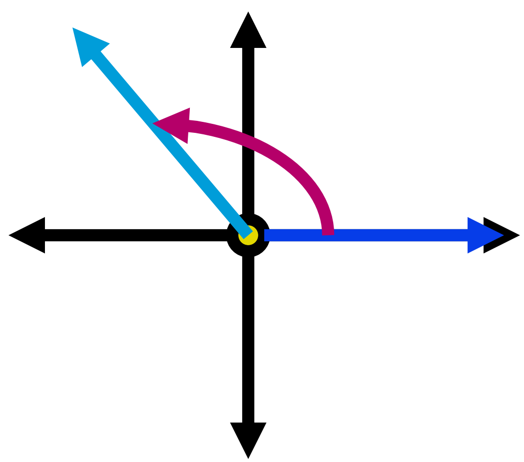
Unit Circle 1

Reference Angles, Coterminal Angles, and the Unit Circle

Angles of Rotation and Radian Measure

Previously, we looked at sine, cosine, and tangent with angles that were acute. In this section, we will look at ____________ ____ ____________ whose measure can be bigger than 360 degrees or negative.

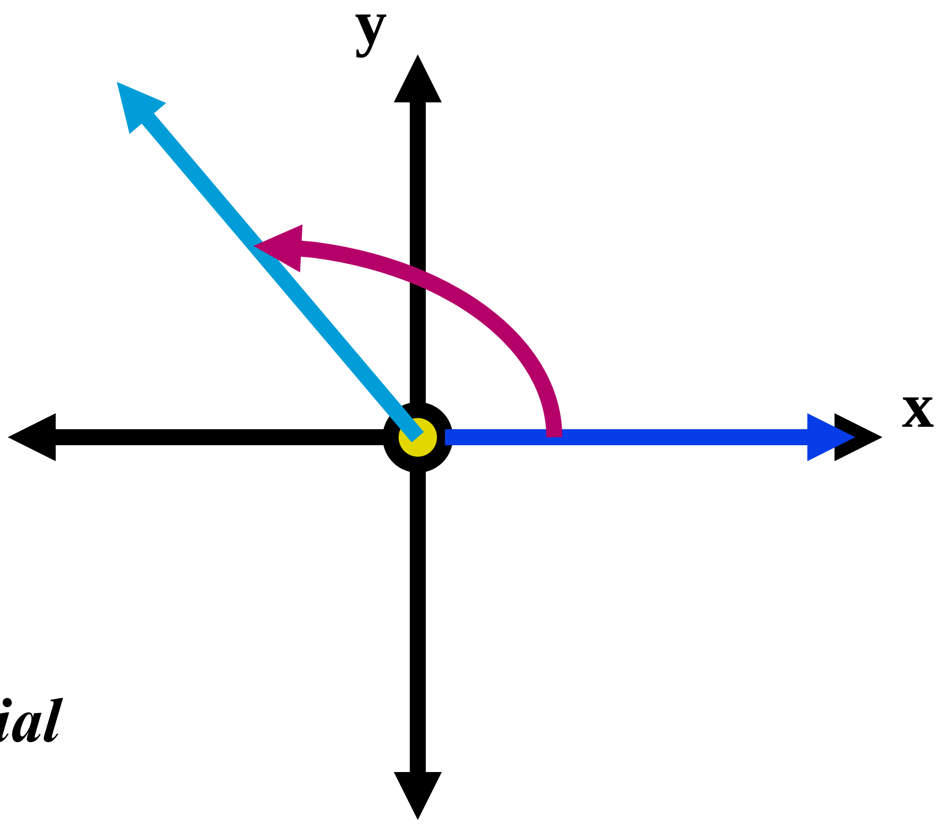
Usually when talking about angles that are either bigger than 90 degrees or negative, we use the coordinate plane to help us. These angles are called angles of rotation



One ray is called the

______________________.

The other ray is called the ______________________.

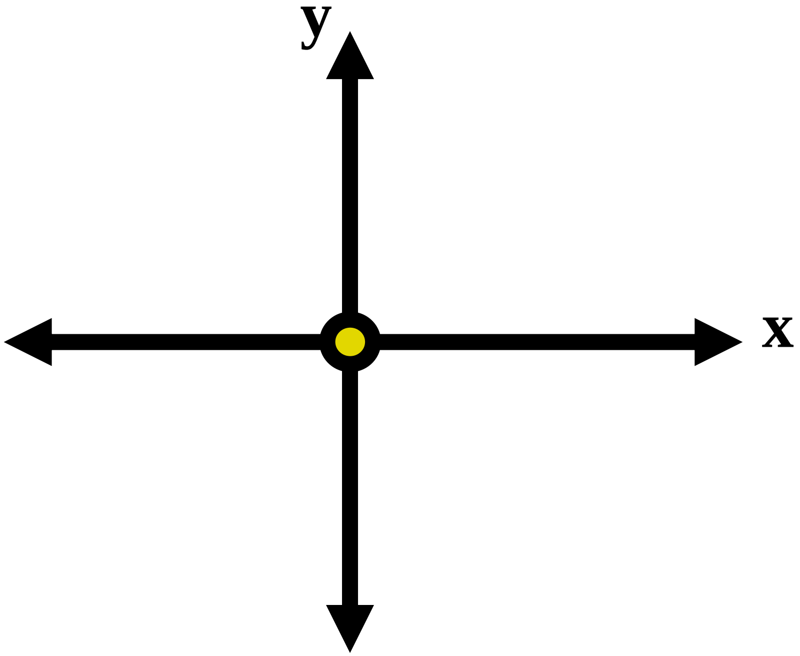


*The **measure of the angle** is determined by the amount and direction of rotation from the initial side to the terminal side.*

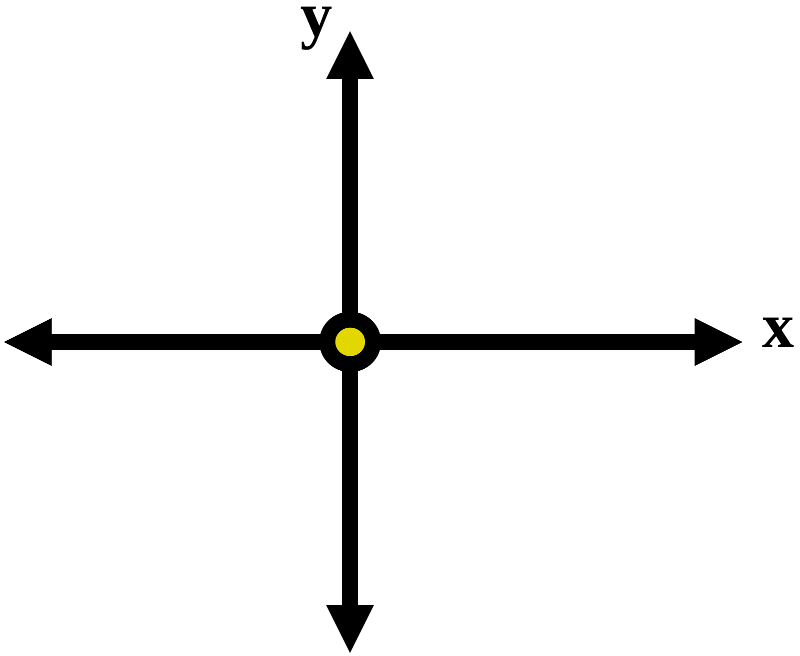
The angle measure is positive if the rotation is counterclockwise, and negative if the rotation is clockwise.
A full revolution (counterclockwise) corresponds to 360º.

Examples

1) 45º

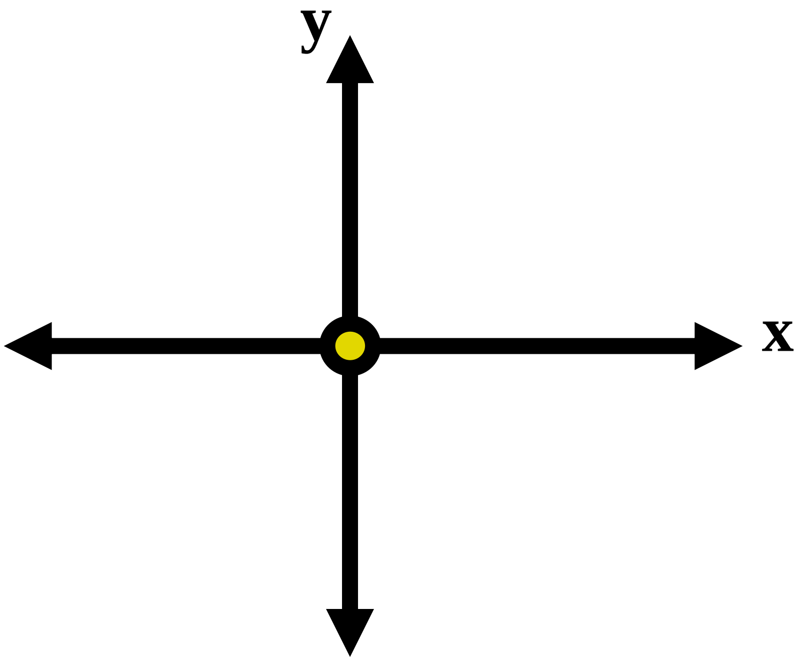


2) 90º

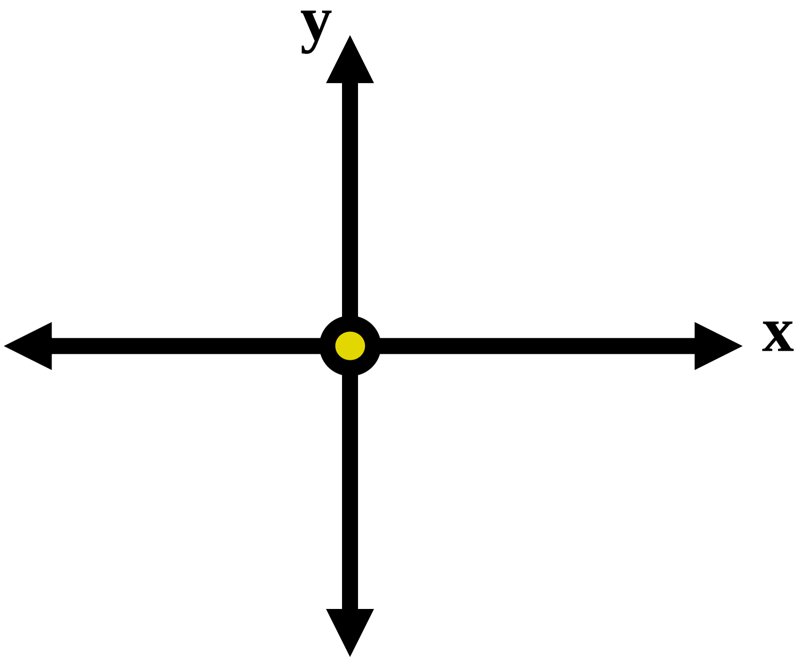


Examples

3) 120º

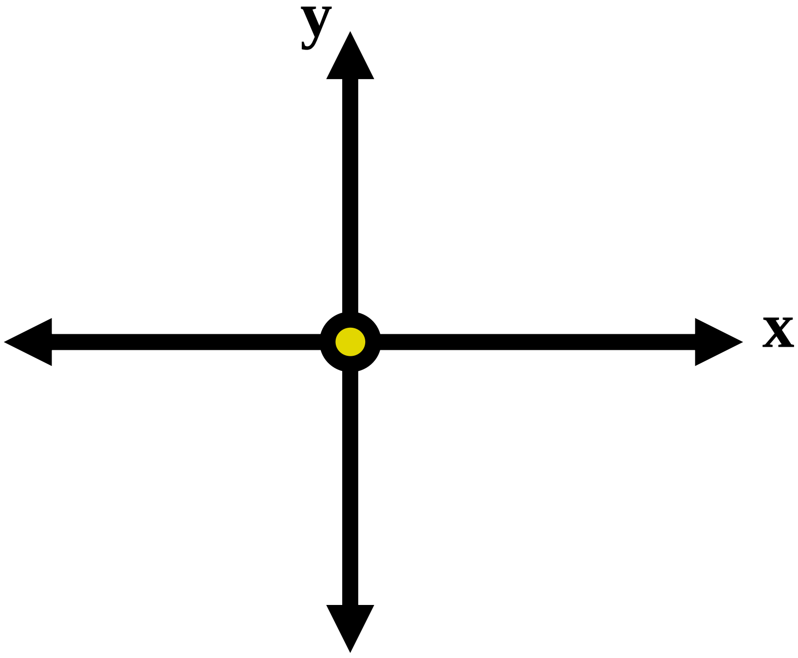


4) 225º

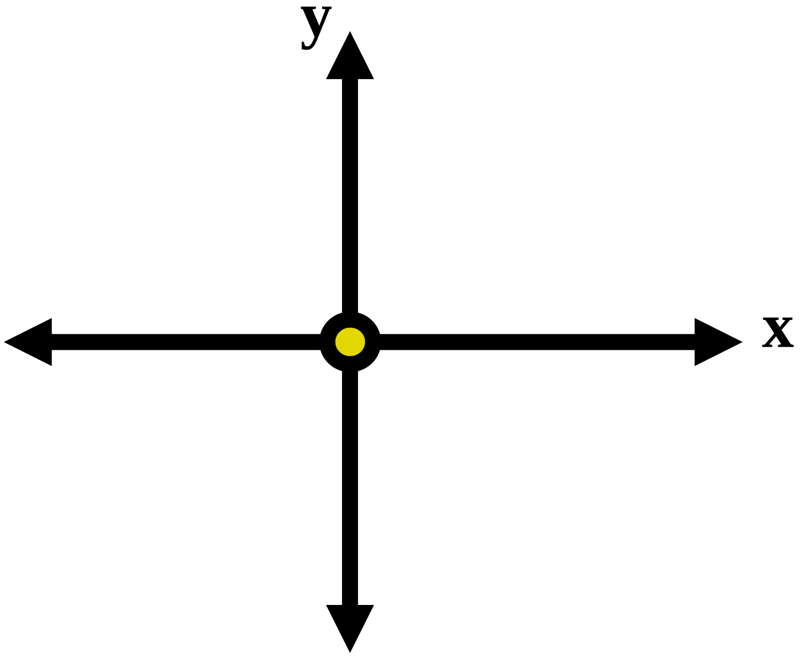


Examples

5) 330º

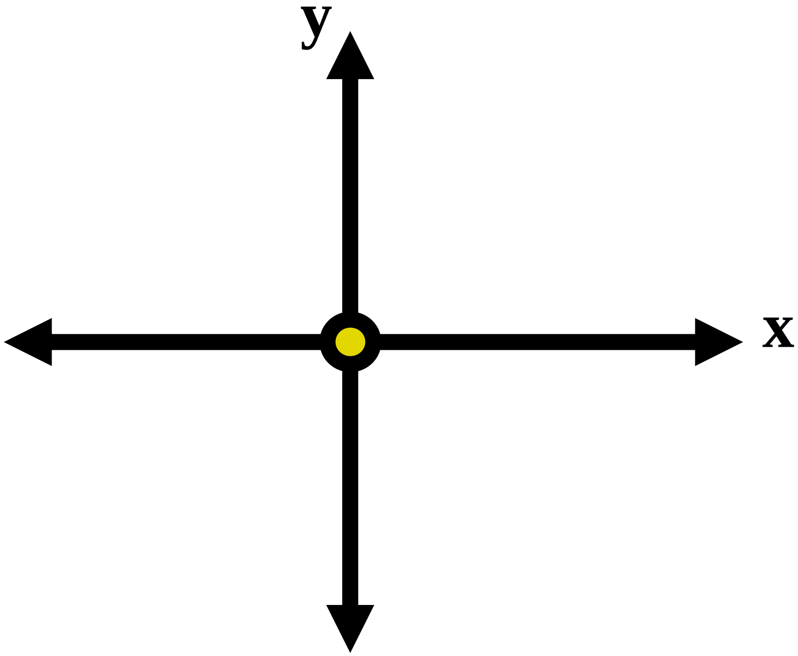


6) -60º

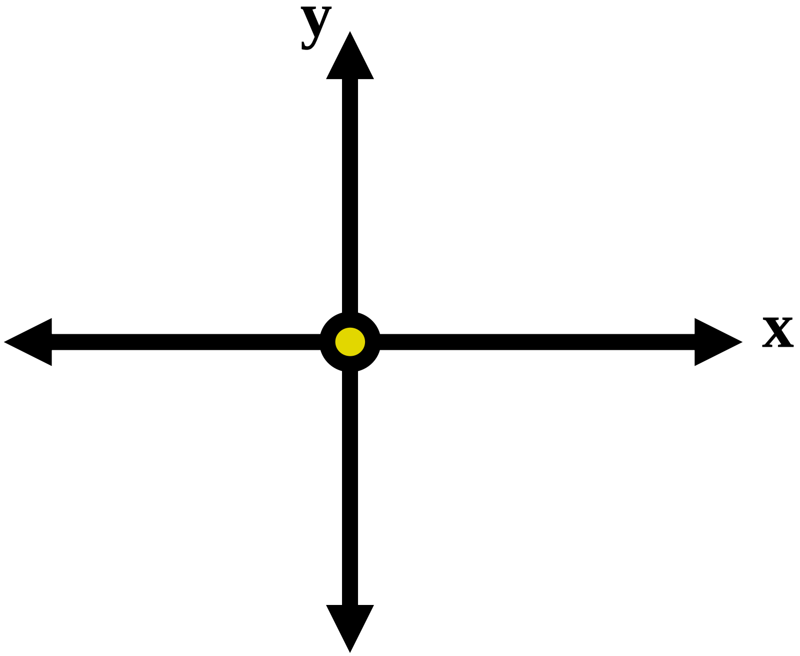


Examples

7) -210º

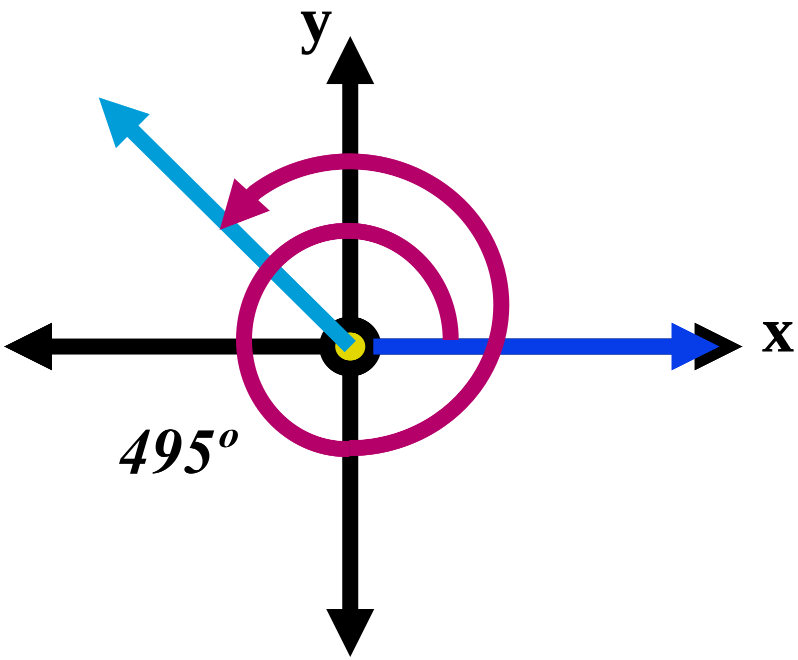


8) 495º



COTERMINAL ANGLES

8) 495°



Note that 495° and 135° are basically the same terminal side.

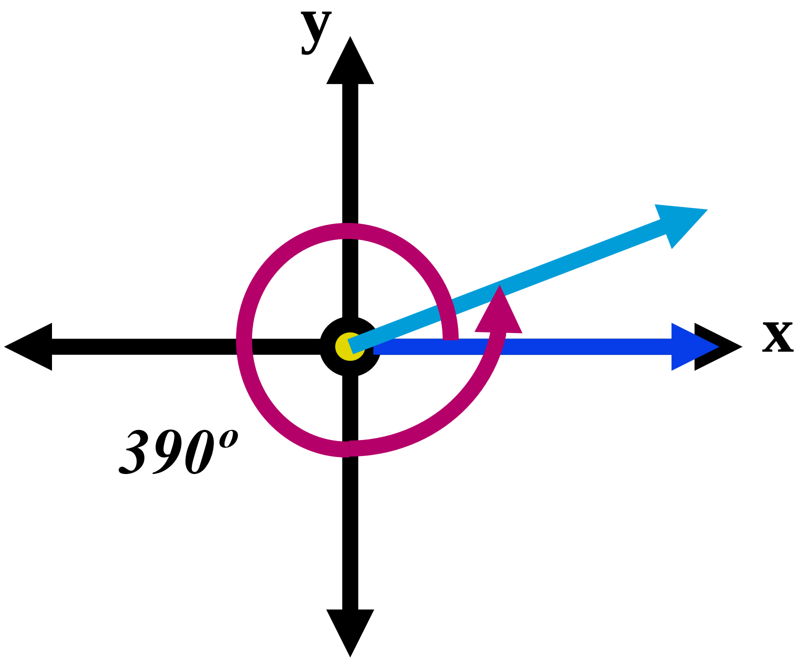
495° and 135° are called coterminal (their terminal sides coincide).

510° and 150° likewise are coterminal.

An angle coterminal with a given angle can be found by adding or subtracting multiples of 360°.

Examples

9) 390°



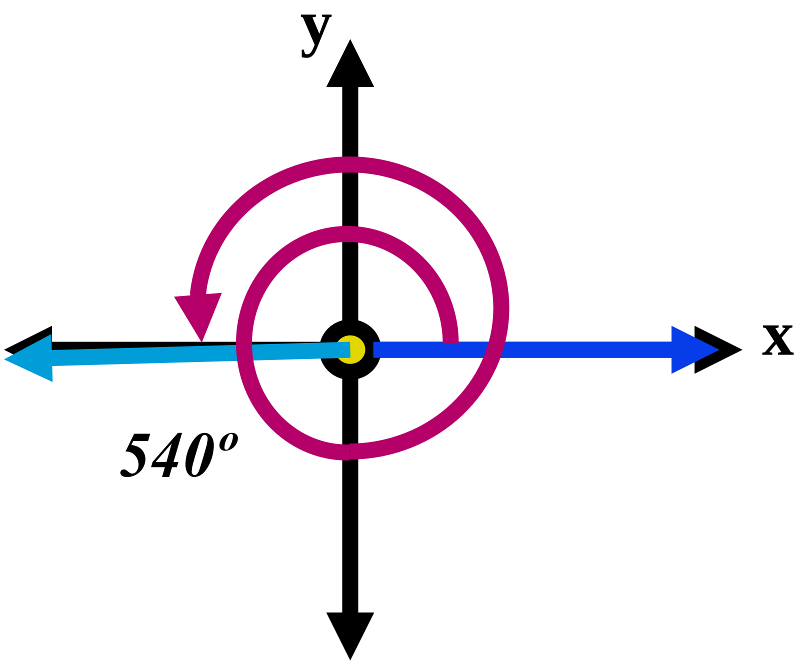
A coterminal angle also must be between -360 and 360°

Find the coterminal angle of the given angle to the left.

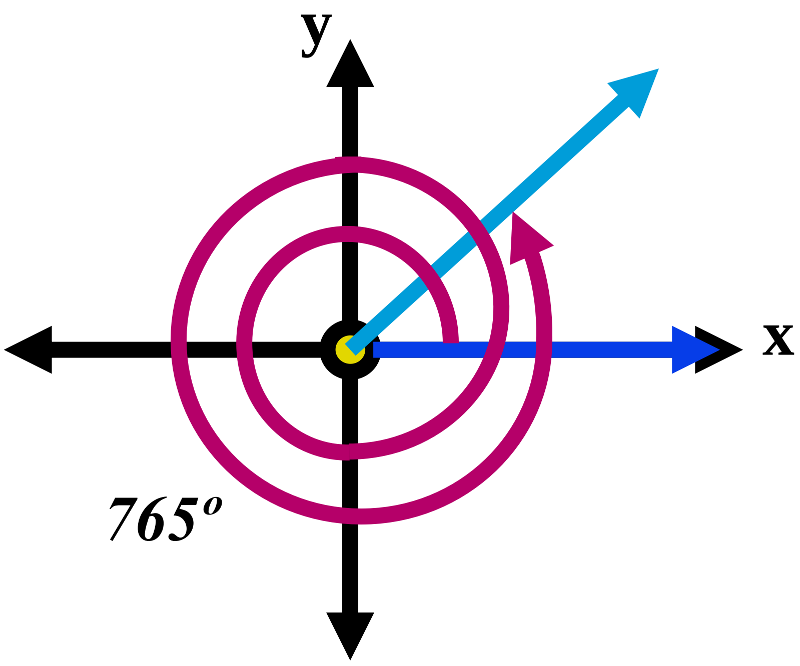
Examples

Find the coterminal angles of the following:

10) 540°



11) 765°



REFERENCE ANGLES

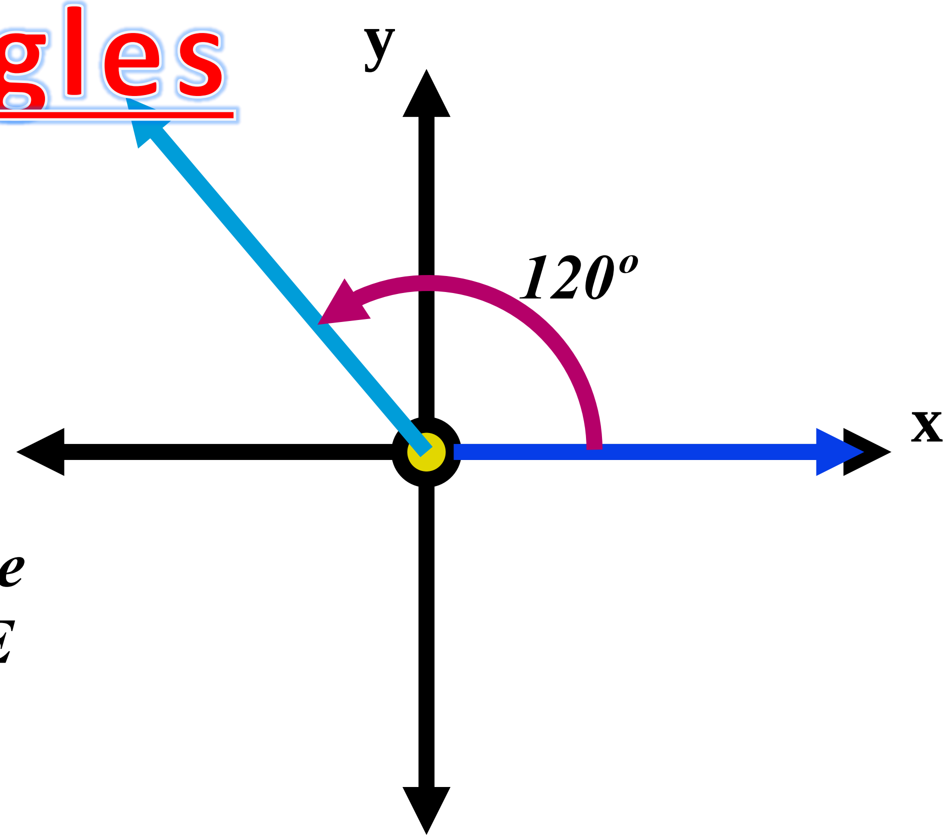
***Reference angles* are very useful things to use when we are find the sine, cosine, and tangent of an angle.**

This is a little different from coterminal angles.

Reference Angles

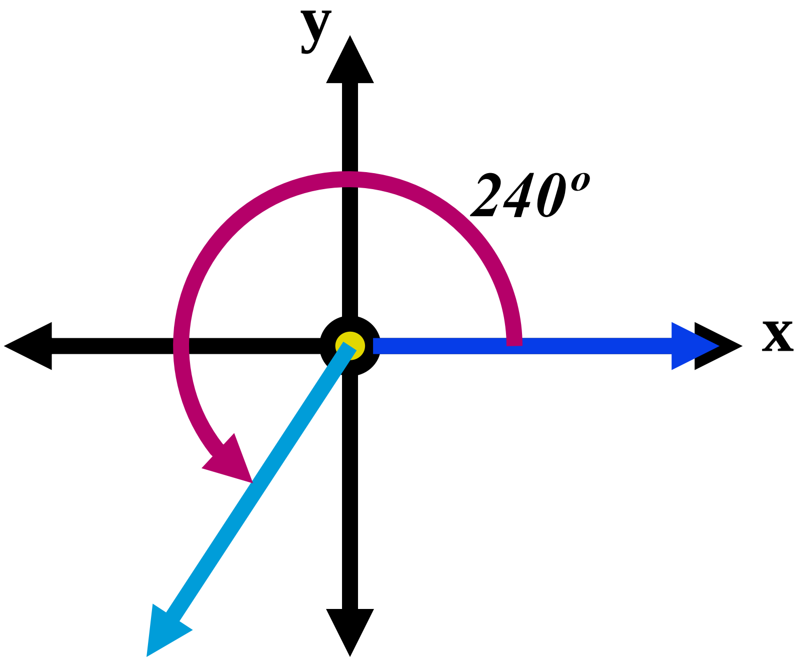
You only need reference angles if the angle is greater than 90º.

*The **reference angle** for a given angle is made by making an angle that is the CLOSEST DISTANCE THAT YOU NEED TO TRAVEL TO GET FROM THE X-AXES ON EITHER THE POSITIVE OR NEGATIVE SIDE TO THE TERMINAL SIDE OF THE ANGLE.*

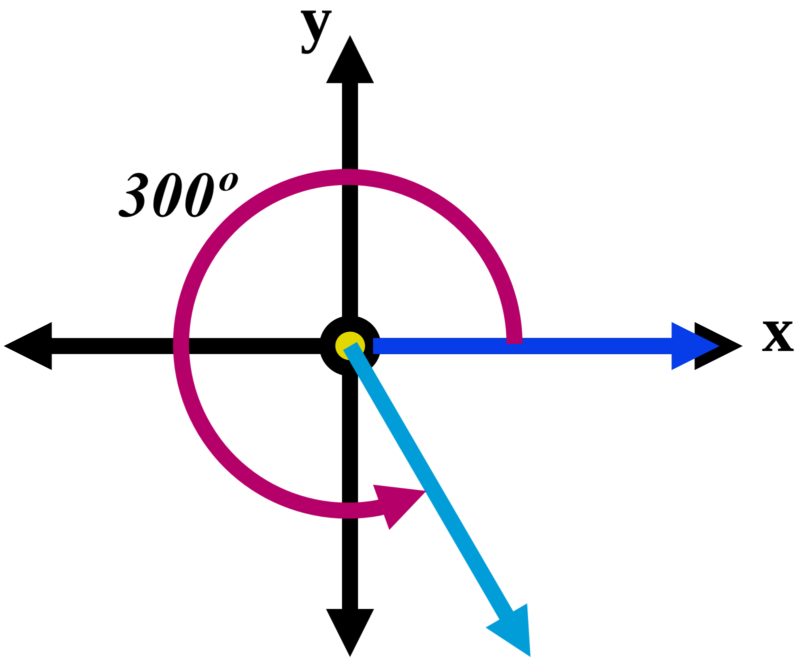


Examples

1) 240º

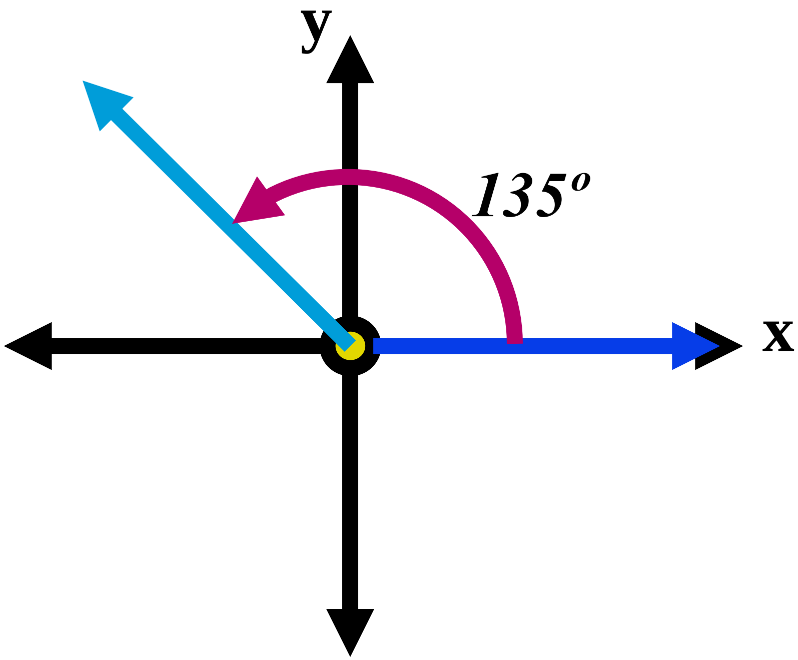


2) 300º

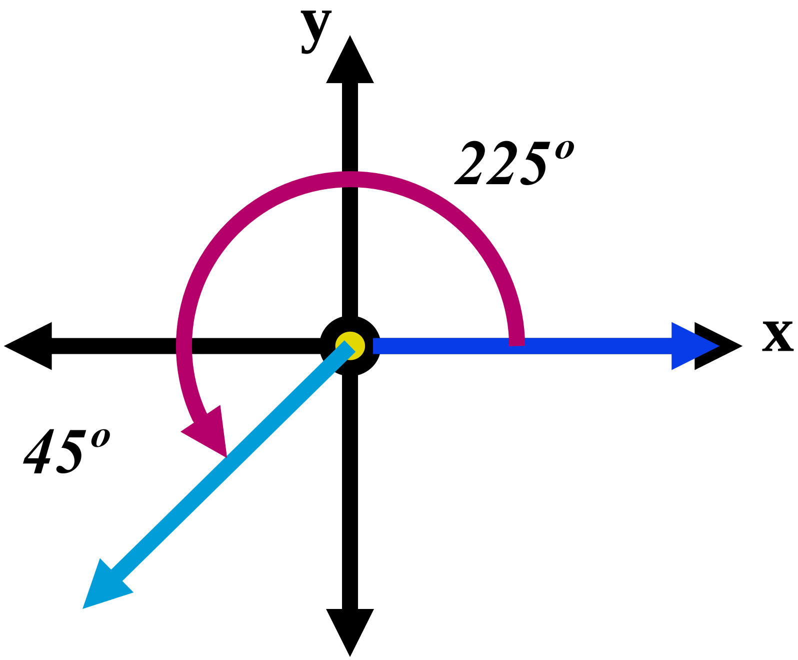


Examples

3) 135°

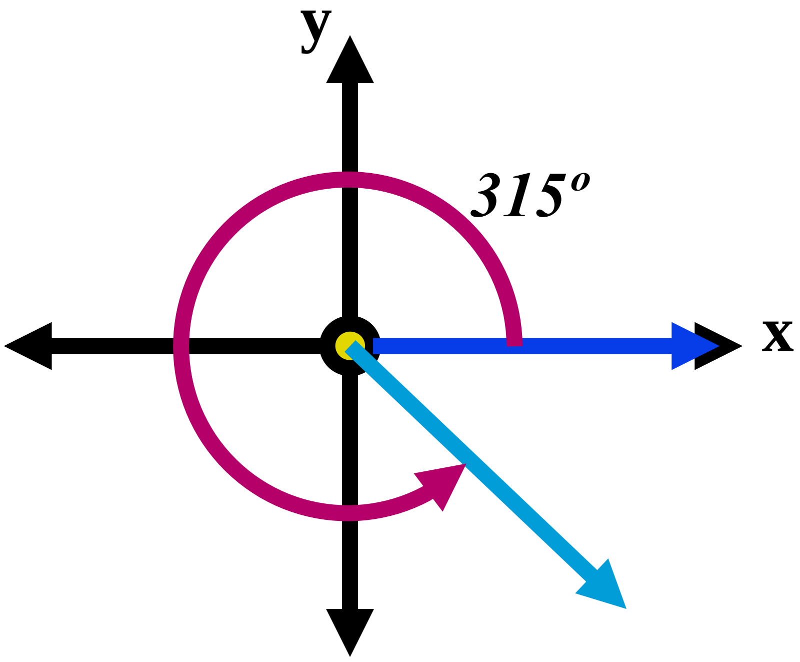


4) 225°

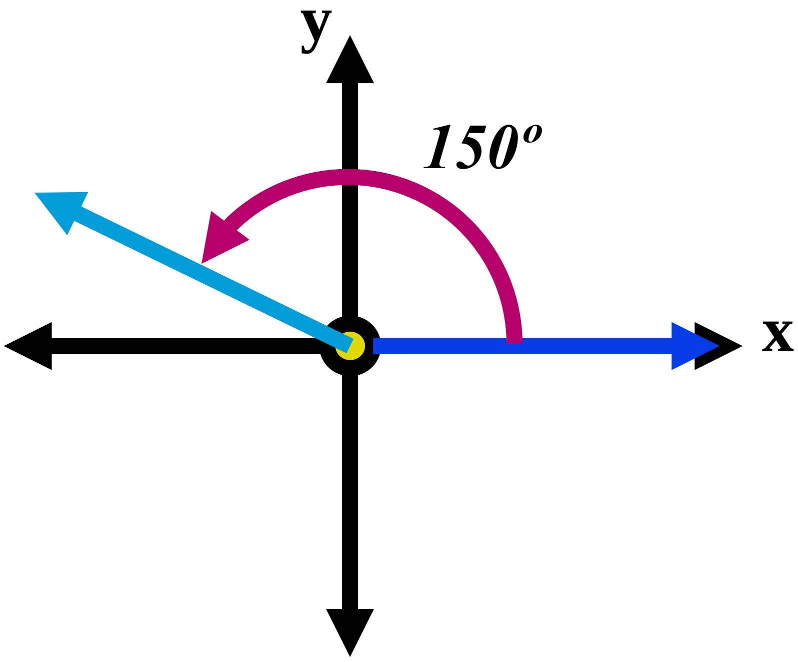


Examples

5) 315º

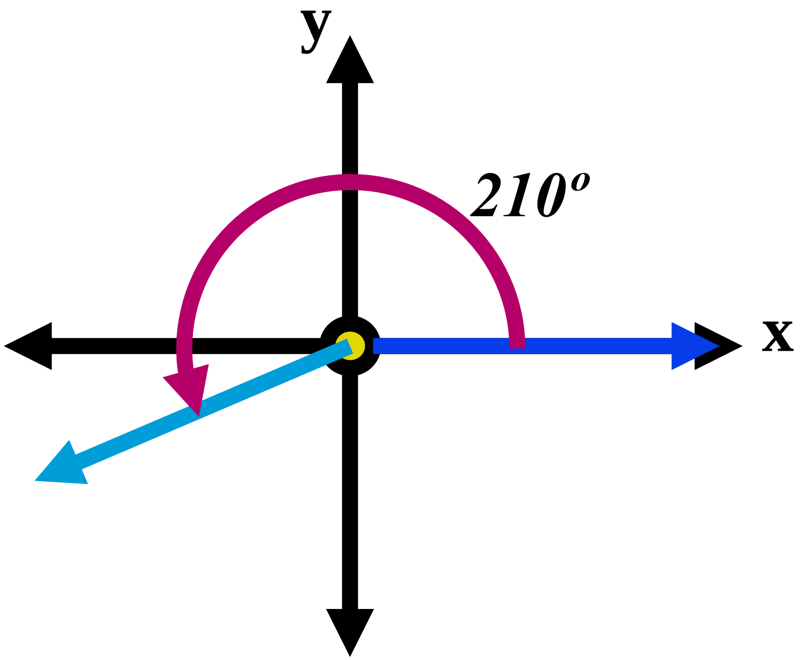


6) 150º

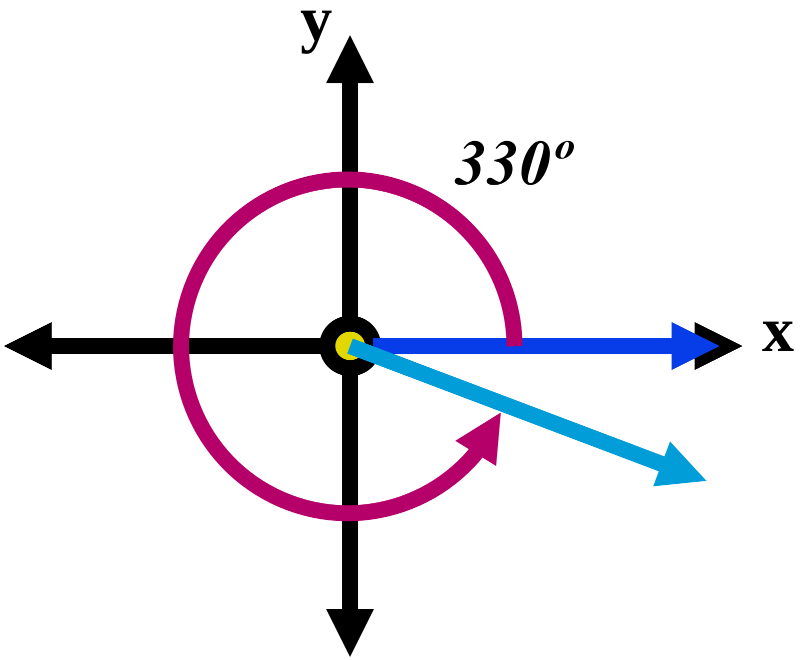


Examples

7) 210°

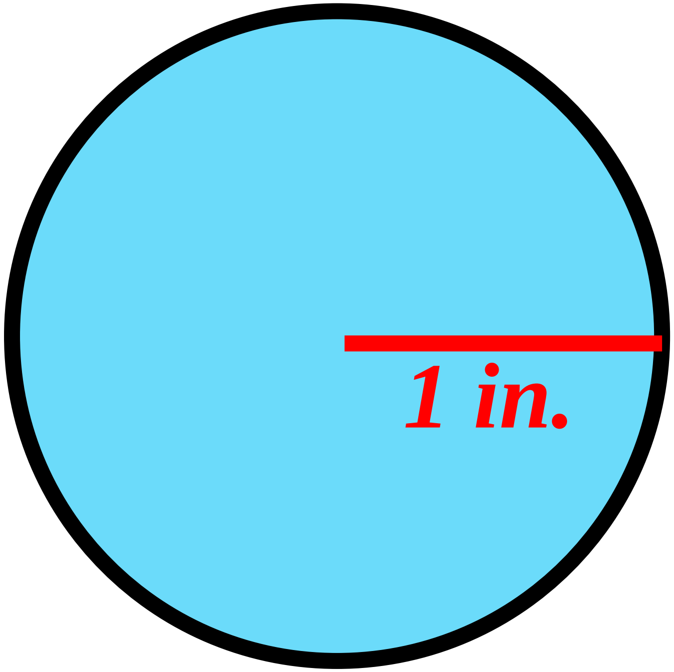


8) 330°



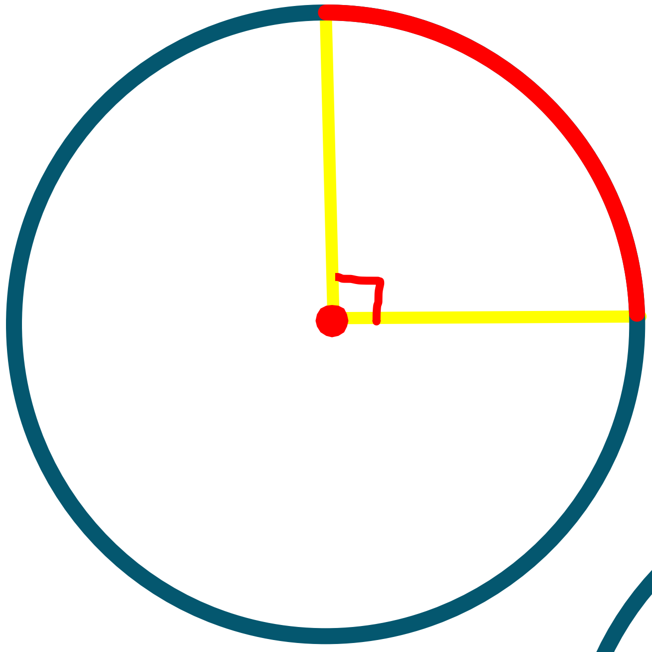
REVIEW

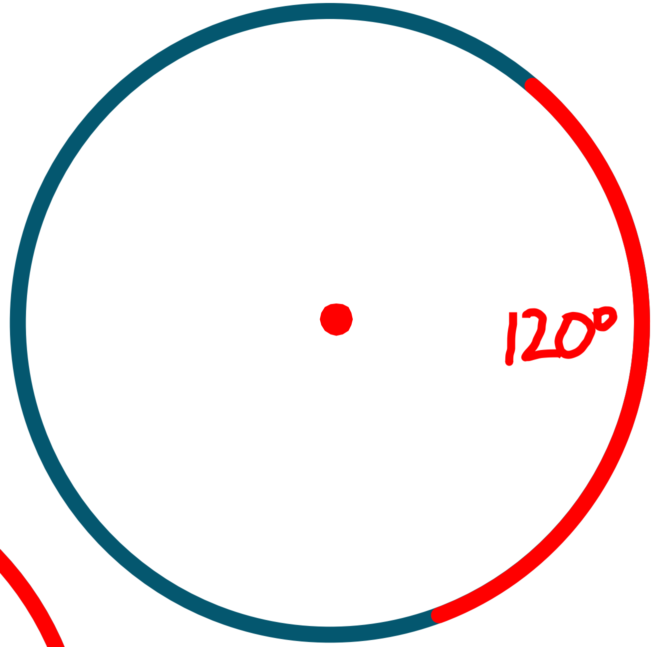
1) Find the circumference of the following circle in exact form:

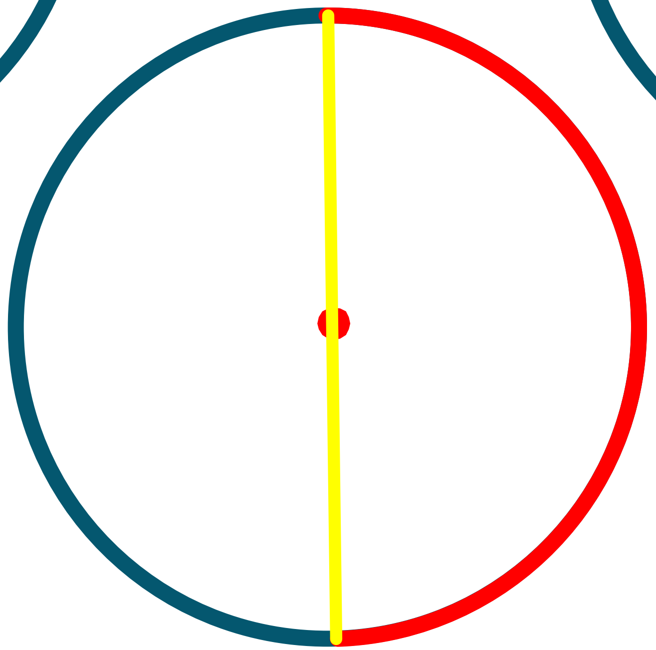


Understanding previous concepts

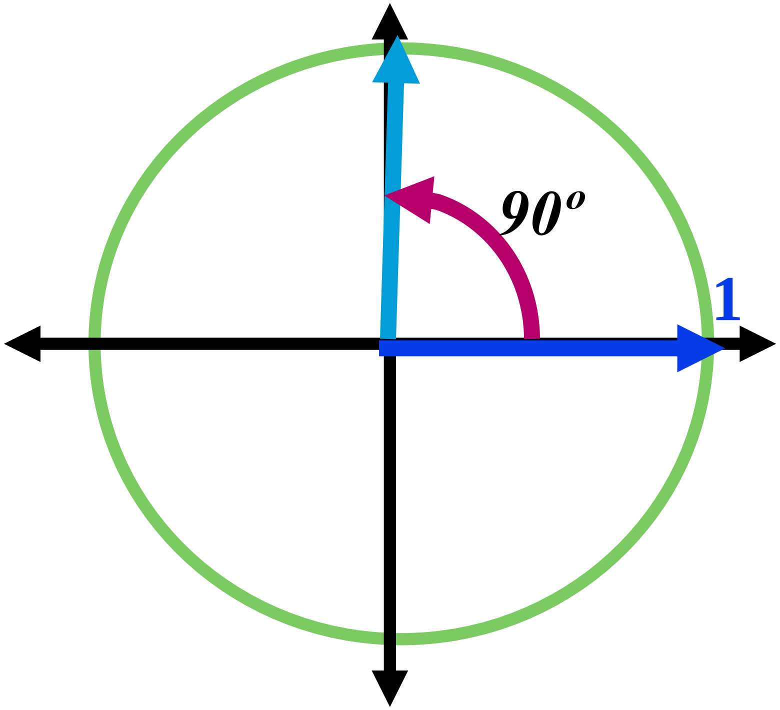
In the following, the radius of each circle is 1 unit. Find the arc length of each arc in exact form.







Unit Circle



***Radians* are basically the arc length of an angle on a circle that has a radius of 1.**

How many radians is 90°?

What is the circumference of the circle to the left?

Since the circumference of the circle is 2π, there are 2π radians in a full circle.

Degree measure and radian measure are therefore related by the following:

$360^{\circ} = 2\pi$ *radians*

Converting Degrees to Radians

One way to convert degrees into radians is simply to find the arclength of a circle with a radius of 1.

a) ***Convert 60º into radians***

There is another short cut. It's very similar however you do less work. It basically skips some reducing.

$$\text{Radians} = \text{Degrees} \times \frac{\pi}{180}$$

Examples

Rewrite each in radians

12. 30^o

13. 90^o

14. 45^o

15. 180^o

16. -120^o

17. 135^o

Converting Radians to Degrees

$$\textbf{Degrees} = \textbf{Radians X } \frac{180}{\pi}$$

Examples

Rewrite each in degrees.

18. $\dfrac{\pi}{2}$

19. $\dfrac{\pi}{6}$

20. $\dfrac{3\pi}{4}$

21. $\dfrac{5\pi}{3}$

22. $\dfrac{3\pi}{2}$

23. $\dfrac{5\pi}{6}$