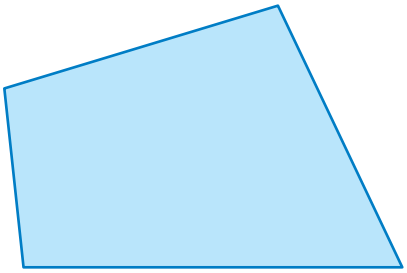
3.3 Angles of Polygons

Essential Question How can you find the sum of the interior angle measures and the sum of the exterior angle measures of a polygon?

1 ACTIVITY: Exploring the Interior Angles of a Polygon

Work with a partner. In parts (a)–(e), identify each polygon and the number of sides *n*. Then find the sum of the interior angle measures of the polygon.

a. Polygon: ____ Number of sides: $n =$ ____

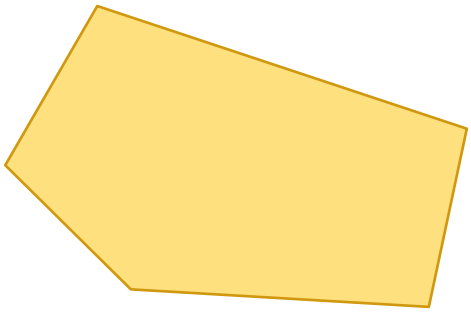


Draw a line segment on the figure that divides it into two triangles. Is there more than one way to do this? Explain.

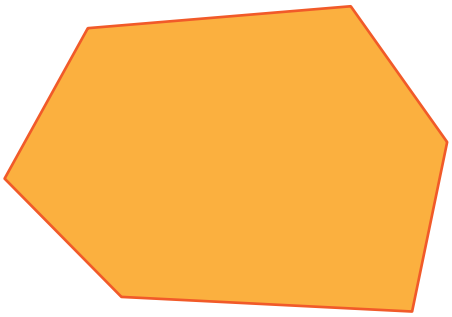
What is the sum of the interior angle measures of each triangle?

What is the sum of the interior angle measures of the figure?

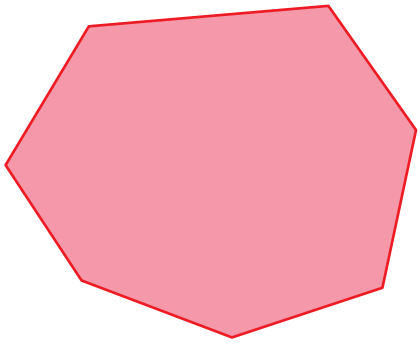
b.



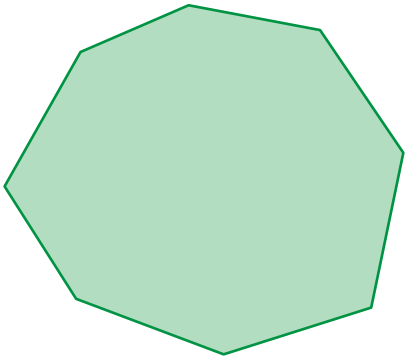
c.



d.



e.



f. **REPEATED REASONING** Use your results to complete the table. Then find the sum of the interior angle measures of a polygon with 12 sides.

Number of Sides, *n*	3	4	5	6	7	8
Number of Triangles						
Angle Sum, *S*						

Common Core

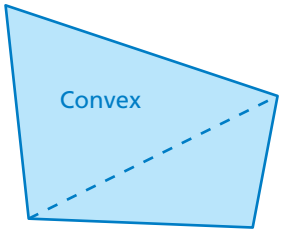
Geometry

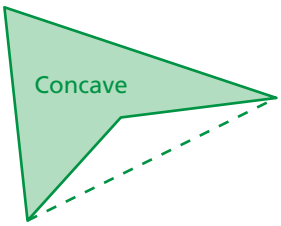
In this lesson, you will

- find the sum of the interior angle measures of polygons.
- understand that the sum of the exterior angle measures of a polygon is 360°.
- find the measures of interior and exterior angles of polygons.

Applying Standard 8.G.5

A polygon is **convex** when every line segment connecting any two vertices lies entirely inside the polygon. A polygon is **concave** when at least one line segment connecting any two vertices lies outside the polygon.

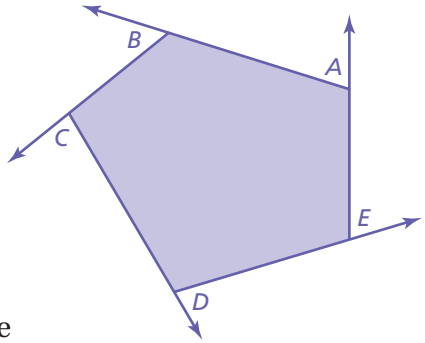




2 ACTIVITY: Exploring the Exterior Angles of a Polygon

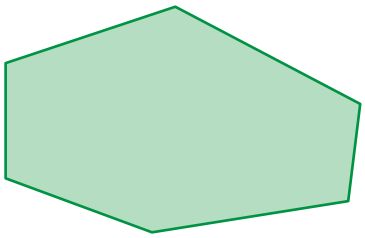
Work with a partner.

a. Draw a convex pentagon. Extend the sides to form the exterior angles. Label one exterior angle at each vertex A, B, C, D, and E, as shown.



b. Cut out the exterior angles. How can you join the vertices to determine the sum of the angle measures? What do you notice?

c. **REPEATED REASONING** Repeat the procedure in parts (a) and (b) for each figure below.



What can you conclude about the sum of the measures of the exterior angles of a convex polygon? Explain.

Math Practice 3

Analyze Conjectures

Do your observations about the sum of the exterior angles make sense? Do you think they would hold true for any convex polygon? Explain.

What Is Your Answer?

3. **STRUCTURE** Use your results from Activity 1 to write an expression that represents the sum of the interior angle measures of a polygon.

4. **IN YOUR OWN WORDS** How can you find the sum of the interior angle measures and the sum of the exterior angle measures of a polygon?

Practice Use what you learned about angles of polygons to complete Exercises 4–6 on page 123.

3.3 Lesson

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Lesson Tutorials
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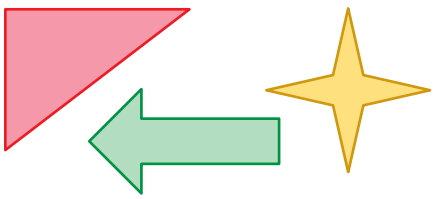
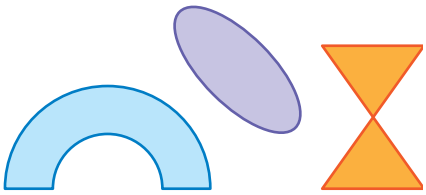
Key Vocabulary
convex polygon, *p. 119*
concave polygon, *p. 119*
regular polygon, *p. 121*

A *polygon* is a closed plane figure made up of three or more line segments that intersect only at their endpoints.



Polygons



Not polygons

Key Idea

Interior Angle Measures of a Polygon

The sum S of the interior angle measures of a polygon with n sides is

$$S = (n - 2) \cdot 180°.$$

EXAMPLE 1 Finding the Sum of Interior Angle Measures

Find the sum of the interior angle measures of the school crossing sign.

The sign is in the shape of a pentagon. It has 5 sides.

$S = (n - 2) \cdot 180°$ Write the formula.

$= (5 - 2) \cdot 180°$ Substitute 5 for n.

$= 3 \cdot 180°$ Subtract.

$= 540°$ Multiply.

∴ The sum of the interior angle measures is 540°.

Reading

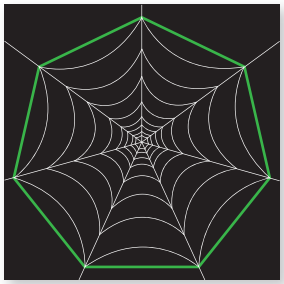
For polygons whose names you have not learned, you can use the phrase "n-gon," where n is the number of sides. For example, a 15-gon is a polygon with 15 sides.

On Your Own

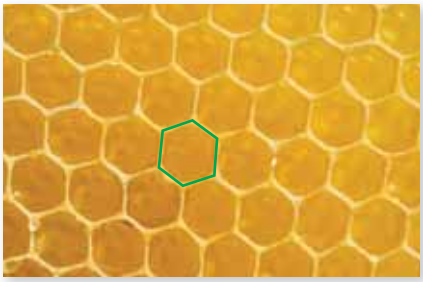
Now You're Ready
Exercises 7–9

Find the sum of the interior angle measures of the green polygon.

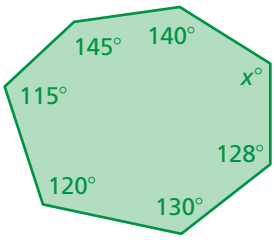
1.



2.



EXAMPLE 2 Finding an Interior Angle Measure of a Polygon



Find the value of x.

Step 1: The polygon has 7 sides. Find the sum of the interior angle measures.

$S = (n - 2) \cdot 180°$ Write the formula.

$= (7 - 2) \cdot 180°$ Substitute 7 for n.

$= 900°$ Simplify. The sum of the interior angle measures is 900°.

Step 2: Write and solve an equation.

$$140 + 145 + 115 + 120 + 130 + 128 + x = 900$$
$$778 + x = 900$$
$$x = 122$$

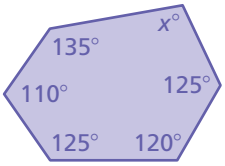
∴ The value of x is 122.

On Your Own

Now You're Ready
Exercises 12–14

Find the value of x.

3.



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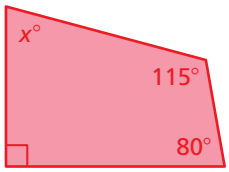
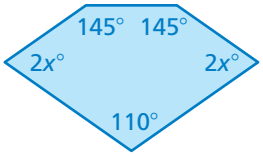
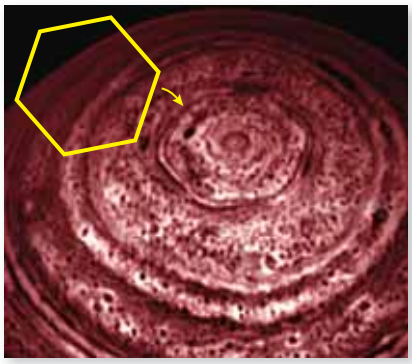


5.



In a **regular polygon**, all the sides are congruent, and all the interior angles are congruent.

EXAMPLE 3 Real-Life Application



The hexagon is about 15,000 miles across. Approximately four Earths could fit inside it.

A cloud system discovered on Saturn is in the approximate shape of a regular hexagon. Find the measure of each interior angle of the hexagon.

Step 1: A hexagon has 6 sides. Find the sum of the interior angle measures.

$S = (n - 2) \cdot 180°$ Write the formula.

$= (6 - 2) \cdot 180°$ Substitute 6 for n.

$= 720°$ Simplify. The sum of the interior angle measures is 720°.

Step 2: Divide the sum by the number of interior angles, 6.

$720° \div 6 = 120°$

∴ The measure of each interior angle is 120°.

On Your Own

Now You're Ready
Exercises 16–18

Find the measure of each interior angle of the regular polygon.

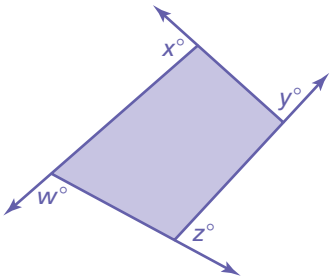
6. octagon **7.** decagon **8.** 18-gon

Key Idea

Exterior Angle Measures of a Polygon

Words The sum of the measures of the exterior angles of a convex polygon is 360°.

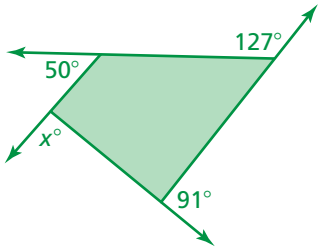
Algebra $w + x + y + z = 360$



EXAMPLE 4 Finding Exterior Angle Measures

Find the measures of the exterior angles of each polygon.

a.

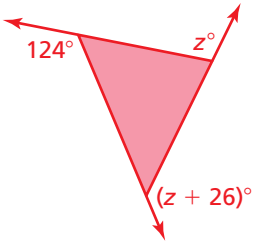


Write and solve an equation for x.

$$x + 50 + 127 + 91 = 360$$
$$x + 268 = 360$$
$$x = 92$$

∴ So, the measures of the exterior angles are 92°, 50°, 127°, and 91°.

b.



Write and solve an equation for z.

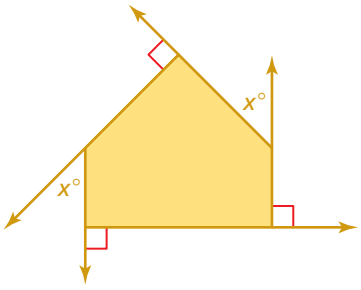
$$124 + z + (z + 26) = 360$$
$$2z + 150 = 360$$
$$z = 105$$

∴ So, the measures of the exterior angles are 124°, 105°, and $(105 + 26)° = 131°$.

On Your Own

Now You're Ready
Exercises 22–28

9. Find the measures of the exterior angles of the polygon.



3.3 Exercises

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Vocabulary and Concept Check

1. **VOCABULARY** Draw a regular polygon that has three sides.

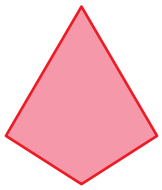
2. **WHICH ONE DOESN'T BELONG?** Which figure does *not* belong with the other three? Explain your reasoning.

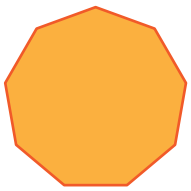
3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What is the measure of an interior angle of a regular pentagon?	What is the sum of the interior angle measures of a convex pentagon?
What is the sum of the interior angle measures of a regular pentagon?	What is the sum of the interior angle measures of a concave pentagon?

Practice and Problem Solving

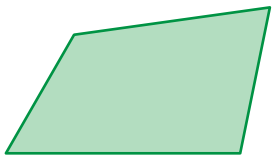
Use triangles to find the sum of the interior angle measures of the polygon.

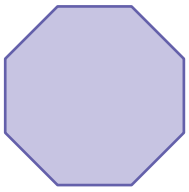
4. 

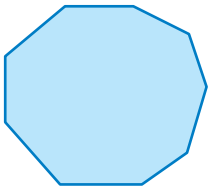
5. 

6. 

Find the sum of the interior angle measures of the polygon.

7. 

8. 

9. 

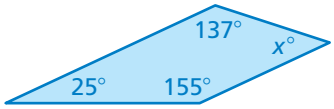
10. **ERROR ANALYSIS** Describe and correct the error in finding the sum of the interior angle measures of a 13-gon.

$S = n \cdot 180°$
$= 13 \cdot 180°$
$= 2340°$

11. **NUMBER SENSE** Can a pentagon have interior angles that measure 120°, 105°, 65°, 150°, and 95°? Explain.

Find the measures of the interior angles.

2 **12.**



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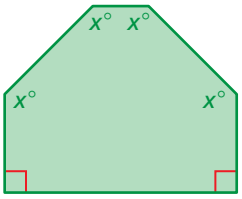

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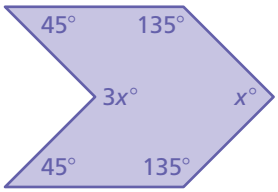
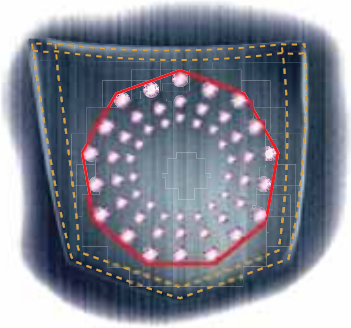


15. **REASONING** The sum of the interior angle measures in a regular polygon is 1260°. What is the measure of one of the interior angles of the polygon?

Find the measure of each interior angle of the regular polygon.

3 **16.**

17.



18.

19. **ERROR ANALYSIS** Describe and correct the error in finding the measure of each interior angle of a regular 20-gon.

$S = (n - 2) \cdot 180°$
$= (20 - 2) \cdot 180°$
$= 18 \cdot 180°$
$= 3240°$
$3240° \div 18 = 180$
The measure of each interior angle is 180°.

20. **FIRE HYDRANT** A fire hydrant bolt is in the shape of a regular pentagon.

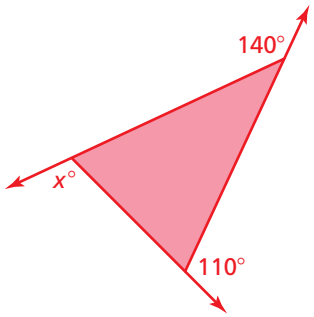
a. What is the measure of each interior angle?

b. Why are fire hydrants made this way?

21. **PROBLEM SOLVING** The interior angles of a regular polygon each measure 165°. How many sides does the polygon have?

Find the measures of the exterior angles of the polygon.

4 **22.**



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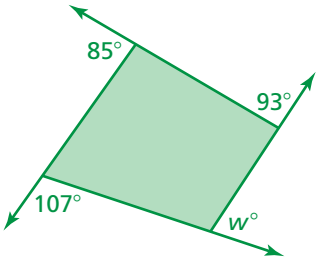

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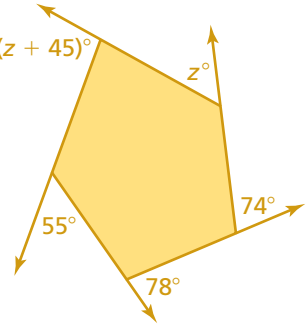
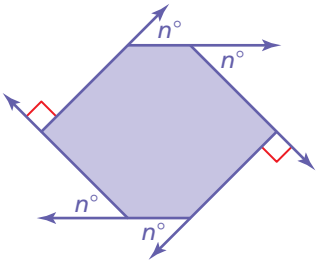


25. **REASONING** What is the measure of an exterior angle of a regular hexagon? Explain.

Find the measures of the exterior angles of the polygon.

26.



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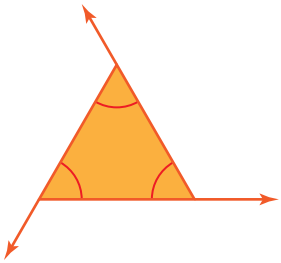

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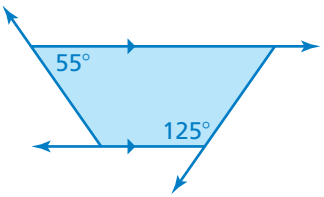


29. **STAINED GLASS** The center of the stained glass window is in the shape of a regular polygon. What is the measure of each interior angle of the polygon? What is the measure of each exterior angle?

30. **PENTAGON** Draw a pentagon that has two right interior angles, two 45° interior angles, and one 270° interior angle.

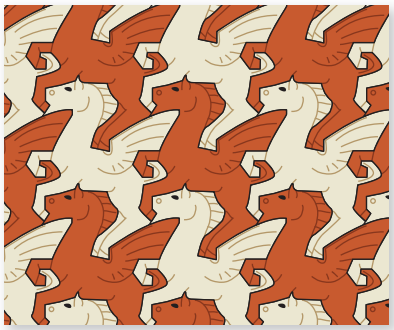
31. **GAZEBO** The floor of a gazebo is in the shape of a heptagon. Four of the interior angles measure 135°. The other interior angles have equal measures. Find their measures.

32. **MONEY** The border of a Susan B. Anthony dollar is in the shape of a regular polygon.

a. How many sides does the polygon have?

b. What is the measure of each interior angle of the border? Round your answer to the nearest degree.

33. Geometry When tiles can be used to cover a floor with no empty spaces, the collection of tiles is called a *tessellation*.



a. Create a tessellation using equilateral triangles.

b. Find two more regular polygons that form tessellations.

c. Create a tessellation that uses two different regular polygons.

d. Use what you know about interior and exterior angles to explain why the polygons in part (c) form a tessellation.

Fair Game Review What you learned in previous grades & lessons

Solve the proportion. *(Skills Review Handbook)*

34. $\frac{x}{12} = \frac{3}{4}$ **35.** $\frac{14}{21} = \frac{x}{3}$ **36.** $\frac{9}{x} = \frac{6}{2}$ **37.** $\frac{10}{4} = \frac{15}{x}$

38. **MULTIPLE CHOICE** The ratio of tulips to daisies is 3 : 5. Which of the following could be the total number of tulips and daisies? *(Skills Review Handbook)*

(A) 6 (B) 10 (C) 15 (D) 16