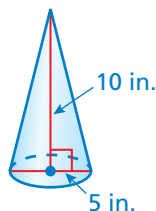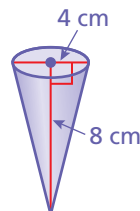
7 yd

10.
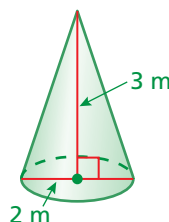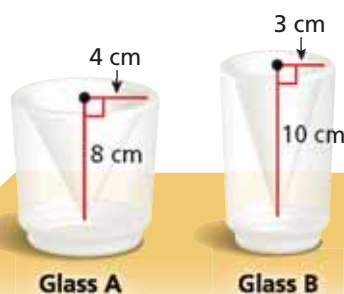
7 ft

4 ft

11.

10 in.

5 in.

12.

4 cm

8 cm
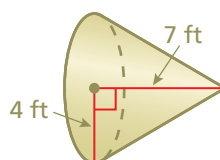
13. **ERROR ANALYSIS** Describe and correct the error in finding the volume of the cone.
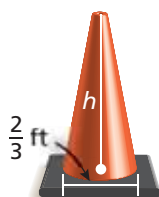
3 m

2 m

$$\times \quad V = \frac{1}{3}Bh$$

$$= \frac{1}{3}(\pi)(2)^2(3)$$

$$= 4\pi \text{ m}^3$$

3 cm

4 cm

8 cm

10 cm

Glass A   Glass B

14. **GLASS** The inside of each glass is shaped like a cone. Which glass can hold more liquid? How much more?

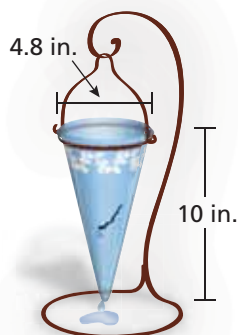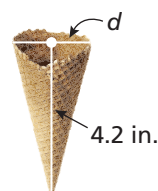**Find the missing dimension of the cone. Round your answer to the nearest tenth.**

2 **15.** Volume $= \frac{1}{18}\pi$ ft^3

$\frac{2}{3}$ ft

16. Volume $= 225$ cm^3

|— 10 cm —|

**17.** Volume $= 3.6$ in.^{3}

d

4.2 in.

4.8 in.

10 in.

18. REASONING The volume of a cone is 20π cubic meters. What is the volume of a cylinder with the same base and height?

19. VASE Water leaks from a crack in a vase at a rate of 0.5 cubic inch per minute. How long does it take for 20% of the water to leak from a full vase?

20. LEMONADE STAND You have 10 gallons of lemonade to sell. (1 gal $\approx$ 3785 cm^3)

|— 8 cm —|

11 cm

a. Each customer uses one paper cup. How many paper cups will you need?

b. The cups are sold in packages of 50. How many packages should you buy?

c. How many cups will be left over if you sell 80% of the lemonade?
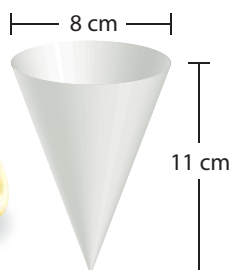
21. STRUCTURE The cylinder and the cone have the same volume. What is the height of the cone?

x

y

?

2x

22. **Critical Thinking** In Example 3, you use a different timer with the same dimensions. The sand in this timer has a height of 30 millimeters. How much time do you have to answer the question?
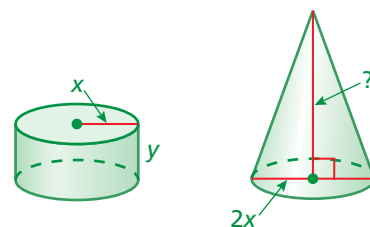
Fair Game Review *What you learned in previous grades & lessons*

The vertices of a figure are given. Rotate the figure as described. Find the coordinates of the image. *(Section 2.4)*

23. $A(-1, 1), B(2, 3), C(2, 1)$
90° counterclockwise about vertex A

24. $E(-4, 1), F(-3, 3), G(-2, 3), H(-1, 1)$
180° about the origin

25. MULTIPLE CHOICE $\triangle ABC \sim \triangle XYZ$ by a scale factor of 3. How many times greater is the area of $\triangle XYZ$ than the area of $\triangle ABC$? *(Section 2.6)*

Ⓐ $\frac{1}{9}$

Ⓑ $\frac{1}{3}$

Ⓒ 3

Ⓓ 9

B

A C

Y

X Z