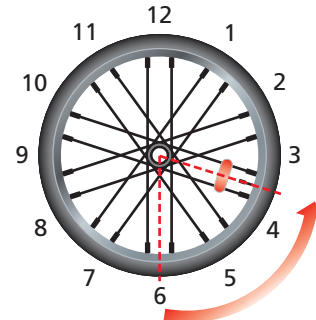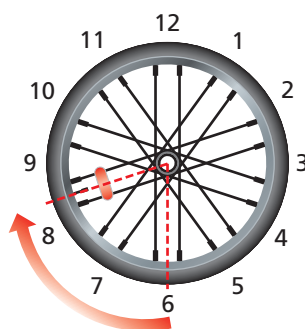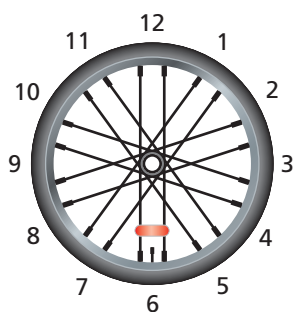# 2.4 Rotations

**Essential Question** What are the three basic ways to move an object in a plane?

## The Meaning of a Word  ● Rotate

A bicycle wheel          can **rotate** clockwise          or counterclockwise.
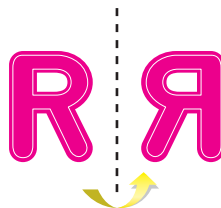
### 1 ACTIVITY: Three Basic Ways to Move Things

There are three basic ways to move objects on a flat surface.

[____] the object.          [____] the object.          [____] the object.
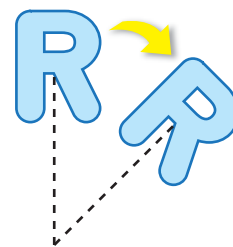
R ➤ R          R ꓤ          R ꓤ

Work with a partner.

a. What type of triangle is the blue triangle? Is it congruent to the red triangles? Explain.

b. Decide how you can move the blue triangle to obtain each red triangle.

c. Is each move a *translation*, a *reflection*, or a *rotation*?
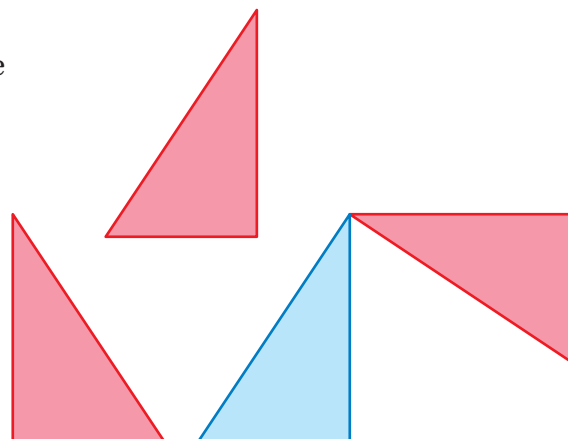
**Work with a partner.**

**a.** Draw a rectangle in Quadrant II of a coordinate plane. Find the dimensions of the rectangle.

**b.** Copy the axes and the rectangle onto a piece of transparent paper.

Align the origin and the vertices of the rectangle on the transparent paper with the coordinate plane. Turn the transparent paper so that the rectangle is in Quadrant I and the axes align.

Draw the new figure in the coordinate plane. List the vertices.

**c.** Compare the dimensions and the angle measures of the new figure to those of the original rectangle.

**d.** Are the opposite sides of the new figure still parallel? Explain.

**e.** Can you conclude that the two figures are congruent? Explain.

**f.** Turn the transparent paper so that the original rectangle is in Quadrant IV. Draw the new figure in the coordinate plane. List the vertices. Then repeat parts (c)–(e).

**g.** Compare your results with those of other students in your class. Do you think the results are true for any type of figure?

**Math Practice 6**

**Calculate Accurately**

What must you do to rotate the figure correctly?

# What Is Your Answer?

**3. IN YOUR OWN WORDS** What are the three basic ways to move an object in a plane? Draw an example of each.

**4. PRECISION** Use the results of Activity 2(b).

**a.** Draw four angles using the conditions below.

● The origin is the vertex of each angle.

● One side of each angle passes through a vertex of the original rectangle.

● The other side of each angle passes through the corresponding vertex of the rotated rectangle.

**b.** Measure each angle in part (a). For each angle, measure the distances between the origin and the vertices of the rectangles. What do you notice?

**c.** How can the results of part (b) help you rotate a figure?

**5. PRECISION** Repeat the procedure in Question 4 using the results of Activity 2(f).

**Practice**

Use what you learned about transformations to complete Exercises 7–9 on page 65.

Check It Out
Lesson Tutorials
BigIdeasMath.com

**Key Vocabulary**
rotation, *p. 62*
center of rotation,
    *p. 62*
angle of rotation,
    *p. 62*

## Key Idea

**Rotations**

A **rotation**, or *turn*, is a transformation in which a figure is rotated about a point called the **center of rotation**. The number of degrees a figure rotates is the **angle of rotation**.

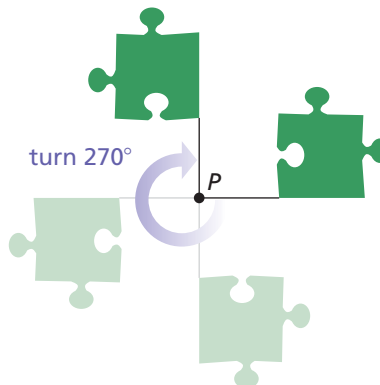In a rotation, the original figure and its image are congruent.

Turn

Angle
of rotation

Center of
rotation
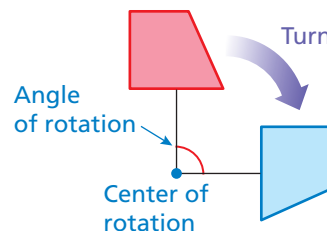
**EXAMPLE 1  Identifying a Rotation**

**You must rotate the puzzle piece 270° clockwise about point *P* to fit it into a puzzle. Which piece fits in the puzzle as shown?**

• *P*

Ⓐ      Ⓑ      Ⓒ      Ⓓ

Rotate the puzzle piece 270° clockwise about point *P*.

**Study Tip**

When rotating figures, it may help to sketch the rotation in several steps, as shown in Example 1.

turn 270°

*P*

∴  So, the correct answer is Ⓒ.

## On Your Own

Now You're Ready
Exercises 10–12

**1.** Which piece is a 90° counterclockwise rotation about point *P*?

**2.** Is Choice D a rotation of the original puzzle piece? If not, what kind of transformation does the image show?
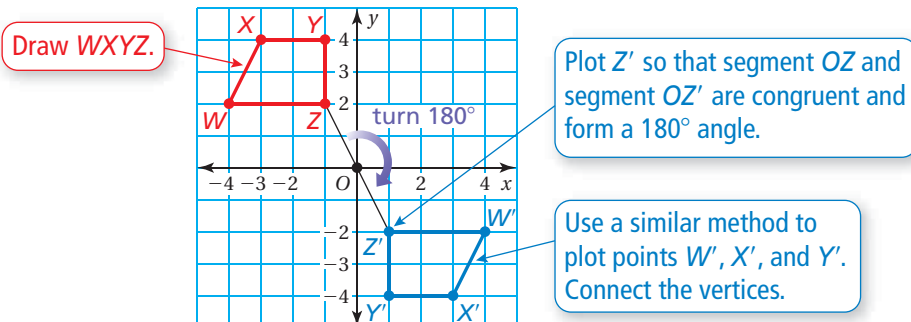
Multi-Language Glossary at BigIdeasMath.com

**EXAMPLE** **2** **Rotating a Figure**

The vertices of a trapezoid are $W(-4, 2), X(-3, 4), Y(-1, 4)$, and $Z(-1, 2)$. Rotate the trapezoid 180° about the origin. What are the coordinates of the image?

**Study Tip**

A 180° clockwise rotation and a 180° counterclockwise rotation have the same image. So, you do not need to specify direction when rotating a figure 180°.
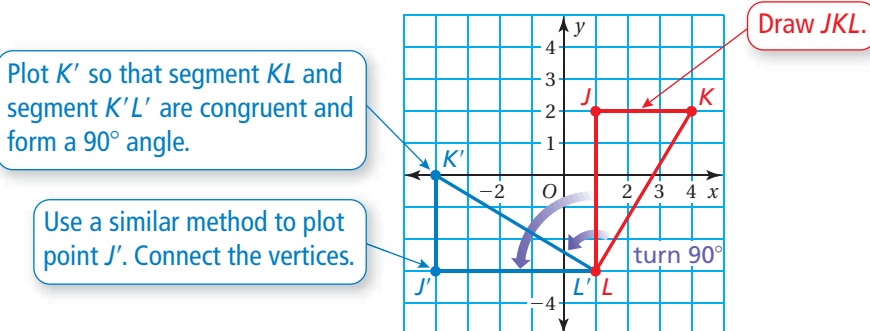
Draw *WXYZ*.

Plot *Z'* so that segment *OZ* and segment *OZ'* are congruent and form a 180° angle.

turn 180°

Use a similar method to plot points *W'*, *X'*, and *Y'*. Connect the vertices.

The coordinates of the image are $W'(4, -2), X'(3, -4)$, $Y'(1, -4)$, and $Z'(1, -2)$.

**EXAMPLE** **3** **Rotating a Figure**

The vertices of a triangle are $J(1, 2), K(4, 2)$, and $L(1, -3)$. Rotate the triangle 90° counterclockwise about vertex $L$. What are the coordinates of the image?

**Common Error**

Be sure to pay attention to whether a rotation is clockwise or counterclockwise.

Plot *K'* so that segment *KL* and segment *K'L'* are congruent and form a 90° angle.

Use a similar method to plot point *J'*. Connect the vertices.

Draw *JKL*.

turn 90°

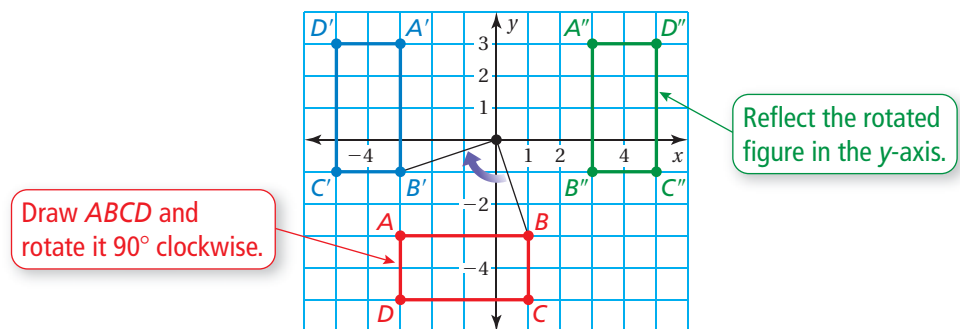The coordinates of the image are $J'(-4, -3), K'(-4, 0)$, and $L'(1, -3)$.

**On Your Own**

**Now You're Ready**
Exercises 13–18

**3.** A triangle has vertices $Q(4, 5), R(4, 0)$, and $S(1, 0)$.

   **a.** Rotate the triangle 90° counterclockwise about the origin.

   **b.** Rotate the triangle 180° about vertex $S$.

   **c.** Are the images in parts (a) and (b) congruent? Explain.

EXAMPLE 4 Using More than One Transformation

The vertices of a rectangle are $A(-3, -3)$, $B(1, -3)$, $C(1, -5)$, and $D(-3, -5)$. Rotate the rectangle 90° clockwise about the origin, and then reflect it in the $y$-axis. What are the coordinates of the image?

Reflect the rotated figure in the $y$-axis.

Draw *ABCD* and rotate it 90° clockwise.

⋮ The coordinates of the image are $A''(3, 3)$, $B''(3, -1)$, $C''$ $(5, -1)$ and $D''$ $(5, 3)$.

The image of a translation, reflection, or rotation is congruent to the original figure. So, two figures are congruent when one can be obtained from the other by a sequence of translations, reflections, and rotations.

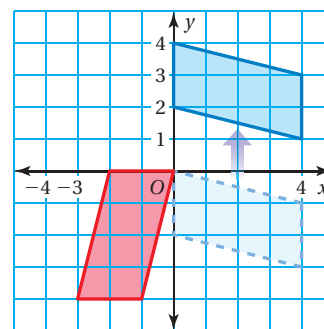EXAMPLE 5 Describing a Sequence of Transformations
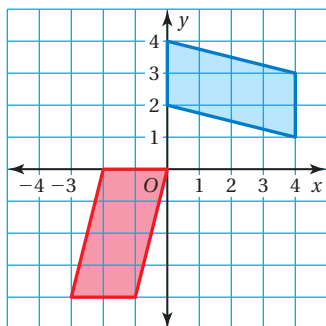
The red figure is congruent to the blue figure. Describe a sequence of transformations in which the blue figure is the image of the red figure.

You can turn the red figure 90° so that it has the same orientation as the blue figure. So, begin with a rotation.

After rotating, you need to slide the figure up.

⋮ So, one possible sequence of transformations is a 90° counterclockwise rotation about the origin followed by a translation 4 units up.

## On Your Own

Now You're Ready
Exercises 22–25

4. The vertices of a triangle are $P(-1, 2)$, $Q(-1, 0)$, and $R(2, 0)$. Rotate the triangle 180° about vertex $R$, and then reflect it in the $x$-axis. What are the coordinates of the image?

5. In Example 5, describe a different sequence of transformations in which the blue figure is the image of the red figure.

## ✓ Vocabulary and Concept Check

1. **VOCABULARY** What are the coordinates of the center of rotation in Example 2? Example 3?

**MENTAL MATH A figure lies entirely in Quadrant II. In which quadrant will the figure lie after the given clockwise rotation about the origin?**

2. 90° 　　　　　 3. 180° 　　　　　 4. 270° 　　　　　 5. 360°

6. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What are the coordinates of the figure after a 90° clockwise rotation about the origin?

What are the coordinates of the figure after a 270° clockwise rotation about the origin?

What are the coordinates of the figure after turning the figure 90° to the right about the origin?
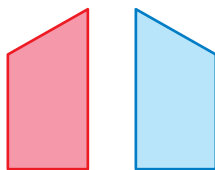
What are the coordinates of the figure after a 270° counterclockwise rotation about the origin?
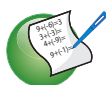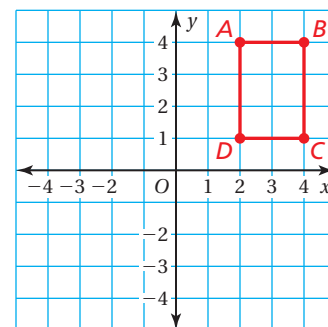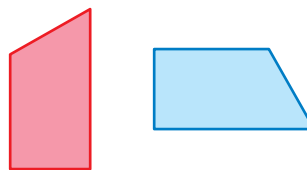
## Practice and Problem Solving

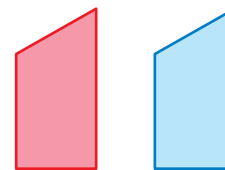**Identify the transformation.**

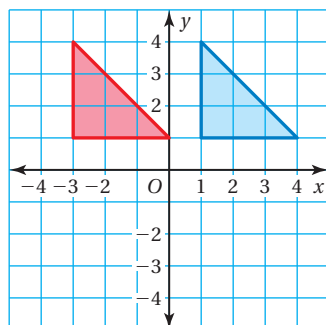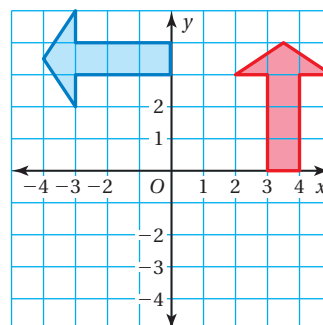7. 　　　　　　　 8. 　　　　　　　 9.

**Tell whether the blue figure is a rotation of the red figure about the origin. If so, give the angle and direction of rotation.**
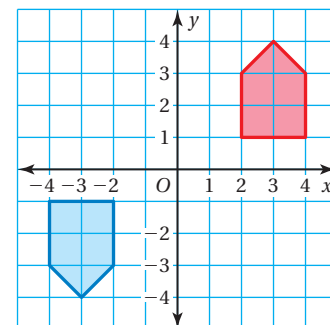
① 10. 　　　　　　 11. 　　　　　　 12.

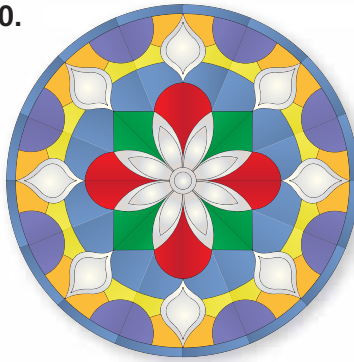**The vertices of a figure are given. Rotate the figure as described. Find the coordinates of the image.**

② ③ **13.** $A(2, -2), B(4, -1), C(4, -3), D(2, -4)$
90° counterclockwise about the origin

**14.** $F(1, 2), G(3, 5), H(3, 2)$
180° about the origin

**15.** $J(-4, 1), K(-2, 1), L(-4, -3)$
90° clockwise about vertex $L$

**16.** $P(-3, 4), Q(-1, 4), R(-2, 1), S(-4, 1)$
180° about vertex $R$

**17.** $W(-6, -2), X(-2, -2), Y(-2, -6), Z(-5, -6)$
270° counterclockwise about the origin

**18.** $A(1, -1), B(5, -6), C(1, -6)$
90° counterclockwise about vertex $A$

**A figure has *rotational symmetry* if a rotation of 180° or less produces an image that fits exactly on the original figure. Explain why the figure has rotational symmetry.**
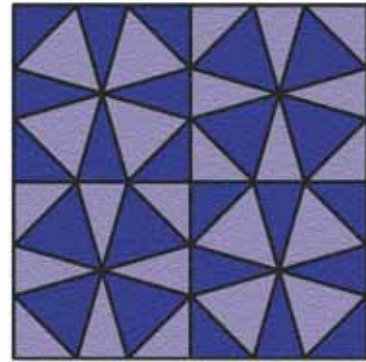
**19.**
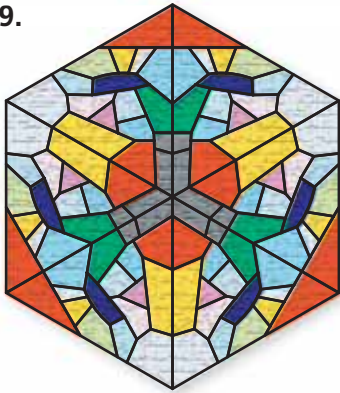


**20.**



**21.**



**The vertices of a figure are given. Find the coordinates of the figure after the transformations given.**
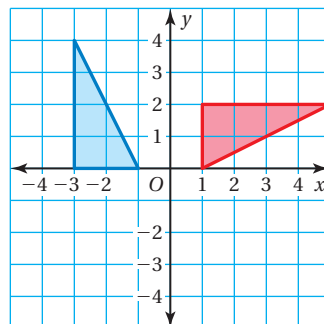
④ **22.** $R(-7, -5), S(-1, -2), T(-1, -5)$

Rotate 90° counterclockwise about the origin. Then translate 3 units left and 8 units up.

**23.** $J(-4, 4), K(-3, 4), L(-1, 1), M(-4, 1)$

Reflect in the *x*-axis, and then rotate 180° about the origin.

**The red figure is congruent to the blue figure. Describe two different sequences of transformations in which the blue figure is the image of the red figure.**
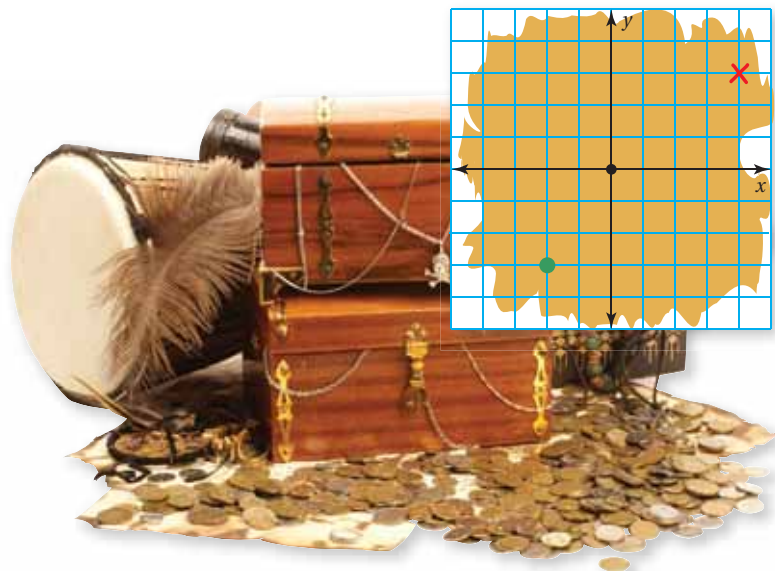
⑤ **24.**
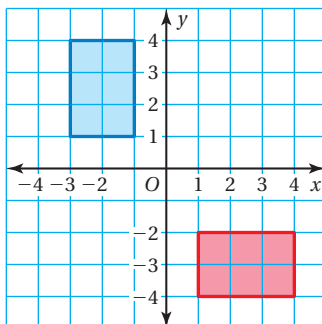


**25.**

**26. REASONING** A trapezoid has vertices $A(-6, -2)$, $B(-3, -2)$, $C(-1, -4)$, and $D(-6, -4)$.

    **a.** Rotate the trapezoid 180° about the origin. What are the coordinates of the image?

    **b.** Describe a way to obtain the same image without using rotations.

**27. TREASURE MAP** You want to find the treasure located on the map at ✕. You are located at ●. The following transformations will lead you to the treasure, but they are not in the correct order. Find the correct order. Use each transformation exactly once.

- Rotate 180° about the origin.
- Reflect in the $y$-axis.
- Rotate 90° counterclockwise about the origin.
- Translate 1 unit right and 1 unit up.

**28. CRITICAL THINKING** Consider $\triangle JKL$.

    **a.** Rotate $\triangle JKL$ 90° clockwise about the origin. How are the $x$- and $y$-coordinates of $\triangle J'K'L'$ related to the $x$- and $y$-coordinates of $\triangle JKL$?

    **b.** Rotate $\triangle JKL$ 180° about the origin. How are the $x$- and $y$-coordinates of $\triangle J'K'L'$ related to the $x$- and $y$-coordinates of $\triangle JKL$?

    **c.** Do you think your answers to parts (a) and (b) hold true for any figure? Explain.
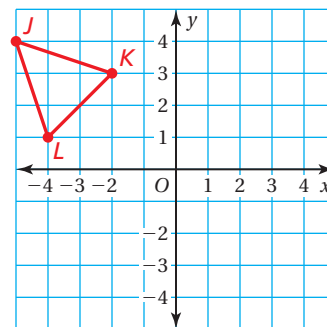
**29. Reasoning** You rotate a triangle 90° counterclockwise about the origin. Then you translate its image 1 unit left and 2 units down. The vertices of the final image are $(-5, 0)$, $(-2, 2)$, and $(-2, -1)$. What are the vertices of the original triangle?

## Fair Game Review *What you learned in previous grades & lessons*

**Tell whether the ratios form a proportion.** *(Skills Review Handbook)*

**30.** $\dfrac{3}{5}, \dfrac{15}{20}$      **31.** $\dfrac{2}{3}, \dfrac{12}{18}$      **32.** $\dfrac{7}{28}, \dfrac{12}{48}$      **33.** $\dfrac{54}{72}, \dfrac{36}{45}$

**34. MULTIPLE CHOICE** What is the solution of the equation $x + 6 \div 2 = 5$? *(Section 1.1)*

    (**A**) $x = -16$      (**B**) $x = 2$      (**C**) $x = 4$      (**D**) $x = 16$