3.2 Angles of Triangles

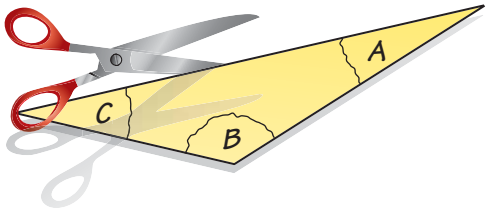
Essential Question How can you describe the relationships among the angles of a triangle?

1 ACTIVITY: Exploring the Interior Angles of a Triangle

Work with a partner.

a. Draw a triangle. Label the interior angles A, B, and C.

b. Carefully cut out the triangle. Tear off the three corners of the triangle.



c. Arrange angles A and B so that they share a vertex and are adjacent.

d. How can you place the third angle to determine the sum of the measures of the interior angles? What is the sum?

e. Compare your results with those of others in your class.

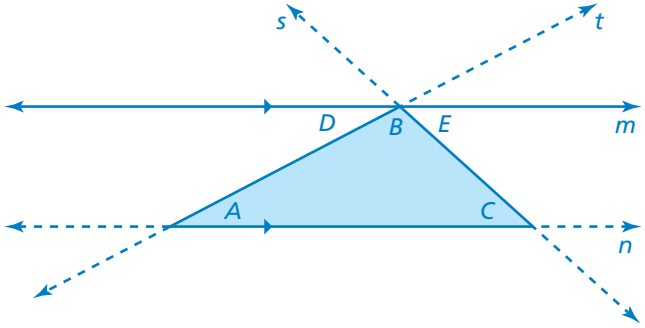
f. **STRUCTURE** How does your result in part (d) compare to the rule you wrote in Lesson 1.1, Activity 2?

2 ACTIVITY: Exploring the Interior Angles of a Triangle

Work with a partner.

a. Describe the figure.

b. **LOGIC** Use what you know about parallel lines and transversals to justify your result in part (d) of Activity 1.



Common Core

Geometry

In this lesson, you will

- understand that the sum of the interior angle measures of a triangle is 180°.
- find the measures of interior and exterior angles of triangles.

Learning Standard 8.G.5

3 ACTIVITY: Exploring an Exterior Angle of a Triangle

Math Practice 8

Maintain Oversight

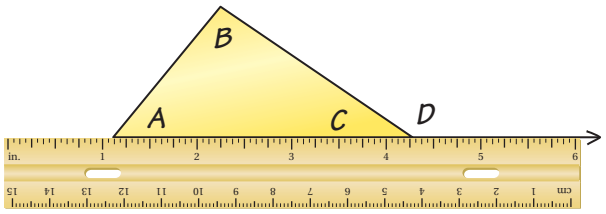
Do you think your conclusion will be true for the exterior angle of any triangle? Explain.

Work with a partner.

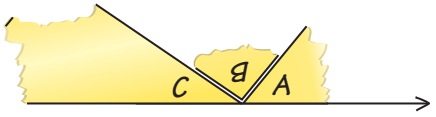
a. Draw a triangle. Label the interior angles A, B, and C.

b. Carefully cut out the triangle.

c. Place the triangle on a piece of paper and extend one side to form *exterior angle D*, as shown.



d. Tear off the corners that are not adjacent to the exterior angle. Arrange them to fill the exterior angle, as shown. What does this tell you about the measure of exterior angle D?

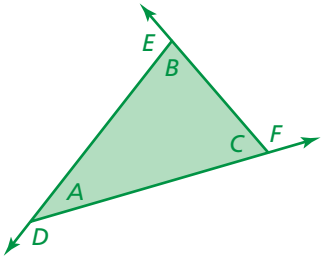


4 ACTIVITY: Measuring the Exterior Angles of a Triangle

Work with a partner.

a. Draw a triangle and label the interior and exterior angles, as shown.

b. Use a protractor to measure all six angles. Copy and complete the table to organize your results. What does the table tell you about the measure of an exterior angle of a triangle?



Exterior Angle	$D =$ ____°	$E =$ ____°	$F =$ ____°
Interior Angle	$B =$ ____°	$A =$ ____°	$A =$ ____°
Interior Angle	$C =$ ____°	$C =$ ____°	$B =$ ____°

What Is Your Answer?

5. **REPEATED REASONING** Draw three triangles that have different shapes. Repeat parts (b)–(d) from Activity 1 for each triangle. Do you get the same results? Explain.

6. **IN YOUR OWN WORDS** How can you describe the relationships among angles of a triangle?

Practice Use what you learned about angles of a triangle to complete Exercises 4–6 on page 114.

3.2 Lesson

Check It Out
Lesson Tutorials
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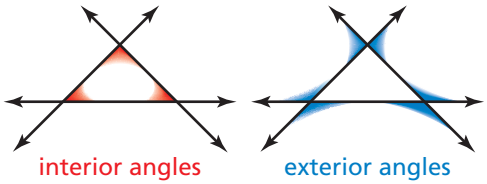
Key Vocabulary
interior angles of a polygon, *p. 112*
exterior angles of a polygon, *p. 112*

The angles inside a polygon are called **interior angles**. When the sides of a polygon are extended, other angles are formed. The angles outside the polygon that are adjacent to the interior angles are called **exterior angles**.

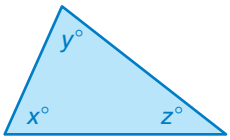


Key Idea

Interior Angle Measures of a Triangle

Words The sum of the interior angle measures of a triangle is 180°.

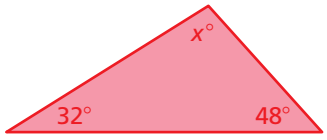
Algebra $x + y + z = 180$



EXAMPLE 1 Using Interior Angle Measures

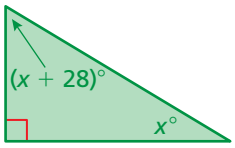
Find the value of x.

a.



$x + 32 + 48 = 180$
$x + 80 = 180$
$x = 100$

b.



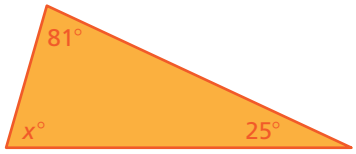
$x + (x + 28) + 90 = 180$
$2x + 118 = 180$
$2x = 62$
$x = 31$

On Your Own

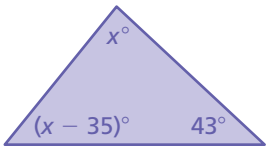
Now You're Ready
Exercises 4–9

Find the value of x.

1.



2.

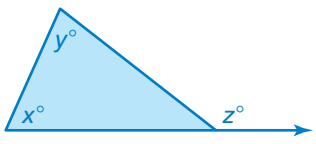


Key Idea

Exterior Angle Measures of a Triangle

Words The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

Algebra $z = x + y$



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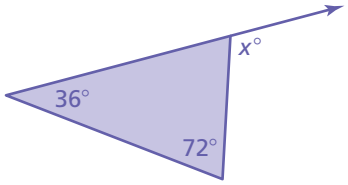
EXAMPLE 2 Finding Exterior Angle Measures

Study Tip

Each vertex has a pair of congruent exterior angles. However, it is common to show only one exterior angle at each vertex.

Find the measure of the exterior angle.

a.

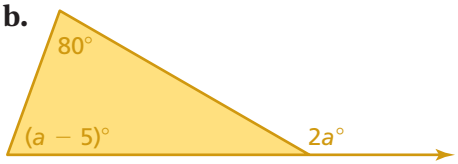


$x = 36 + 72$

$x = 108$

∴ So, the measure of the exterior angle is 108°.

b.



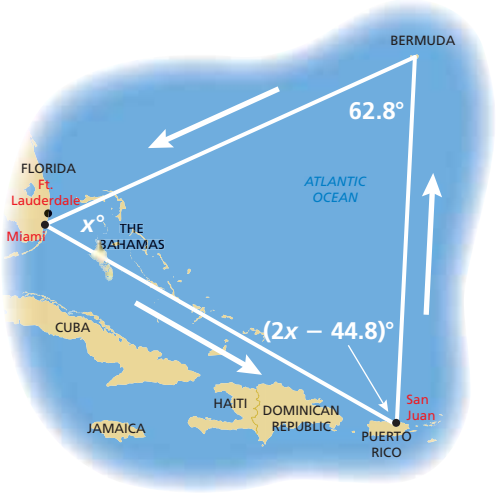
$2a = (a - 5) + 80$

$2a = a + 75$

$a = 75$

∴ So, the measure of the exterior angle is $2(75)° = 150°$.

EXAMPLE 3 Real-Life Application



An airplane leaves from Miami and travels around the Bermuda Triangle. What is the value of x?

Ⓐ 26.8 Ⓑ 27.2 Ⓒ 54 Ⓓ 64

Use what you know about the interior angle measures of a triangle to write an equation.

$x + (2x - 44.8) + 62.8 = 180$ Write equation.

$3x + 18 = 180$ Combine like terms.

$3x = 162$ Subtract 18 from each side.

$x = 54$ Divide each side by 3.

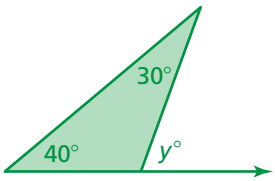
∴ The value of x is 54. The correct answer is Ⓒ.

On Your Own

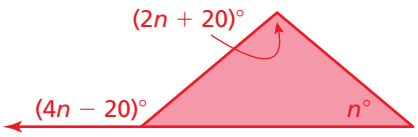
Now You're Ready
Exercises 12–14

Find the measure of the exterior angle.

3.



4.



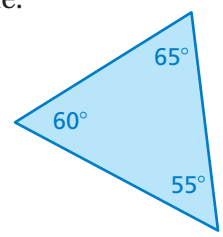
5. In Example 3, the airplane leaves from Fort Lauderdale. The interior angle measure at Bermuda is 63.9°. The interior angle measure at San Juan is $(x + 7.5)°$. Find the value of x.

3.2 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** You know the measures of two interior angles of a triangle. How can you find the measure of the third interior angle?
2. **VOCABULARY** How many exterior angles does a triangle have at each vertex? Explain.
3. **NUMBER SENSE** List the measures of the exterior angles for the triangle shown at the right.

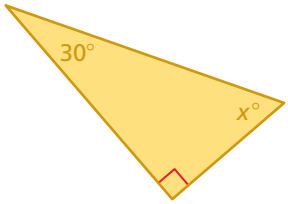


Practice and Problem Solving

Find the measures of the interior angles.

1

4.



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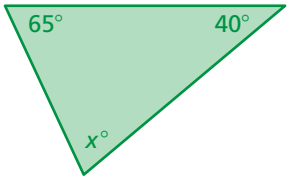

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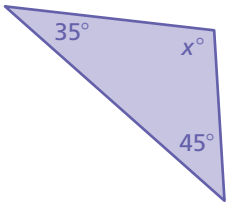

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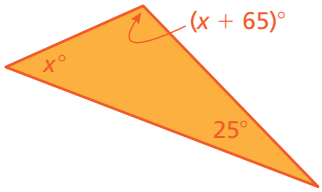

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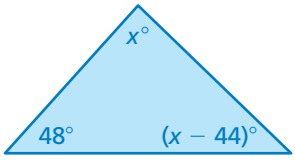

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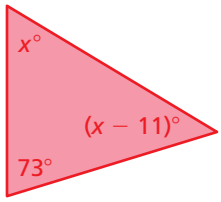
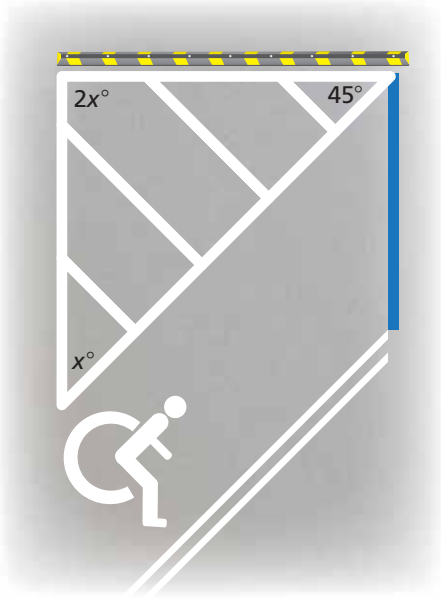
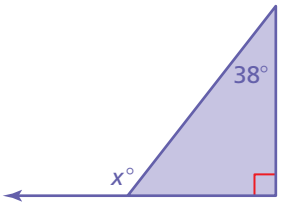


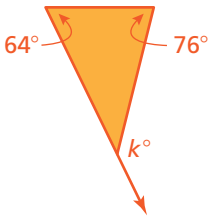
10. **BILLIARD RACK** Find the value of x in the billiard rack.

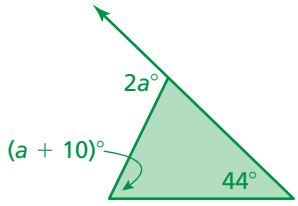
11. **NO PARKING** The triangle with lines through it designates a no parking zone. What is the value of x?



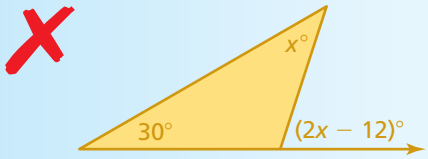
Find the measure of the exterior angle.

12. 

13. 

14. 

15. ERROR ANALYSIS Describe and correct the error in finding the measure of the exterior angle.

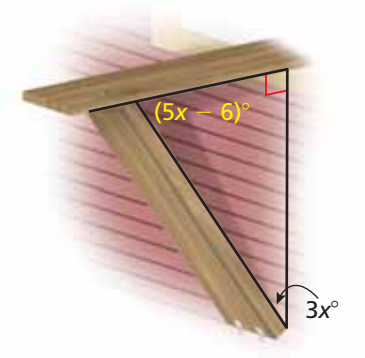


$(2x - 12) + x + 30 = 180$

$3x + 18 = 180$

$x = 54$

The exterior angle is $(2(54) - 12)° = 96°$.



16. RATIO The ratio of the interior angle measures of a triangle is 2 : 3 : 5. What are the angle measures?

17. CONSTRUCTION The support for a window air-conditioning unit forms a triangle and an exterior angle. What is the measure of the exterior angle?

18. REASONING A triangle has an exterior angle with a measure of 120°. Can you determine the measures of the interior angles? Explain.

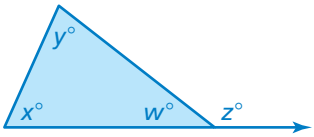
Determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

19. Given three angle measures, you can construct a triangle.

20. The acute interior angles of a right triangle are complementary.

21. A triangle has more than one vertex with an acute exterior angle.

22. **Precision** Using the figure at the right, show that $z = x + y$. (*Hint:* Find two equations involving w.)



Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. *(Section 1.2)*

23. $-4x + 3 = 19$ **24.** $2(y - 1) + 6y = -10$ **25.** $5 + 0.5(6n + 14) = 3$

26. MULTIPLE CHOICE Which transformation moves every point of a figure the same distance and in the same direction? *(Section 2.2)*

(A) translation (B) reflection (C) rotation (D) dilation