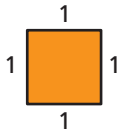
2.6 Perimeters and Areas of Similar Figures

Essential Question How do changes in dimensions of similar geometric figures affect the perimeters and the areas of the figures?

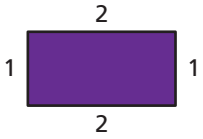
1 ACTIVITY: Creating Similar Figures

Work with a partner. Use pattern blocks to make a figure whose dimensions are 2, 3, and 4 times greater than those of the original figure.

a. Square



b. Rectangle



2 ACTIVITY: Finding Patterns for Perimeters

Work with a partner. Copy and complete the table for the perimeter P of each figure in Activity 1. Describe the pattern.

Figure	Original Side Lengths	Double Side Lengths	Triple Side Lengths	Quadruple Side Lengths
	$P =$			
	$P =$			

3 ACTIVITY: Finding Patterns for Areas

Work with a partner. Copy and complete the table for the area A of each figure in Activity 1. Describe the pattern.

Figure	Original Side Lengths	Double Side Lengths	Triple Side Lengths	Quadruple Side Lengths
	$A =$			
	$A =$			

Common Core

Geometry

In this lesson, you will

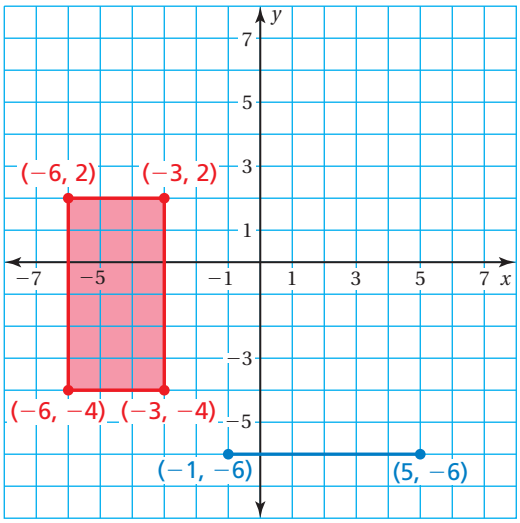
- understand the relationship between perimeters of similar figures.
- understand the relationship between areas of similar figures.
- find ratios of perimeters and areas for similar figures.

Preparing for Standard 8.G.4

4 ACTIVITY: Drawing and Labeling Similar Figures

Work with a partner.

a. Find a blue rectangle that is similar to the red rectangle and has one side from $(-1, -6)$ to $(5, -6)$. Label the vertices.



Check that the two rectangles are similar by showing that the ratios of corresponding sides are equal.

$$\frac{\text{Red Length}}{\text{Blue Length}} \stackrel{?}{=} \frac{\text{Red Width}}{\text{Blue Width}}$$

$$\frac{\text{change in } y}{\text{change in } y} \stackrel{?}{=} \frac{\text{change in } x}{\text{change in } x}$$

$$\frac{\square}{\square} \stackrel{?}{=} \frac{\square}{\square}$$

$$\frac{\square}{\square} \stackrel{?}{=} \frac{\square}{\square}$$

∴ The ratios are equal. So, the rectangles are similar.

Math Practice 1

Analyze Givens

What values should you use to fill in the proportion? Does it matter where each value goes? Explain.

b. Compare the perimeters and the areas of the figures. Are the results the same as your results from Activities 2 and 3? Explain.

c. There are three other blue rectangles that are similar to the red rectangle and have the given side.

- Draw each one. Label the vertices of each.
- Show that each is similar to the original red rectangle.

What Is Your Answer?

5. **IN YOUR OWN WORDS** How do changes in dimensions of similar geometric figures affect the perimeters and the areas of the figures?

6. What information do you need to know to find the dimensions of a figure that is similar to another figure? Give examples to support your explanation.

Practice

Use what you learned about perimeters and areas of similar figures to complete Exercises 8 and 9 on page 80.

2.6 Lesson

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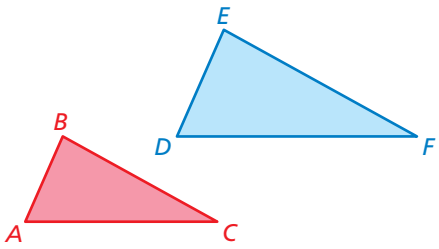
Key Idea

Perimeters of Similar Figures

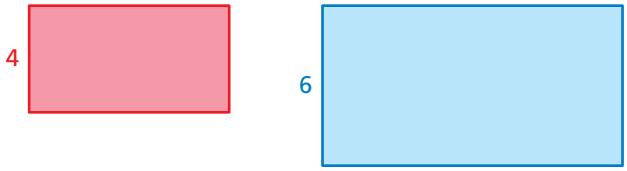
When two figures are similar, the ratio of their perimeters is equal to the ratio of their corresponding side lengths.



$$\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

EXAMPLE 1 Finding Ratios of Perimeters

Find the ratio (red to blue) of the perimeters of the similar rectangles.



$$\frac{\text{Perimeter of red rectangle}}{\text{Perimeter of blue rectangle}} = \frac{4}{6} = \frac{2}{3}$$

∴ The ratio of the perimeters is $\frac{2}{3}$.

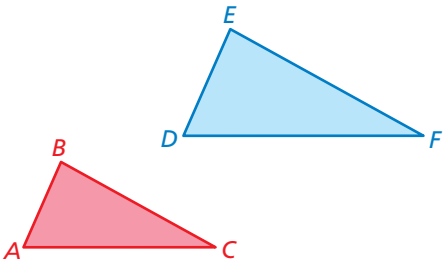
On Your Own

1. The height of Figure A is 9 feet. The height of a similar Figure B is 15 feet. What is the ratio of the perimeter of A to the perimeter of B?

Key Idea

Areas of Similar Figures

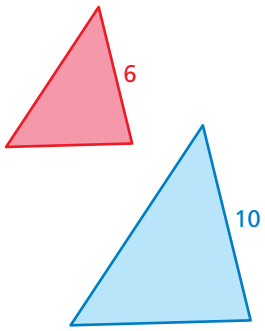
When two figures are similar, the ratio of their areas is equal to the *square* of the ratio of their corresponding side lengths.



$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

EXAMPLE 2 Finding Ratios of Areas

Find the ratio (red to blue) of the areas of the similar triangles.



$$\frac{\text{Area of red triangle}}{\text{Area of blue triangle}} = \left(\frac{6}{10}\right)^2$$

$$= \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

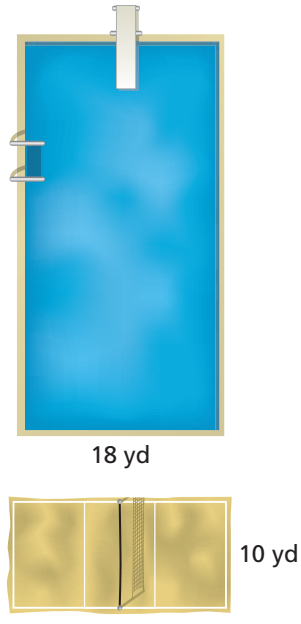
∴ The ratio of the areas is $\frac{9}{25}$.

On Your Own

Now You're Ready
Exercises 4–7

2. The base of Triangle P is 8 meters. The base of a similar Triangle Q is 7 meters. What is the ratio of the area of P to the area of Q?

EXAMPLE 3 Using Proportions to Find Perimeters and Areas



A swimming pool is similar in shape to a volleyball court. Find the perimeter P and the area A of the pool.

The rectangular pool and the court are similar. So, use the ratio of corresponding side lengths to write and solve proportions to find the perimeter and the area of the pool.

Perimeter

$$\frac{\text{Perimeter of court}}{\text{Perimeter of pool}} = \frac{\text{Width of court}}{\text{Width of pool}}$$

$$\frac{60}{P} = \frac{10}{18}$$

$$1080 = 10P$$

$$108 = P$$

Area

$$\frac{\text{Area of court}}{\text{Area of pool}} = \left(\frac{\text{Width of court}}{\text{Width of pool}}\right)^2$$

$$\frac{200}{A} = \left(\frac{10}{18}\right)^2$$

$$\frac{200}{A} = \frac{100}{324}$$

$$64{,}800 = 100A$$

$$648 = A$$

∴ So, the perimeter of the pool is 108 yards, and the area is 648 square yards.

On Your Own

3. **WHAT IF?** The width of the pool is 16 yards. Find the perimeter P and the area A of the pool.

2.6 Exercises

Check It Out
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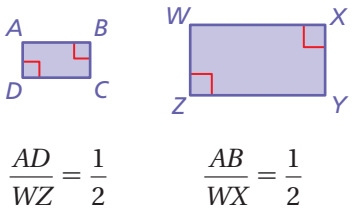
Vocabulary and Concept Check

1. **WRITING** How are the perimeters of two similar figures related?

2. **WRITING** How are the areas of two similar figures related?

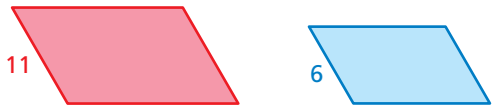
3. **NUMBER SENSE** Rectangle $ABCD$ is similar to Rectangle $WXYZ$. The area of $ABCD$ is 30 square inches. Explain how to find the area of $WXYZ$.

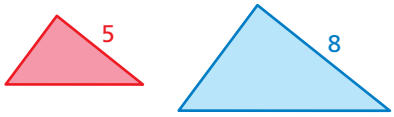


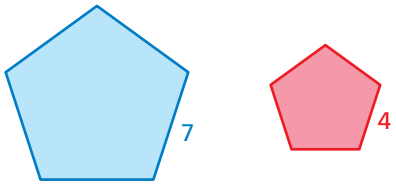
$\frac{AD}{WZ} = \frac{1}{2}$ $\frac{AB}{WX} = \frac{1}{2}$

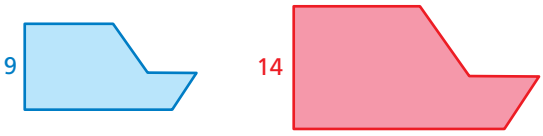
Practice and Problem Solving

The two figures are similar. Find the ratios (red to blue) of the perimeters and of the areas.

4. 

5. 

6. 

7. 

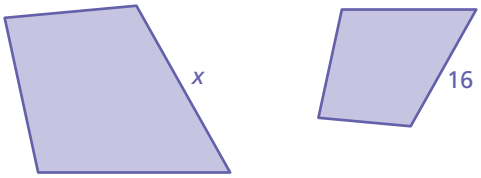
8. **PERIMETER** How does doubling the side lengths of a right triangle affect its perimeter?

9. **AREA** How does tripling the side lengths of a right triangle affect its area?

The figures are similar. Find x.

10. The ratio of the perimeters is 7 : 10.

11. The ratio of the perimeters is 8 : 5.



12. **FOOSBALL** The playing surfaces of two foosball tables are similar. The ratio of the corresponding side lengths is 10 : 7. What is the ratio of the areas?

13. **CHEERLEADING** A rectangular school banner has a length of 44 inches, a perimeter of 156 inches, and an area of 1496 square inches. The cheerleaders make signs similar to the banner. The length of a sign is 11 inches. What is its perimeter and its area?

14. REASONING The vertices of two rectangles are $A(-5, -1)$, $B(-1, -1)$, $C(-1, -4)$, $D(-5, -4)$ and $W(1, 6)$, $X(7, 6)$, $Y(7, -2)$, $Z(1, -2)$. Compare the perimeters and the areas of the rectangles. Are the rectangles similar? Explain.

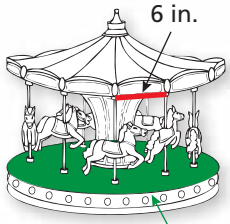
15. SQUARE The ratio of the side length of Square A to the side length of Square B is 4 : 9. The side length of Square A is 12 yards. What is the perimeter of Square B?

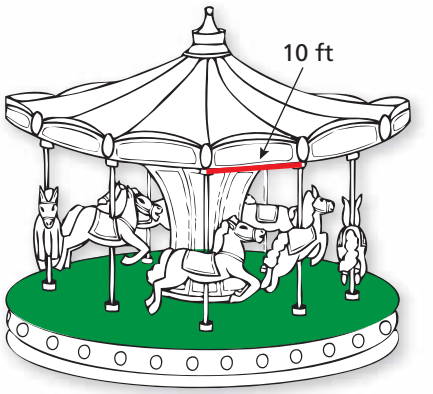
16. FABRIC The cost of the fabric is $1.31. What would you expect to pay for a similar piece of fabric that is 18 inches by 42 inches?

17. AMUSEMENT PARK A scale model of a merry-go-round and the actual merry-go-round are similar.

a. How many times greater is the base area of the actual merry-go-round than the base area of the scale model? Explain.

b. What is the base area of the actual merry-go-round in square feet?

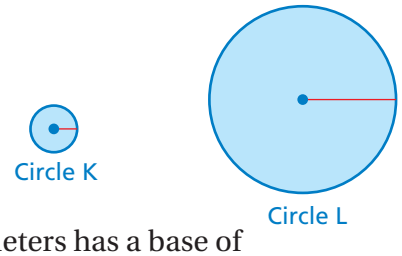




18. STRUCTURE The circumference of Circle K is π. The circumference of Circle L is 4π.

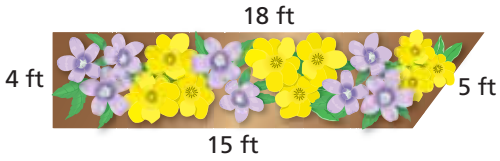
a. What is the ratio of their circumferences? of their radii? of their areas?

b. What do you notice?



19. GEOMETRY A triangle with an area of 10 square meters has a base of 4 meters. A similar triangle has an area of 90 square meters. What is the *height* of the larger triangle?

20. Problem Solving You need two bottles of fertilizer to treat the flower garden shown. How many bottles do you need to treat a similar garden with a perimeter of 105 feet?



Fair Game Review What you learned in previous grades & lessons

Solve the equation. Check your solution. *(Section 1.3)*

21. $4x + 12 = -2x$ **22.** $2b + 6 = 7b - 2$ **23.** $8(4n + 13) = 6n$

24. MULTIPLE CHOICE Last week, you collected 20 pounds of cans for recycling. This week, you collect 25 pounds of cans for recycling. What is the percent of increase? *(Skills Review Handbook)*

(A) 20% (B) 25% (C) 80% (D) 125%