1.3 Solving Equations with Variables on Both Sides

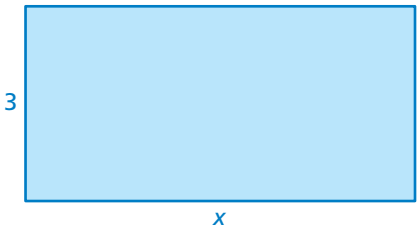
Essential Question How can you solve an equation that has variables on both sides?

1 ACTIVITY: Perimeter and Area

Work with a partner.

- **Each figure has the unusual property that the value of its perimeter (in feet) is equal to the value of its area (in square feet). Write an equation for each figure.**
- **Solve each equation for x.**
- **Use the value of x to find the perimeter and the area of each figure.**
- **Describe how you can check your solution.**

a.



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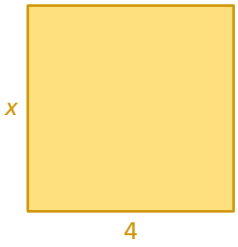

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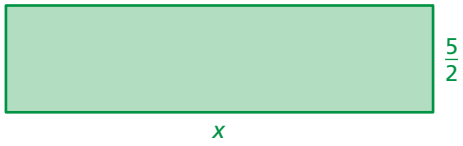

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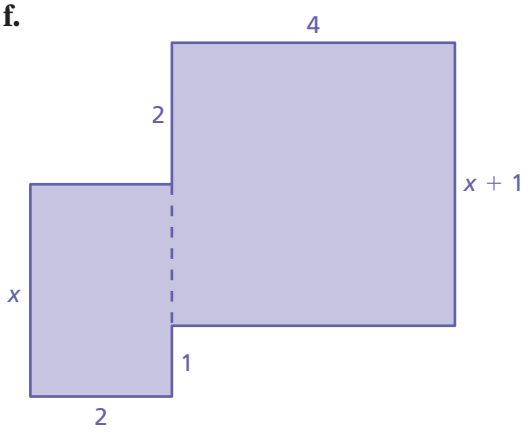

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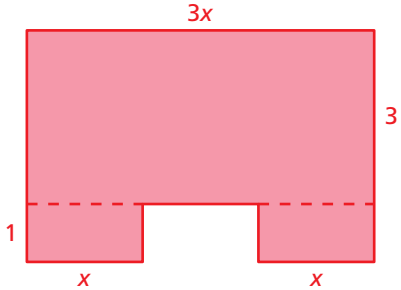


Common Core

Solving Equations

In this lesson, you will

- solve equations with variables on both sides.
- determine whether equations have no solution or infinitely many solutions.

Learning Standards
8.EE.7a
8.EE.7b

2 ACTIVITY: Surface Area and Volume

Math Practice 2

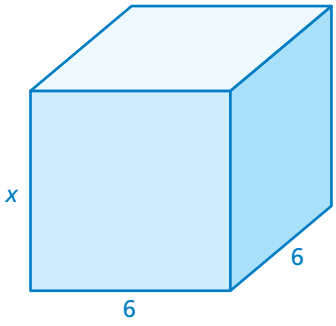
Use Operations

What properties of operations do you need to use in order to find the value of x?

Work with a partner.

- **Each solid has the unusual property that the value of its surface area (in square inches) is equal to the value of its volume (in cubic inches). Write an equation for each solid.**
- **Solve each equation for x.**
- **Use the value of x to find the surface area and the volume of each solid.**
- **Describe how you can check your solution.**

a.



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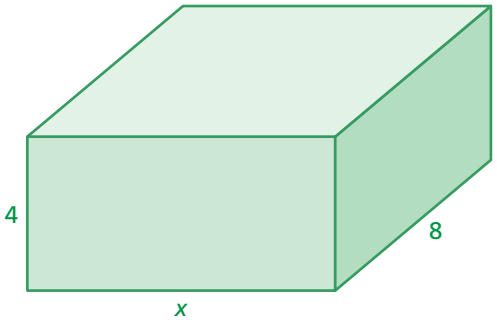
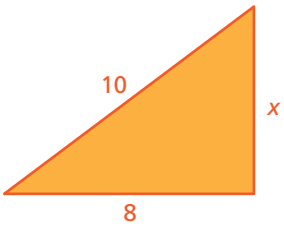
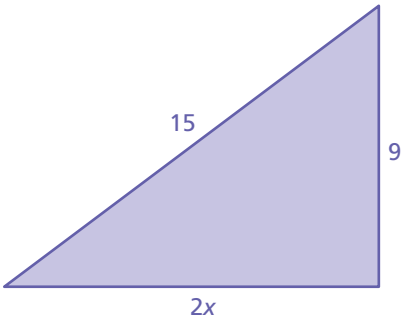


3 ACTIVITY: Puzzle

Work with a partner. The perimeter of the larger triangle is 150% of the perimeter of the smaller triangle. Find the dimensions of each triangle.





What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you solve an equation that has variables on both sides? How do you move a variable term from one side of the equation to the other?

5. Write an equation that has variables on both sides. Solve the equation.

Practice Use what you learned about solving equations with variables on both sides to complete Exercises 3–5 on page 23.

1.3 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Idea

Solving Equations with Variables on Both Sides

To solve equations with variables on both sides, collect the variable terms on one side and the constant terms on the other side.

EXAMPLE 1 Solving an Equation with Variables on Both Sides

Solve $15 - 2x = -7x$. Check your solution.

	$15 - 2x = -7x$	Write the equation.
Undo the subtraction. →	$+ 2x \quad + 2x$	Addition Property of Equality
	$15 = -5x$	Simplify.
Undo the multiplication. →	$\frac{15}{-5} = \frac{-5x}{-5}$	Division Property of Equality
	$-3 = x$	Simplify.

∴ The solution is $x = -3$.

Check

$15 - 2x = -7x$

$15 - 2(-3) \stackrel{?}{=} -7(-3)$

$21 = 21$ ✓

EXAMPLE 2 Using the Distributive Property to Solve an Equation

Solve $-2(x - 5) = 6\left(2 - \frac{1}{2}x\right)$.

	$-2(x - 5) = 6\left(2 - \frac{1}{2}x\right)$	Write the equation.
	$-2x + 10 = 12 - 3x$	Distributive Property
Undo the subtraction. →	$+ 3x \quad + 3x$	Addition Property of Equality
	$x + 10 = 12$	Simplify.
Undo the addition. →	$- 10 \quad - 10$	Subtraction Property of Equality
	$x = 2$	Simplify.

∴ The solution is $x = 2$.

On Your Own

Now You're Ready
Exercises 6–14

Solve the equation. Check your solution.

1. $-3x = 2x + 19$ **2.** $2.5y + 6 = 4.5y - 1$ **3.** $6(4 - z) = 2z$

Some equations do not have one solution. Equations can also have no solution or infinitely many solutions.

When solving an equation that has no solution, you will obtain an equivalent equation that is not true for any value of the variable, such as $0 = 2$.

EXAMPLE 3 Solving Equations with No Solution

Solve $3 - 4x = -7 - 4x$.

$3 - 4x = -7 - 4x$ Write the equation.

Undo the subtraction. → $+ 4x \quad + 4x$ Addition Property of Equality

$3 = -7$ ✗ Simplify.

∴ The equation $3 = -7$ is never true. So, the equation has no solution.

When solving an equation that has infinitely many solutions, you will obtain an equivalent equation that is true for all values of the variable, such as $-5 = -5$.

EXAMPLE 4 Solving Equations with Infinitely Many Solutions

Solve $6x + 4 = 4\left(\frac{3}{2}x + 1\right)$.

$6x + 4 = 4\left(\frac{3}{2}x + 1\right)$ Write the equation.

$6x + 4 = 6x + 4$ Distributive Property

Undo the addition. → $- 6x \quad - 6x$ Subtraction Property of Equality

$4 = 4$ Simplify.

∴ The equation $4 = 4$ is always true. So, the equation has infinitely many solutions.

On Your Own

Now You're Ready
Exercises 18–29

Solve the equation.

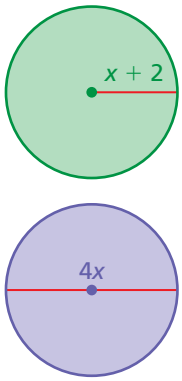
4. $2x + 1 = 2x - 1$

5. $\frac{1}{2}(6t - 4) = 3t - 2$

6. $\frac{1}{3}(2b + 9) = \frac{2}{3}\left(b + \frac{9}{2}\right)$

7. $6(5 - 2v) = -4(3v + 1)$

EXAMPLE 5 Writing and Solving an Equation



The circles are identical. What is the area of each circle?

Ⓐ 2 Ⓑ 4 Ⓒ 16π Ⓓ 64π

The circles are identical, so the radius of each circle is the same.

$x + 2 = 2x$ Write an equation. The radius of the purple circle is $\frac{4x}{2} = 2x$.

$-x \quad -x$ Subtraction Property of Equality

$2 = x$ Simplify.

Because the radius of each circle is 4, the area of each circle is $\pi r^2 = \pi(4)^2 = 16\pi$.

∴ So, the correct answer is Ⓒ.

EXAMPLE 6 Real-Life Application

A boat travels x miles per hour upstream on the Mississippi River. On the return trip, the boat travels 2 miles per hour faster. How far does the boat travel upstream?

The speed of the boat on the return trip is $(x + 2)$ miles per hour.

Distance upstream = Distance of return trip

$3x = 2.5(x + 2)$ Write an equation.

$3x = 2.5x + 5$ Distributive Property

$-2.5x \quad -2.5x$ Subtraction Property of Equality

$0.5x = 5$ Simplify.

$\frac{0.5x}{0.5} = \frac{5}{0.5}$ Division Property of Equality

$x = 10$ Simplify.

∴ The boat travels 10 miles per hour for 3 hours upstream. So, it travels 30 miles upstream.

On Your Own

8. **WHAT IF?** In Example 5, the diameter of the purple circle is $3x$. What is the area of each circle?

9. A boat travels x miles per hour from one island to another island in 2.5 hours. The boat travels 5 miles per hour faster on the return trip of 2 hours. What is the distance between the islands?

1.3 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

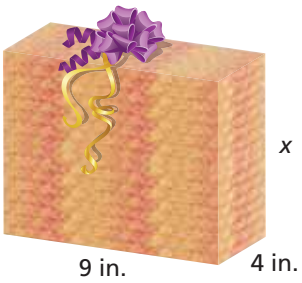
Vocabulary and Concept Check

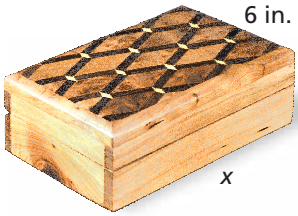
1. **WRITING** Is $x = 3$ a solution of the equation $3x - 5 = 4x - 9$? Explain.
2. **OPEN-ENDED** Write an equation that has variables on both sides and has a solution of -3.

Practice and Problem Solving

The value of the solid's surface area is equal to the value of the solid's volume. Find the value of x.

3. 

4. 

5. 

Solve the equation. Check your solution.

1 2 6. $m - 4 = 2m$

7. $3k - 1 = 7k + 2$

8. $6.7x = 5.2x + 12.3$

9. $-24 - \frac{1}{8}p = \frac{3}{8}p$

10. $12(2w - 3) = 6w$

11. $2(n - 3) = 4n + 1$

12. $2(4z - 1) = 3(z + 2)$

13. $0.1x = 0.2(x + 2)$

14. $\frac{1}{6}d + \frac{2}{3} = \frac{1}{4}(d - 2)$

15. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

✗
$$3x - 4 = 2x + 1$$
$$3x - 4 - 2x = 2x + 1 - 2x$$
$$x - 4 = 1$$
$$x - 4 + 4 = 1 - 4$$
$$x = -3$$

16. **TRAIL MIX** The equation $4.05p + 14.40 = 4.50(p + 3)$ represents the number p of pounds of peanuts you need to make trail mix. How many pounds of peanuts do you need for the trail mix?

17. **CARS** Write and solve an equation to find the number of miles you must drive to have the same cost for each of the car rentals.


$15 plus $0.50 per mile


$25 plus $0.25 per mile

Solve the equation. Check your solution, if possible.

18. $x + 6 = x$

19. $3x - 1 = 1 - 3x$

20. $4x - 9 = 3.5x - 9$

21. $\frac{1}{2}x + \frac{1}{2}x = x + 1$

22. $3x + 15 = 3(x + 5)$

23. $\frac{1}{3}(9x + 3) = 3x + 1$

24. $5x - 7 = 4x - 1$

25. $2x + 4 = -(-7x + 6)$

26. $5.5 - x = -4.5 - x$

27. $10x - \frac{8}{3} - 4x = 6x$

28. $-3(2x - 3) = -6x + 9$

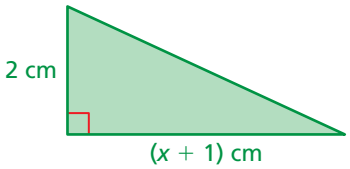
29. $6(7x + 7) = 7(6x + 6)$

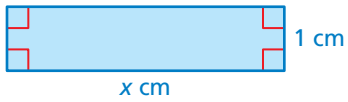
30. ERROR ANALYSIS Describe and correct the error in solving the equation.

$$-4(2n - 3) = 12 - 8n$$
$$-8n + 12 = 12 - 8n$$
$$-8n = -8n$$
$$0 = 0$$
The solution is $n = 0$.

31. OPEN-ENDED Write an equation with variables on both sides that has no solution. Explain why it has no solution.

32. GEOMETRY Are there any values of x for which the areas of the figures are the same? Explain.

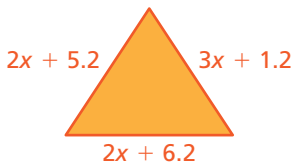




33. SATELLITE TV Provider A charges $75 for installation and charges $39.95 per month for the basic package. Provider B offers free installation and charges $39.95 per month for the basic package. Your neighbor subscribes to Provider A the same month you subscribe to Provider B. After how many months is your neighbor's total cost the same as your total cost for satellite TV?

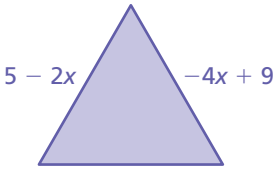
34. PIZZA CRUST Pepe's Pizza makes 52 pizza crusts the first week and 180 pizza crusts each subsequent week. Dianne's Delicatessen makes 26 pizza crusts the first week and 90 pizza crusts each subsequent week. In how many weeks will the total number of pizza crusts made by Pepe's Pizza equal twice the total number of pizza crusts made by Dianne's Delicatessen?

35. PRECISION Is the triangle an equilateral triangle? Explain.



A polygon is *regular* if each of its sides has the same length. Find the perimeter of the regular polygon.

36.



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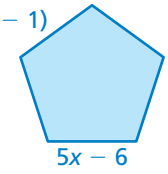

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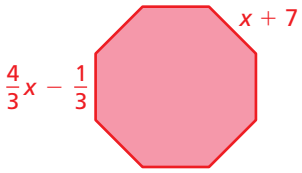
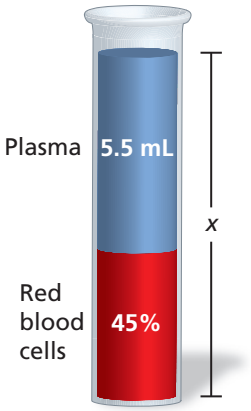


39. PRECISION Sending a DVD in an express delivery service envelope costs the same as sending the DVD in a priority service box. What is the weight of the DVD with its packing material? Round your answer to the nearest hundredth.

	Packing Material	Priority	Express
Box	$2.25	$2.50 per lb	$8.50 per lb
Envelope	$1.10	$2.50 per lb	$8.50 per lb

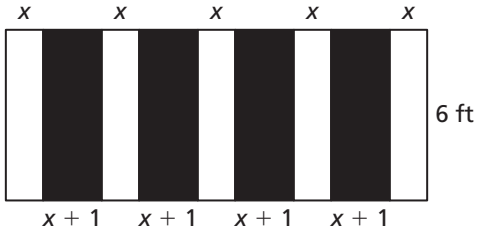


40. PROBLEM SOLVING Would you solve the equation $0.25x + 7 = \frac{1}{3}x - 8$ using fractions or decimals? Explain.

41. BLOOD SAMPLE The amount of red blood cells in a blood sample is equal to the total amount in the sample minus the amount of plasma. What is the total amount x of blood drawn?

42. NUTRITION One serving of oatmeal provides 16% of the fiber you need daily. You must get the remaining 21 grams of fiber from other sources. How many grams of fiber should you consume daily?

43. Geometry A 6-foot-wide hallway is painted as shown, using equal amounts of white and black paint.



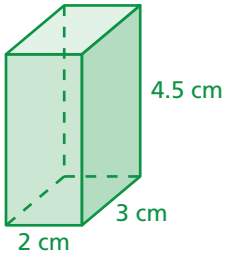
a. How long is the hallway?

b. Can this same hallway be painted with the same pattern, but using twice as much black paint as white paint? Explain.

Fair Game Review What you learned in previous grades & lessons

Find the volume of the solid. *(Skills Review Handbook)*

44.



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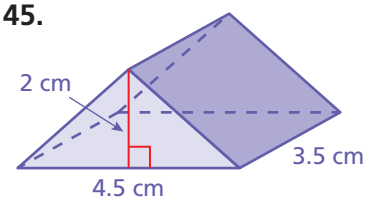

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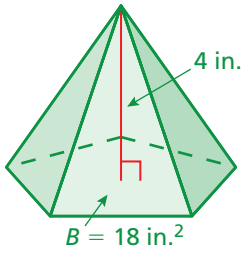

47. MULTIPLE CHOICE A car travels 480 miles on 15 gallons of gasoline. How many miles does the car travel per gallon? *(Skills Review Handbook)*

(A) 28 mi/gal (B) 30 mi/gal (C) 32 mi/gal (D) 35 mi/gal