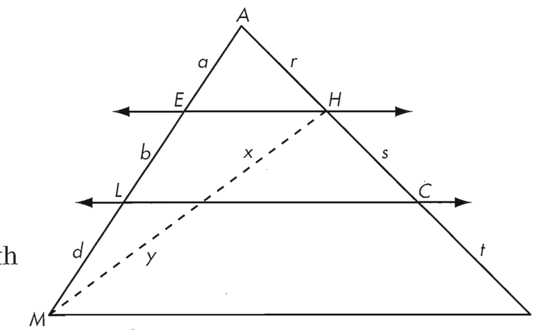
3.* The Extended Parallel Proportionality Conjecture can be proven with the help of an additional line, as shown below.

The conjecture states: If two or more lines pass through two sides of a triangle parallel to the third side, then they divide the two sides proportionally.



Conjecture: If $\overleftrightarrow{EH} \parallel \overleftrightarrow{LC} \parallel \overline{MI}$, then $\frac{b}{d} = \frac{s}{t}$.

Given: $\overleftrightarrow{EH} \parallel \overleftrightarrow{LC} \parallel \overline{MI}$

Show: $\frac{b}{d} = \frac{s}{t}$

Because you must connect *b*, *d*, *s*, and *t* into one equation, you construct $\overline{MH}$. This gives you ΔHEM with $\overline{LC} \parallel \overline{EH}$ and ΔMIH with $\overline{LC} \parallel \overline{MI}$. Write a proof for the Extended Parallel Proportionality Conjecture.

4.* Is the converse of the Extended Parallel Proportionality Conjecture true? That is, if two lines intersect two sides of a triangle, dividing the two sides proportionally, must the two lines be parallel to the third side? Prove it is true or find a counterexample proving it is not true.

5. Sketch and label a plane intersecting a pyramid parallel to the base of the pyramid. State a conjecture about planes intersecting pyramids parallel to the base of the pyramid. Test your conjecture. Can you explain why you think your conjecture is true?

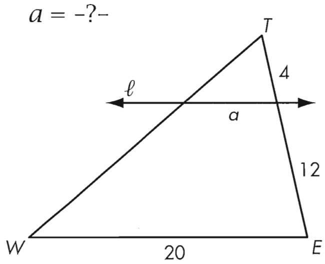
6. If the three sides of one triangle are parallel to the three sides of another triangle, what might be true about the two triangles? Use a geometry computer program to investigate. Make a conjecture and explain why you think your conjecture is true.

Exercise Set 12.7

All measurements are in centimeters.

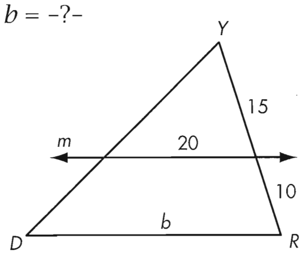
1.* $\ell \parallel \overline{WE}$

$a = $ -?-



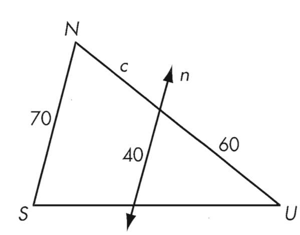
2. $m \parallel \overline{DR}$

$b = $ -?-



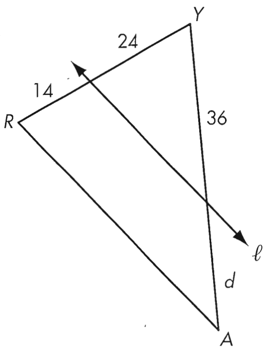
3.* $n \parallel \overline{SN}$

$c = $ -?-



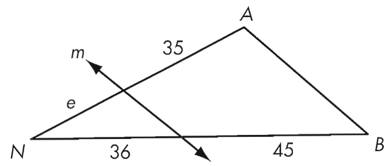
4.* $\ell \parallel \overline{RA}$

d = -?-

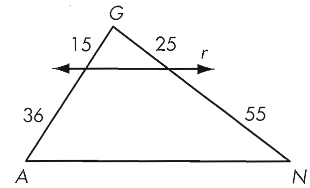


5. $m \parallel \overline{BA}$

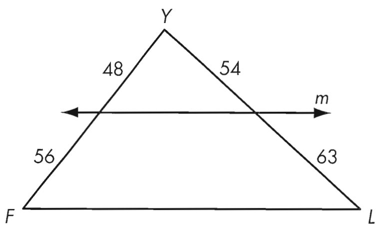
e = -?-



6.* Is $r \parallel \overline{AN}$?

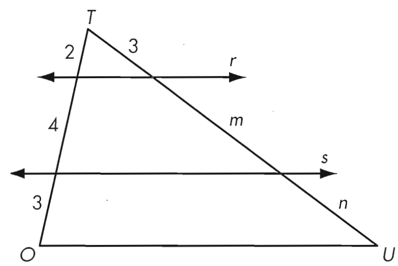


7. Is $m \parallel \overline{FL}$?



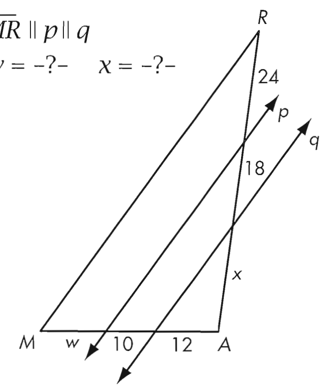
8. $r \parallel s \parallel \overline{OU}$

m = -?- n = -?-



9. $\overline{MR} \parallel p \parallel q$

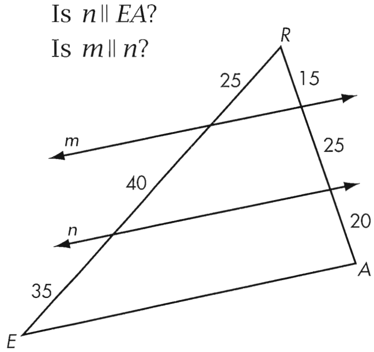
w = -?- x = -?-



10. Is $m \parallel \overline{EA}$?

Is $n \parallel \overline{EA}$?

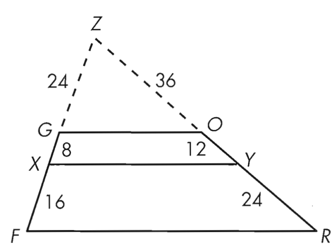
Is $m \parallel n$?



11. Is $\overline{XY} \parallel \overline{OG}$?

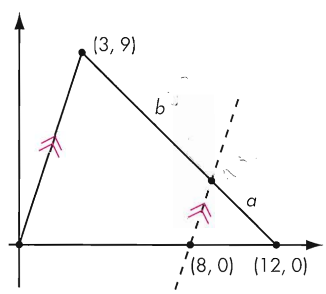
Is $\overline{XY} \parallel \overline{FR}$?

Is *FROG* a trapezoid?



12.* a = -?-

b = -?-



In Exercises 13 and 14, use your compass and straightedge to complete each task.

13. Draw $\overline{EF}$. Then divide it into five equal parts.

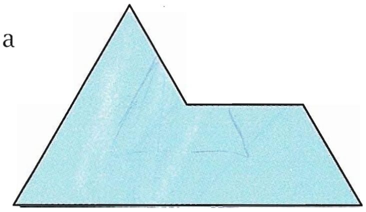
14.* Draw $\overline{IJ}$. Then construct a regular hexagon with IJ as the perimeter.

15. The ratio of the volumes of two similar cylinders is 27:64. What is the ratio of the diameters of their similar bases?

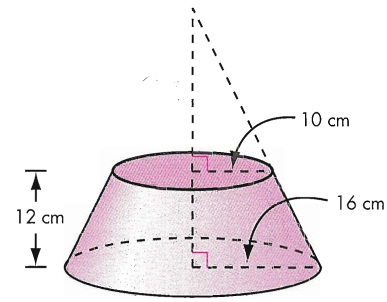
16.* The surface areas of two cubes are in the ratio of 49:81. What is the ratio of their volumes?

17.* A sheet of lined paper or graph paper can be used to divide a segment into equal parts. Draw a segment on a piece of patty paper and divide the segment into five equal parts by positioning it onto lined paper or graph paper. What conjecture explains why this works?

18. In Lesson 12.5, you saw how it is possible to divide a triangle into four similar triangles, a parallelogram into four similar parallelograms, and a special trapezoid into four similar trapezoids. These figures are called **rep-tiles**. A rep-tile is a figure that can be repeated to form a larger similar figure (a replica). A rep-tile can also be divided into figures smaller than but similar to itself. Copy the figure at right onto your paper and show how it can be divided into four similar figures.

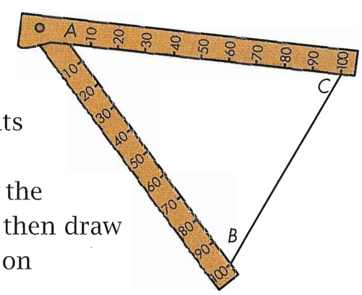


19.* The truncated cone shown at right was formed by cutting off the top of a cone with a slice parallel to the base of the cone. What is the volume of the truncated cone?

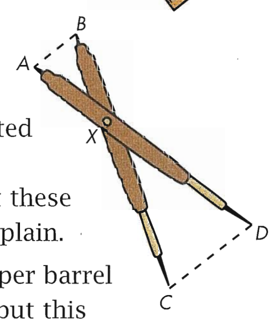


20. Romunda is preparing her specialty, *les cannonballs chocolates,* for this evening's guests of honor, Brian and Maggie. In the original recipe, her special cookie dough is rolled into 36 cannonballs (spheres with 4-cm diameters), then covered in finely ground hazelnuts. However, Romunda has decided to double the recipe. With twice the amount of dough, she reasons that she can still make 36 spheres but they will now be spheres with 8-cm diameters. Is she correct? If not, how many 8-cm diameter spheres can she make by doubling the recipe?

21.* Galileo Galilei (1564–1642) used the drafting instrument shown at right. Called a sector compass, it consists of two sides, each of which displays equal scale. The sector compass is used to construct segments that are some fraction of a given segment. If you wish to construct a segment that is three fourths (or 75%) of a given segment, you adjust the sector compass so that the segment fits between the 100-marks. You then draw the segment connecting the two —?— on the two scales. What points on the compass should you connect? Explain why this works.



22.* Another drafting instrument used to construct segments is the pair of proportional dividers, shown at right. This instrument consists of two styluses of equal length connected by a set screw. The dividers are adjusted for different proportions by loosening and sliding the set screw along grooves in the dividers. Where should the set screw be positioned so that these dividers can make segments that are three fourths of given segments? Explain.



23. The graph on the following page shows one-dimensional data—the price per barrel of light crude oil leaving Saudi Arabia each January from 1973 to 1979—but this data is represented by a three-dimensional display. The heights are supposed to represent the price per barrel, but instead the volumes of the barrels seem to represent the data, which is misleading because the barrels are growing in height, width, and depth. It's even difficult to correctly perceive the changes in height because the barrels are drawn in perspective.

The ratio of the light crude prices in 1979 and 1975 is $\frac{\$13.34}{\$10.48}$, which is