2 Solving Linear Inequalities

Camel Physiology *(p. 91)*

Mountain Plant Life *(p. 85)*

Digital Camera *(p. 70)*

Microwave Electricity *(p. 64)*

Natural Arch *(p. 59)*

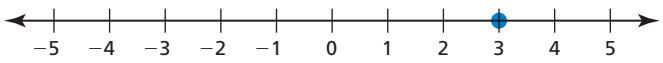
Mathematical Practices: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*

Maintaining Mathematical Proficiency

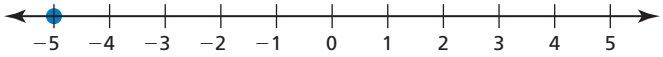
Graphing Numbers on a Number Line (6.NS.C.6c)

Example 1 **Graph each number.**

a. 3

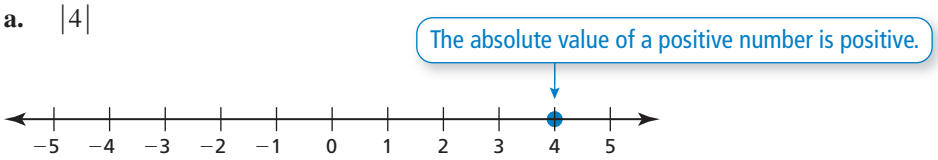


b. -5

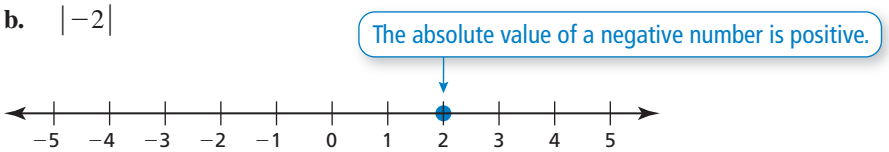


Example 2 **Graph each number.**

a. $|4|$



b. $|-2|$



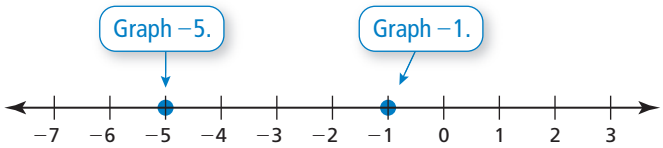
Graph the number.

1. 6 **2.** $|2|$ **3.** $|-1|$

4. $2 + |-2|$ **5.** $1 - |-4|$ **6.** $-5 + |3|$

Comparing Real Numbers (6.NS.C.7a)

Example 3 **Complete the statement -1 ☐ -5 with <, >, or =.**



▶ -1 is to the right of -5. So, $-1 > -5$.

Complete the statement with <, >, or =.

7. 2 ☐ 9 **8.** -6 ☐ 5 **9.** -12 ☐ -4

10. -7 ☐ -13 **11.** $|-8|$ ☐ $|8|$ **12.** -10 ☐ $|-18|$

13. **ABSTRACT REASONING** A number a is to the left of a number b on the number line. How do the numbers $-a$ and $-b$ compare?

Mathematical Practices

Mathematically proficient students ***use technology tools*** *to explore and deepen their understanding of concepts.* **(MP5)**

Using a Graphing Calculator

Core Concept

Solving an Inequality in One Variable

You can use a graphing calculator to solve an inequality.

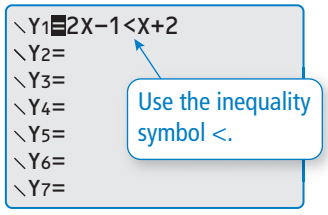
1. Enter the inequality into a graphing calculator.
2. Graph the inequality.
3. Use the graph to write the solution.

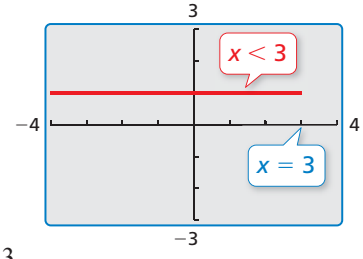
EXAMPLE 1 Using a Graphing Calculator

Use a graphing calculator to solve (a) $2x - 1 < x + 2$ and (b) $2x - 1 \leq x + 2$.

SOLUTION

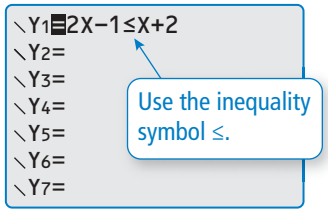
a. Enter the inequality $2x - 1 < x + 2$ into a graphing calculator. Press *graph*.

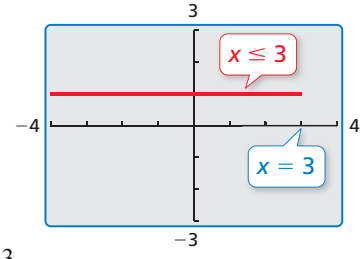




▶ The solution of the inequality is $x < 3$.

b. Enter the inequality $2x - 1 \leq x + 2$ into a graphing calculator. Press *graph*.





▶ The solution of the inequality is $x \leq 3$.

Notice that the graphing calculator does not distinguish between the solutions $x < 3$ and $x \leq 3$. You must distinguish between these yourself, based on the inequality symbol used in the original inequality.

Monitoring Progress

Use a graphing calculator to solve the inequality.

1. $2x + 3 < x - 1$ **2.** $-x - 1 > -2x + 2$ **3.** $\frac{1}{2}x + 1 \leq \frac{3}{2}x + 3$

2.1 Writing and Graphing Inequalities

Common Core

Learning Standard
HSA-CED.A.1

Essential Question How can you use an inequality to describe a real-life statement?

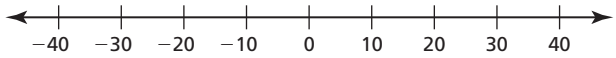
EXPLORATION 1 Writing and Graphing Inequalities

Work with a partner. Write an inequality for each statement. Then sketch the graph of the numbers that make each inequality true.

a. Statement The temperature t in Sweden is at least $-10°C$.

Inequality

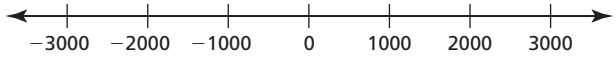
Graph



b. Statement The elevation e of Alabama is at most 2407 feet.

Inequality

Graph



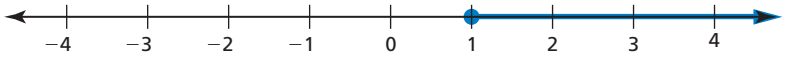
ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely. You also need to state the meanings of the symbols you use.

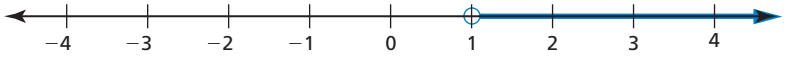
EXPLORATION 2 Writing Inequalities

Work with a partner. Write an inequality for each graph. Then, in words, describe all the values of x that make each inequality true.

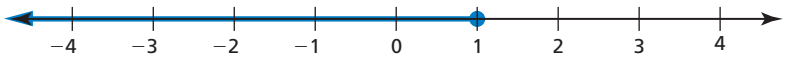
a.



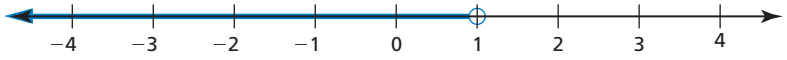
b.



c.



d.



Communicate Your Answer

3. How can you use an inequality to describe a real-life statement?

4. Write a real-life statement that involves each inequality.

a. $x < 3.5$

b. $x \leq 6$

c. $x > -2$

d. $x \geq 10$

2.1 Lesson

What You Will Learn

- Write linear inequalities.
- Sketch the graphs of linear inequalities.
- Write linear inequalities from graphs.

Core Vocabulary

inequality, *p. 54*
solution of an inequality, *p. 55*
solution set, *p. 55*
graph of an inequality, *p. 56*

Previous
expression

Writing Linear Inequalities

An **inequality** is a mathematical sentence that compares expressions. An inequality contains the symbol $<$, $>$, $\leq$, or $\geq$. To write an inequality, look for the following phrases to determine what inequality symbol to use.

Inequality Symbols				
Symbol	$<$	$>$	$\leq$	$\geq$
Key Phrases	• is less than • is fewer than	• is greater than • is more than	• is less than or equal to • is at most • is no more than	• is greater than or equal to • is at least • is no less than

EXAMPLE 1 Writing Inequalities

Write each sentence as an inequality.

a. A number w minus 3.5 is less than or equal to -2.

b. Three is less than a number n plus 5.

c. Zero is greater than or equal to twice a number x plus 1.

SOLUTION

a. A number w minus 3.5 is less than or equal to -2.

$w - 3.5$ $\leq$ -2

▶ An inequality is $w - 3.5 \leq -2$.

READING

The inequality $3 < n + 5$ is the same as $n + 5 > 3$.

b. Three is less than a number n plus 5.

3 $<$ $n + 5$

▶ An inequality is $3 < n + 5$.

c. Zero is greater than or equal to twice a number x plus 1.

0 $\geq$ $2x + 1$

▶ An inequality is $0 \geq 2x + 1$.

Monitoring Progress

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Write the sentence as an inequality.

1. A number b is fewer than 30.4.
2. $-\frac{7}{10}$ is at least twice a number k minus 4.

Sketching the Graphs of Linear Inequalities

A **solution of an inequality** is a value that makes the inequality true. An inequality can have more than one solution. The set of all solutions of an inequality is called the **solution set**.

Value of x	$x + 5 \geq -2$	Is the inequality true?
-6	$-6 + 5 \stackrel{?}{\geq} -2$ $-1 \geq -2$ ✓	yes
-7	$-7 + 5 \stackrel{?}{\geq} -2$ $-2 \geq -2$ ✓	yes
-8	$-8 + 5 \stackrel{?}{\geq} -2$ $-3 \not\geq -2$ ✗	no

Recall that a diagonal line through an inequality symbol means the inequality is *not* true. For instance, the symbol $\not\geq$ means "is not greater than or equal to."

EXAMPLE 2 Checking Solutions

Tell whether -4 is a solution of each inequality.

a. $x + 8 < -3$

b. $-4.5x > -21$

SOLUTION

a. $x + 8 < -3$ Write the inequality.

$-4 + 8 \stackrel{?}{<} -3$ Substitute -4 for x.

$4 \not< -3$ ✗ Simplify.

4 is *not* less than -3.

▶ So, -4 is *not* a solution of the inequality.

b. $-4.5x > -21$ Write the inequality.

$-4.5(-4) \stackrel{?}{>} -21$ Substitute -4 for x.

$18 > -21$ ✓ Simplify.

18 is greater than -21.

▶ So, -4 is a solution of the inequality.

Monitoring Progress

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Tell whether -6 is a solution of the inequality.

3. $c + 4 < -1$

4. $10 \leq 3 - m$

5. $21 \div x \geq -3.5$

6. $4x - 25 > -2$

The **graph of an inequality** shows the solution set of the inequality on a number line. An open circle, ○, is used when a number is *not* a solution. A closed circle, ●, is used when a number is a solution. An arrow to the left or right shows that the graph continues in that direction.

EXAMPLE 3 Graphing Inequalities

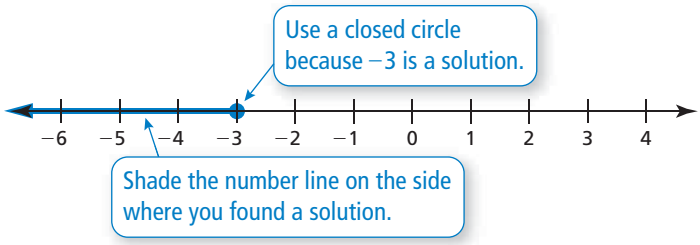
Graph each inequality.

a. $y \leq -3$ **b.** $2 < x$ **c.** $x > 0$

SOLUTION

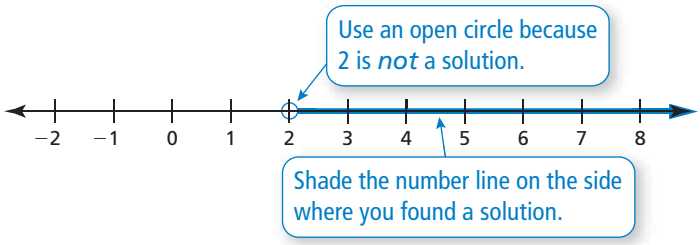
a. Test a number to the left of -3. $y = -4$ is a solution.

Test a number to the right of -3. $y = 0$ is *not* a solution.

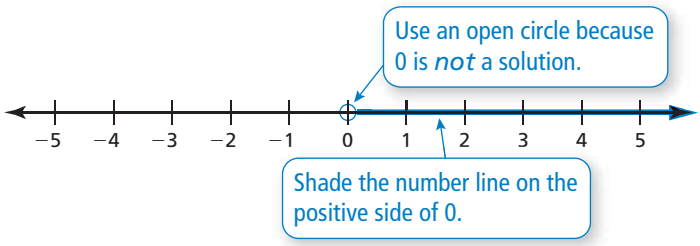


b. Test a number to the left of 2. $x = 0$ is *not* a solution.

Test a number to the right of 2. $x = 4$ is a solution.



c. Just by looking at the inequality, you can see that it represents the set of all positive numbers.



ANOTHER WAY

Another way to represent the solutions of an inequality is to use *set-builder notation*. In Example 3b, the solutions can be written as $\{x \mid x > 2\}$, which is read as "the set of all numbers x such that x is greater than 2."

Monitoring Progress

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Graph the inequality.

7. $b > -8$ **8.** $1.4 \geq g$

9. $r < \frac{1}{2}$ **10.** $v \geq \sqrt{36}$

Writing Linear Inequalities from Graphs

EXAMPLE 4 **Writing Inequalities from Graphs**

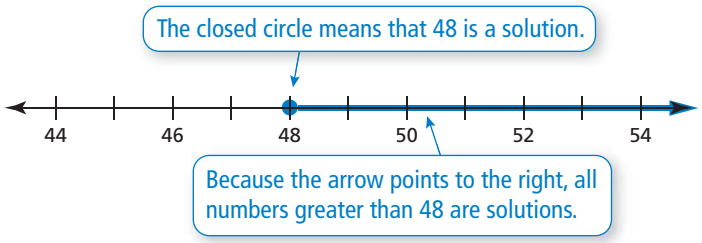
The graphs show the height restrictions h (in inches) for two rides at an amusement park. Write an inequality that represents the height restriction of each ride.

Ride A

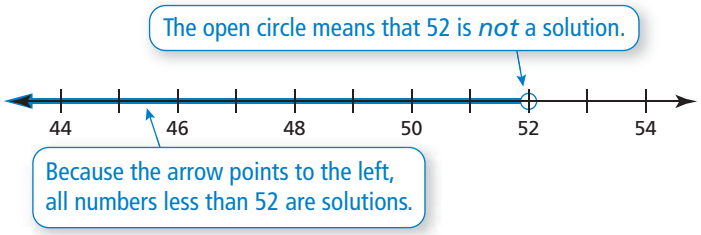
Ride B

SOLUTION

Ride A



Ride B



So, $h \geq 48$ represents the height restriction for Ride A, and $h < 52$ represents the height restriction for Ride B.

Monitoring Progress

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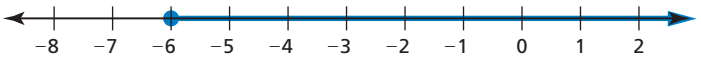
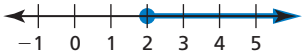
11. Write an inequality that represents the graph.



Concept Summary

Representing Linear Inequalities

Words	Algebra	Graph
x is less than 2	$x < 2$	
x is greater than 2	$x > 2$	
x is less than or equal to 2	$x \leq 2$	
x is greater than or equal to 2	$x \geq 2$	

2.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A mathematical sentence using the symbols $<$, $>$, $\leq$, or $\geq$ is called a(n)_____.
2. **VOCABULARY** Is 5 in the solution set of $x + 3 > 8$? Explain.
3. **ATTENDING TO PRECISION** Describe how to graph an inequality.
4. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write "both" inequalities.

w is greater than or equal to -7.

w is no less than -7.

w is no more than -7.

w is at least -7.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, write the sentence as an inequality. *(See Example 1.)*

5. A number x is greater than 3.
6. A number n plus 7 is less than or equal to 9.
7. Fifteen is no more than a number t divided by 5.
8. Three times a number w is less than 18.
9. One-half of a number y is more than 22.
10. Three is less than the sum of a number s and 4.
11. Thirteen is at least the difference of a number v and 1.
12. Four is no less than the quotient of a number x and 2.1.
13. **MODELING WITH MATHEMATICS** On a fishing trip, you catch two fish. The weight of the first fish is shown. The second fish weighs at least 0.5 pound more than the first fish. Write an inequality that represents the possible weights of the second fish.

14. **MODELING WITH MATHEMATICS** There are 430 people in a wave pool. Write an inequality that represents how many more people can enter the pool.

In Exercises 15–24, tell whether the value is a solution of the inequality. *(See Example 2.)*

15. $r + 4 > 8$; $r = 2$
16. $5 - x < 8$; $x = -3$
17. $3s \leq 19$; $s = -6$
18. $17 \geq 2y$; $y = 7$
19. $-1 > -\frac{x}{2}$; $x = 3$
20. $\frac{4}{z} \geq 3$; $z = 2$
21. $14 \geq -2n + 4$; $n = -5$
22. $-5 \div (2s) < -1$; $s = 10$
23. $20 \leq \frac{10}{2z} + 20$; $z = 5$
24. $\frac{3m}{6} - 2 > 3$; $m = 8$
25. **MODELING WITH MATHEMATICS** The tallest person who ever lived was approximately 8 feet 11 inches tall.
    a. Write an inequality that represents the heights of every other person who has ever lived.
    b. Is 9 feet a solution of the inequality? Explain.

26. DRAWING CONCLUSIONS The winner of a weight-lifting competition bench-pressed 400 pounds. The other competitors all bench-pressed at least 23 pounds less.

a. Write an inequality that represents the weights that the other competitors bench-pressed.

b. Was one of the other competitors able to bench-press 379 pounds? Explain.

ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in determining whether 8 is in the solution set of the inequality.**

27.

$$-y + 7 < -4$$
$$-8 + 7 \stackrel{?}{<} -4$$
$$-1 < -4$$

8 is in the solution set.

28.

$$\tfrac{1}{2}x + 2 \leq 6$$
$$\tfrac{1}{2}(8) + 2 \stackrel{?}{\leq} 6$$
$$4 + 2 \stackrel{?}{\leq} 6$$
$$6 \leq 6$$

8 is *not* in the solution set.

In Exercises 29–36, graph the inequality. *(See Example 3.)*

29. $x \geq 2$

30. $z \leq 5$

31. $-1 > t$

32. $-2 < w$

33. $v \leq -4$

34. $s < 1$

35. $\frac{1}{4} < p$

36. $r \geq -|5|$

In Exercises 37–40, write and graph an inequality for the given solution set.

37. $\{x \mid x < 7\}$

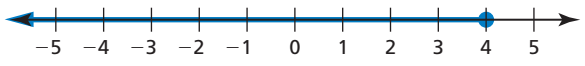
38. $\{n \mid n \geq -2\}$

39. $\{z \mid 1.3 \leq z\}$

40. $\{w \mid 5.2 > w\}$

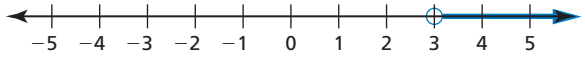
In Exercises 41–44, write an inequality that represents the graph. *(See Example 4.)*

41.



42.

43.



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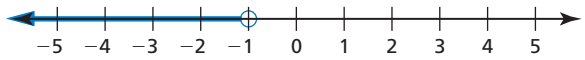

45. ANALYZING RELATIONSHIPS The water temperature of a swimming pool must be no less than 76°F. The temperature is currently 74°F. Which graph correctly shows how much the temperature needs to increase? Explain your reasoning.

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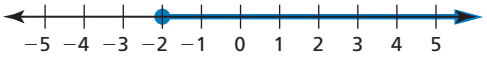

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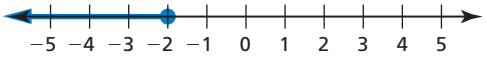

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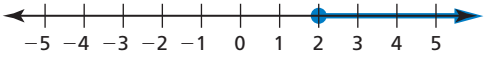

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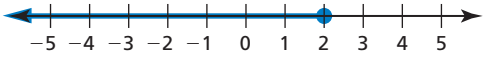


46. MODELING WITH MATHEMATICS According to a state law for vehicles traveling on state roads, the maximum total weight of a vehicle and its contents depends on the number of axles on the vehicle. For each type of vehicle, write and graph an inequality that represents the possible total weights w (in pounds) of the vehicle and its contents.

Maximum Total Weights

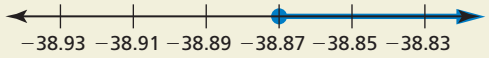
2 axles, 40,000 lb	3 axles, 60,000 lb	4 axles, 80,000 lb

47. PROBLEM SOLVING The Xianren Bridge is located in Guangxi Province, China. This arch is the world's longest natural arch, with a length of 400 feet. Write and graph an inequality that represents the lengths ℓ (in *inches*) of all other natural arches.

48. THOUGHT PROVOKING A student works no more than 25 hours each week at a part-time job. Write an inequality that represents how many hours the student can work each day.

49. WRITING Describe a real-life situation modeled by the inequality $23 + x \leq 31$.

50. HOW DO YOU SEE IT? The graph represents the known melting points of all metallic elements (in degrees Celsius).



a. Write an inequality represented by the graph.

b. Is it possible for a metallic element to have a melting point of −38.87°C? Explain.

51. DRAWING CONCLUSIONS A one-way ride on a subway costs \$0.90. A monthly pass costs \$24. Write an inequality that represents how many one-way rides you can buy before it is cheaper to buy the monthly pass. Is it cheaper to pay the one-way fare for 25 rides? Explain.

Subway Prices	
One-way ride	\$0.90
Monthly pass	\$24.00

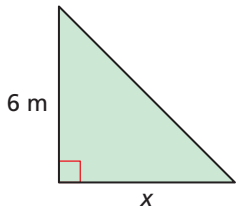
52. MAKING AN ARGUMENT The inequality $x \leq 1324$ represents the weights (in pounds) of all mako sharks ever caught using a rod and reel. Your friend says this means no one using a rod and reel has ever caught a mako shark that weighs 1324 pounds. Your cousin says this means someone using a rod and reel *has* caught a mako shark that weighs 1324 pounds. Who is correct? Explain your reasoning.

53. CRITICAL THINKING Describe a real-life situation that can be modeled by more than one inequality.

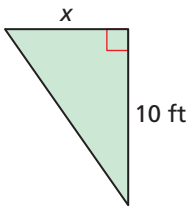
54. MODELING WITH MATHEMATICS In 1997, Superman's cape from the 1978 movie *Superman* was sold at an auction. The winning bid was \$17,000. Write and graph an inequality that represents the amounts all the losing bids.

MATHEMATICAL CONNECTIONS In Exercises 55–58, write an inequality that represents the missing dimension x.

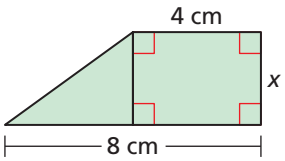
55. The area is less than 42 square meters.



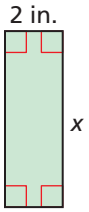
56. The area is greater than or equal to 8 square feet.



57. The area is less than 18 square centimeters.



58. The area is greater than 12 square inches.



59. WRITING A runner finishes a 200-meter dash in 35 seconds. Let r represent any speed (in meters per second) faster than the runner's speed.

a. Write an inequality that represents r. Then graph the inequality.

b. Every point on the graph represents a speed faster than the runner's speed. Do you think every point could represent the speed of a runner? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Section 1.1)*

60. $x + 2 = 3$

61. $y - 9 = 5$

62. $6 = 4 + y$

63. $-12 = y - 11$

Solve the literal equation for x. *(Section 1.5)*

64. $v = x \cdot y \cdot z$

65. $s = 2r + 3x$

66. $w = 5 + 3(x - 1)$

67. $n = \dfrac{2x + 1}{2}$

2.2 Solving Inequalities Using Addition or Subtraction

COMMON CORE

Learning Standards
HSA-CED.A.1
HSA-REI.B.3

Essential Question

How can you use addition or subtraction to solve an inequality?

EXPLORATION 1 Quarterback Passing Efficiency

Work with a partner. The National Collegiate Athletic Association (NCAA) uses the following formula to rank the passing efficiencies P of quarterbacks.

$$P = \frac{8.4Y + 100C + 330T - 200N}{A}$$

Y = total length of all completed passes (in Yards)

C = Completed passes

T = passes resulting in a Touchdown

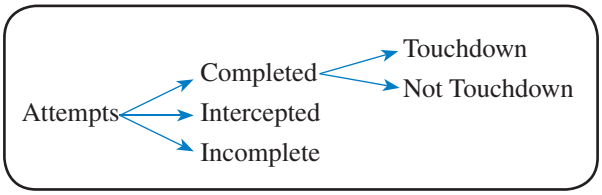
N = iNtercepted passes

A = Attempted passes

M = incoMplete passes

MODELING WITH MATHEMATICS

To be proficient in math, you need to identify and analyze important relationships and then draw conclusions, using tools such as diagrams, flowcharts, and formulas.



Determine whether each inequality must be true. Explain your reasoning.

a. $T < C$ **b.** $C + N \leq A$ **c.** $N < A$ **d.** $A - C \geq M$

EXPLORATION 2 Finding Solutions of Inequalities

Work with a partner. Use the passing efficiency formula to create a passing record that makes each inequality true. Record your results in the table. Then describe the values of P that make each inequality true.

Attempts	Completions	Yards	Touchdowns	Interceptions

a. $P < 0$

b. $P + 100 \geq 250$

c. $P - 250 > -80$

Communicate Your Answer

3. How can you use addition or subtraction to solve an inequality?

4. Solve each inequality.

a. $x + 3 < 4$ **b.** $x - 3 \geq 5$

c. $4 > x - 2$ **d.** $-2 \leq x + 1$

2.2 Lesson

What You Will Learn

- Solve inequalities using addition.
- Solve inequalities using subtraction.
- Use inequalities to solve real-life problems.

Core Vocabulary

equivalent inequalities, *p. 62*

Previous

inequality

Solving Inequalities Using Addition

Just as you used the properties of equality to produce equivalent equations, you can use the properties of inequality to produce equivalent inequalities. **Equivalent inequalities** are inequalities that have the same solutions.

Core Concept

Addition Property of Inequality

Words Adding the same number to each side of an inequality produces an equivalent inequality.

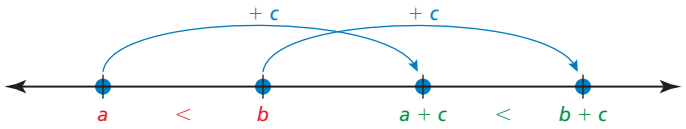
Numbers

$$\begin{array}{rcl} -3 & < & 2 \\ \underline{+4} & & \underline{+4} \\ 1 & < & 6 \end{array} \qquad \begin{array}{rcl} -3 & \geq & -10 \\ \underline{+3} & & \underline{+3} \\ 0 & \geq & -7 \end{array}$$

Algebra If $a > b$, then $a + c > b + c$. If $a \geq b$, then $a + c \geq b + c$.

If $a < b$, then $a + c < b + c$. If $a \leq b$, then $a + c \leq b + c$.

The diagram shows one way to visualize the Addition Property of Inequality when $c > 0$.



EXAMPLE 1 Solving an Inequality Using Addition

Solve $x - 6 \geq -10$. Graph the solution.

SOLUTION

$$\begin{array}{rcl} x - 6 & \geq & -10 \\ \underline{+6} & & \underline{+6} \\ x & \geq & -4 \end{array}$$

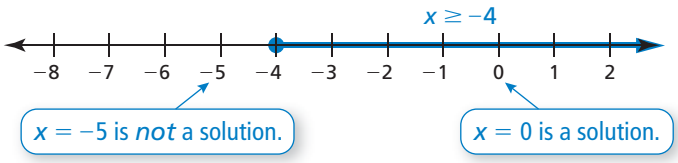
Write the inequality.

Addition Property of Inequality

Add 6 to each side.

Simplify.

▶ The solution is $x \geq -4$.



REMEMBER

To check this solution, substitute a few numbers to the left and right of −4 into the original inequality.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the inequality. Graph the solution.

1. $b - 2 > -9$ **2.** $m - 3 \leq 5$ **3.** $\frac{1}{4} > y - \frac{1}{4}$

Solving Inequalities Using Subtraction

Core Concept

Subtraction Property of Inequality

Words Subtracting the same number from each side of an inequality produces an equivalent inequality.

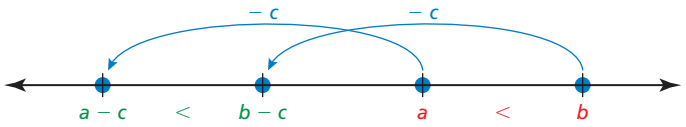
Numbers

$$\begin{array}{rcl} -3 & \leq & 1 \\ \underline{-5} & & \underline{-5} \\ -8 & \leq & -4 \end{array} \qquad \begin{array}{rcl} 7 & > & -20 \\ \underline{-7} & & \underline{-7} \\ 0 & > & -27 \end{array}$$

Algebra If $a > b$, then $a - c > b - c$. If $a \geq b$, then $a - c \geq b - c$.

If $a < b$, then $a - c < b - c$. If $a \leq b$, then $a - c \leq b - c$.

The diagram shows one way to visualize the Subtraction Property of Inequality when $c > 0$.



EXAMPLE 2 Solving an Inequality Using Subtraction

Solve each inequality. Graph the solution.

a. $y + 8 \leq 5$ **b.** $-8 < 1.4 + m$

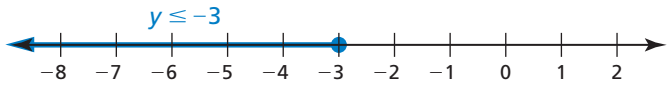
SOLUTION

a. $y + 8 \leq 5$ Write the inequality.

Subtraction Property of Inequality → $\underline{-8} \quad \underline{-8}$ Subtract 8 from each side.

$y \leq -3$ Simplify.

▶ The solution is $y \leq -3$.

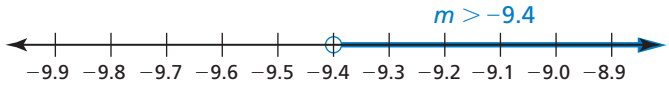


b. $-8 < 1.4 + m$ Write the inequality.

Subtraction Property of Inequality → $\underline{-1.4} \quad \underline{-1.4}$ Subtract 1.4 from each side.

$-9.4 < m$ Simplify.

▶ The solution is $m > -9.4$.



Monitoring Progress

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Solve the inequality. Graph the solution.

4. $k + 5 \leq -3$ **5.** $\frac{5}{6} \leq z + \frac{1}{6}$ **6.** $p + 0.7 > -2.3$

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**

A circuit overloads at 1800 watts of electricity. You plug a microwave oven that uses 1100 watts of electricity into the circuit.

a. Write and solve an inequality that represents how many watts you can add to the circuit without overloading the circuit.

b. In addition to the microwave oven, which of the following appliances can you plug into the circuit at the same time without overloading the circuit?

Appliance	Watts
Clock radio	50
Blender	300
Hot plate	1200
Toaster	800

SOLUTION

1. Understand the Problem You know that the microwave oven uses 1100 watts out of a possible 1800 watts. You are asked to write and solve an inequality that represents how many watts you can add without overloading the circuit. You also know the numbers of watts used by four other appliances. You are asked to identify the appliances you can plug in at the same time without overloading the circuit.

2. Make a Plan Use a verbal model to write an inequality. Then solve the inequality and identify other appliances that you can plug into the circuit at the same time without overloading the circuit.

3. Solve the Problem

Words Watts used by microwave oven + Additional watts < Overload wattage

Variable Let w be the additional watts you can add to the circuit.

Inequality $1100 + w < 1800$ Write the inequality.

Subtraction Property of Inequality → $-1100 \quad -1100$ Subtract 1100 from each side.

$w < 700$ Simplify.

▶ You can add up to 700 watts to the circuit, which means that you can also plug in the clock radio and the blender.

4. Look Back You can check that your answer is correct by adding the numbers of watts used by the microwave oven, clock radio, and blender.

$1100 + 50 + 300 = 1450$

The circuit will not overload because the total wattage is less than 1800 watts.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. The microwave oven uses only 1000 watts of electricity. Does this allow you to have both the microwave oven and the toaster plugged into the circuit at the same time? Explain your reasoning.

2.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Why is the inequality $x \leq 6$ equivalent to the inequality $x - 5 \leq 6 - 5$?
2. **WRITING** Compare solving equations using addition with solving inequalities using addition.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, tell which number you would add to or subtract from each side of the inequality to solve it.

3. $k + 11 < -3$
4. $v - 2 > 14$
5. $-1 \geq b - 9$
6. $-6 \leq 17 + p$

In Exercises 7–20, solve the inequality. Graph the solution. *(See Examples 1 and 2.)*

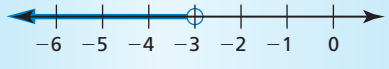
7. $x - 4 < -5$
8. $1 \leq s - 8$
9. $6 \geq m - 1$
10. $c - 12 > -4$
11. $r + 4 < 5$
12. $-8 \leq 8 + y$
13. $9 + w > 7$
14. $15 \geq q + 3$
15. $h - (-2) \geq 10$
16. $-6 > t - (-13)$
17. $j + 9 - 3 < 8$
18. $1 - 12 + y \geq -5$
19. $10 \geq 3p - 2p - 7$
20. $18 - 5z + 6z > 3 + 6$

In Exercises 21–24, write the sentence as an inequality. Then solve the inequality.

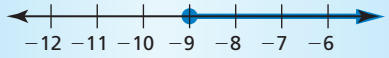
21. A number plus 8 is greater than 11.
22. A number minus 3 is at least -5.
23. The difference of a number and 9 is fewer than 4.
24. Six is less than or equal to the sum of a number and 15.
25. **MODELING WITH MATHEMATICS** You are riding a train. Your carry-on bag can weigh no more than 50 pounds. Your bag weighs 38 pounds. *(See Example 3.)*
    a. Write and solve an inequality that represents how much weight you can add to your bag.
    b. Can you add both a 9-pound laptop and a 5-pound pair of boots to your bag without going over the weight limit? Explain.
26. **MODELING WITH MATHEMATICS** You order the hardcover book shown from a website that offers free shipping on orders of $25 or more. Write and solve an inequality that represents how much more you must spend to get free shipping.

ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in solving the inequality or graphing the solution.**

27.
$$-17 < x - 14$$
$$-17 + 14 < x - 14 + 14$$
$$-3 < x$$



28.
$$-10 + x \geq -9$$
$$-10 + 10 + x \geq -9$$
$$x \geq -9$$



29. **PROBLEM SOLVING** An NHL hockey player has 59 goals so far in a season. What are the possible numbers of additional goals the player can score to match or break the NHL record of 92 goals in a season?

30. MAKING AN ARGUMENT In an aerial ski competition, you perform two acrobatic ski jumps. The scores on the two jumps are then added together.

Ski jump	Competitor's score	Your score
1	117.1	119.5
2	119.8	

a. Describe the score that you must earn on your second jump to beat your competitor.

b. Your coach says that you will beat your competitor if you score 118.4 points. A teammate says that you only need 117.5 points. Who is correct? Explain.

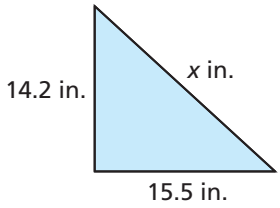
31. REASONING Which of the following inequalities are equivalent to the inequality $x - b < 3$, where b is a constant? Justify your answer.

Ⓐ $x - b - 3 < 0$ Ⓑ $0 > b - x + 3$

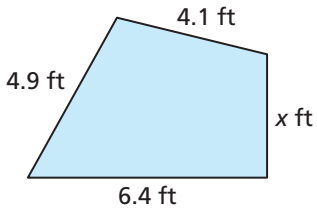
Ⓒ $x < 3 - b$ Ⓓ $-3 < b - x$

MATHEMATICAL CONNECTIONS In Exercises 32 and 33, write and solve an inequality to find the possible values of x.

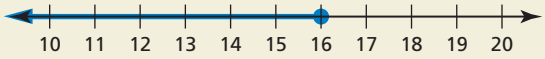
32. Perimeter < 51.3 inches



33. Perimeter ≤ 18.7 feet

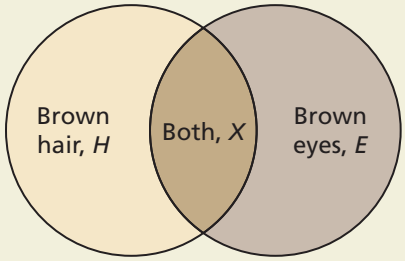


34. THOUGHT PROVOKING Write an inequality that has the solution shown in the graph. Describe a real-life situation that can be modeled by the inequality.



35. WRITING Is it possible to check all the numbers in the solution set of an inequality? When you solve the inequality $x - 11 \geq -3$, which numbers can you check to verify your solution? Explain your reasoning.

36. HOW DO YOU SEE IT? The diagram represents the numbers of students in a school with brown eyes, brown hair, or both.



Determine whether each inequality must be true. Explain your reasoning.

a. $H \geq E$ **b.** $H + 10 \geq E$

c. $H \geq X$ **d.** $H + 10 \geq X$

e. $H > X$ **f.** $H + 10 > X$

37. REASONING Write and graph an inequality that represents the numbers that are *not* solutions of each inequality.

a. $x + 8 < 14$

b. $x - 12 \geq 5.7$

38. PROBLEM SOLVING Use the inequalities $c - 3 \geq d$, $b + 4 < a + 1$, and $a - 2 \leq d - 7$ to order a, b, c, and d from least to greatest.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the product or quotient. *(Skills Review Handbook)*

39. $7 \cdot (-9)$ **40.** $-11 \cdot (-12)$ **41.** $-27 \div (-3)$ **42.** $20 \div (-5)$

Solve the equation. Check your solution. *(Section 1.1)*

43. $6x = 24$ **44.** $-3y = -18$ **45.** $\frac{s}{-8} = 13$ **46.** $\frac{n}{4} = -7.3$

2.3 Solving Inequalities Using Multiplication or Division

COMMON CORE

Learning Standards
HSA-CED.A.1
HSA-REI.B.3

Essential Question

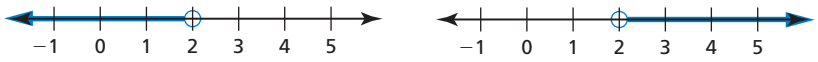
How can you use division to solve an inequality?

EXPLORATION 1 Writing a Rule

Work with a partner.

a. Copy and complete the table. Decide which graph represents the solution of the inequality $6 < 3x$. Write the solution of the inequality.

x	-1	0	1	2	3	4	5
$3x$	-3						
$6 \stackrel{?}{<} 3x$	No						



b. Use a table to solve each inequality. Then write a rule that describes how to use division to solve the inequalities.

i. $2x < 4$ **ii.** $3 \geq 3x$ **iii.** $2x < 8$ **iv.** $6 \geq 3x$

LOOKING FOR A PATTERN

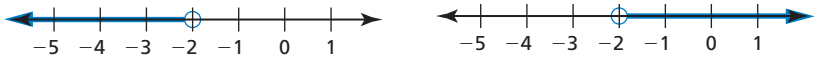
To be proficient in math, you need to investigate relationships, observe patterns, and use your observations to write general rules.

EXPLORATION 2 Writing a Rule

Work with a partner.

a. Copy and complete the table. Decide which graph represents the solution of the inequality $6 < -3x$. Write the solution of the inequality.

x	-5	-4	-3	-2	-1	0	1
$-3x$							
$6 \stackrel{?}{<} -3x$							



b. Use a table to solve each inequality. Then write a rule that describes how to use division to solve the inequalities.

i. $-2x < 4$ **ii.** $3 \geq -3x$ **iii.** $-2x < 8$ **iv.** $6 \geq -3x$

Communicate Your Answer

3. How can you use division to solve an inequality?

4. Use the rules you wrote in Explorations 1(b) and 2(b) to solve each inequality.

a. $7x < -21$ **b.** $12 \leq 4x$ **c.** $10 < -5x$ **d.** $-3x \leq 0$

2.3 Lesson

What You Will Learn

- Solve inequalities by multiplying or dividing by *positive* numbers.
- Solve inequalities by multiplying or dividing by *negative* numbers.
- Use inequalities to solve real-life problems.

Multiplying or Dividing by Positive Numbers

Core Concept

Multiplication and Division Properties of Inequality ($c > 0$)

Words Multiplying or dividing each side of an inequality by the same *positive* number produces an equivalent inequality.

Numbers

$-6 < 8$ $\qquad$ $6 > -8$

$2 \cdot (-6) < 2 \cdot 8$ $\qquad$ $\frac{6}{2} > \frac{-8}{2}$

$-12 < 16$ $\qquad$ $3 > -4$

Algebra If $a > b$ and $c > 0$, then $ac > bc$. If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$.

If $a < b$ and $c > 0$, then $ac < bc$. If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

These properties are also true for $\leq$ and $\geq$.

EXAMPLE 1 Multiplying or Dividing by Positive Numbers

Solve (a) $\frac{x}{8} > -5$ and (b) $-24 \geq 3x$. Graph each solution.

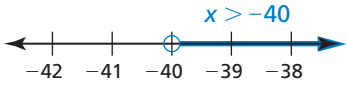
SOLUTION

a. $\frac{x}{8} > -5$ Write the inequality.

Multiplication Property of Inequality → $8 \cdot \frac{x}{8} > 8 \cdot (-5)$ Multiply each side by 8.

$x > -40$ Simplify.

▶ The solution is $x > -40$.

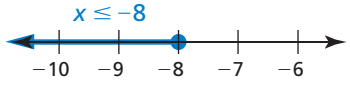


b. $-24 \geq 3x$ Write the inequality.

Division Property of Inequality → $\frac{-24}{3} \geq \frac{3x}{3}$ Divide each side by 3.

$-8 \geq x$ Simplify.

▶ The solution is $x \leq -8$.



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the inequality. Graph the solution.

1. $\frac{n}{7} \geq -1$ **2.** $-6.4 \geq \frac{1}{5}w$ **3.** $4b \geq 36$ **4.** $-18 > 1.5q$

Multiplying or Dividing by Negative Numbers

Core Concept

Multiplication and Division Properties of Inequality ($c < 0$)

Words When multiplying or dividing each side of an inequality by the same *negative* number, the direction of the inequality symbol must be reversed to produce an equivalent inequality.

Numbers

$-6 < 8$ $\qquad$ $6 > -8$

$-2 \cdot (-6) > -2 \cdot 8$ $\qquad$ $\frac{6}{-2} < \frac{-8}{-2}$

$12 > -16$ $\qquad$ $-3 < 4$

Algebra If $a > b$ and $c < 0$, then $ac < bc$. If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.

If $a < b$ and $c < 0$, then $ac > bc$. If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.

These properties are also true for $\leq$ and $\geq$.

COMMON ERROR

A negative sign in an inequality does not necessarily mean you must reverse the inequality symbol, as shown in Example 1.

Only reverse the inequality symbol when you multiply or divide each side by a negative number.

EXAMPLE 2 Multiplying or Dividing by Negative Numbers

Solve each inequality. Graph each solution.

a. $2 < \frac{y}{-3}$ $\qquad$ **b.** $-7y \leq -35$

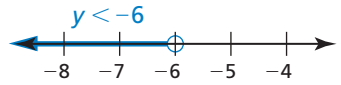
SOLUTION

a. $2 < \frac{y}{-3}$ Write the inequality.

Multiplication Property of Inequality → $-3 \cdot 2 > -3 \cdot \frac{y}{-3}$ Multiply each side by -3. Reverse the inequality symbol.

$-6 > y$ Simplify.

▶ The solution is $y < -6$.

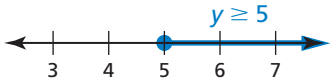


b. $-7y \leq -35$ Write the inequality.

Division Property of Inequality → $\frac{-7y}{-7} \geq \frac{-35}{-7}$ Divide each side by -7. Reverse the inequality symbol.

$y \geq 5$ Simplify.

▶ The solution is $y \geq 5$.



Monitoring Progress

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Solve the inequality. Graph the solution.

5. $\frac{p}{-4} < 7$ $\qquad$ **6.** $\frac{x}{-5} \leq -5$ $\qquad$ **7.** $1 \geq -\frac{1}{10}z$

8. $-9m > 63$ $\qquad$ **9.** $-2r \geq -22$ $\qquad$ **10.** $-0.4y \geq -12$

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**

You earn $9.50 per hour at your summer job. Write and solve an inequality that represents the numbers of hours you need to work to buy a digital camera that costs $247.

SOLUTION

1. **Understand the Problem** You know your hourly wage and the cost of the digital camera. You are asked to write and solve an inequality that represents the numbers of hours you need to work to buy the digital camera.
2. **Make a Plan** Use a verbal model to write an inequality. Then solve the inequality.
3. **Solve the Problem**

Words Hourly wage $\cdot$ Hours worked $\geq$ Cost of camera

Variable Let n be the number of hours worked.

Inequality $9.5 \cdot n \geq 247$

$9.5n \geq 247$ Write the inequality.

Division Property of Inequality → $\frac{9.5n}{9.5} \geq \frac{247}{9.5}$ Divide each side by 9.5.

$n \geq 26$ Simplify.

▶ You need to work at least 26 hours for your gross pay to be at least $247. If you have payroll deductions, such as Social Security taxes, you need to work more than 26 hours.

REMEMBER

Compatible numbers are numbers that are easy to compute mentally.

4. **Look Back** You can use estimation to check that your answer is reasonable.

$\$247 \div \$9.50/\text{h}$

$\downarrow \qquad \downarrow$

$\$250 \div \$10/\text{h} = 25 \text{ h}$ Use compatible numbers.

Your hourly wage is about $10 per hour. So, to earn about $250, you need to work about 25 hours.

Unit Analysis Each time you set up an equation or inequality to represent a real-life problem, be sure to check that the units balance.

$$\frac{\$9.50}{\cancel{\text{h}}} \times 26\cancel{\text{h}} = \$247$$

Monitoring Progress

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11. You have at most $3.65 to make copies. Each copy costs $0.25. Write and solve an inequality that represents the numbers of copies you can make.

12. The maximum speed limit for a school bus is 55 miles per hour. Write and solve an inequality that represents the numbers of hours it takes to travel 165 miles in a school bus.

2.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how solving $2x < -8$ is different from solving $-2x < 8$.

2. **OPEN-ENDED** Write an inequality that is solved using the Division Property of Inequality where the inequality symbol needs to be reversed.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, solve the inequality. Graph the solution. *(See Example 1.)*

3. $4x < 8$
4. $3y \leq -9$
5. $-20 \leq 10n$
6. $35 < 7t$
7. $\frac{x}{2} > -2$
8. $\frac{a}{4} < 10.2$
9. $20 \geq \frac{4}{5}w$
10. $-16 \leq \frac{8}{3}t$

In Exercises 11–18, solve the inequality. Graph the solution. *(See Example 2.)*

11. $-6t < 12$
12. $-9y > 9$
13. $-10 \geq -2z$
14. $-15 \leq -3c$
15. $\frac{n}{-3} \geq 1$
16. $\frac{w}{-5} \leq 16$
17. $-8 < -\frac{1}{4}m$
18. $-6 > -\frac{2}{3}y$

19. **MODELING WITH MATHEMATICS** You have \$12 to buy five goldfish for your new fish tank. Write and solve an inequality that represents the prices you can pay per fish. *(See Example 3.)*

20. **MODELING WITH MATHEMATICS** A weather forecaster predicts that the temperature in Antarctica will decrease 8°F each hour for the next 6 hours. Write and solve an inequality to determine how many hours it will take for the temperature to drop at least 36°F.

USING TOOLS **In Exercises 21–26, solve the inequality. Use a graphing calculator to verify your answer.**

21. $36 < 3y$
22. $17v \geq 51$
23. $2 \leq -\frac{2}{9}x$
24. $4 > \frac{n}{-4}$
25. $2x > \frac{3}{4}$
26. $1.1y < 4.4$

ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in solving the inequality.**

27. ✗

$$-6 > \frac{2}{3}x$$
$$\frac{3}{2} \cdot (-6) < \frac{3}{2} \cdot \frac{2}{3}x$$
$$-\frac{18}{2} < x$$
$$-9 < x$$

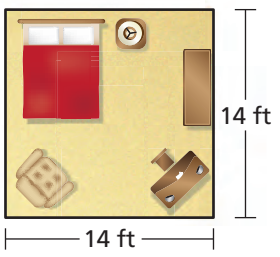
The solution is $x > -9$.

28. ✗

$$-4y \leq -32$$
$$\frac{-4y}{-4} \leq \frac{-32}{-4}$$
$$y \leq 8$$

The solution is $y \leq 8$.

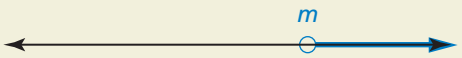
29. **ATTENDING TO PRECISION** You have \$700 to buy new carpet for your bedroom. Write and solve an inequality that represents the costs per square foot that you can pay for the new carpet. Specify the units of measure in each step.



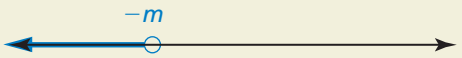
30. HOW DO YOU SEE IT? Let $m > 0$. Match each inequality with its graph. Explain your reasoning.

a. $\frac{x}{m} < -1$ **b.** $\frac{x}{m} > 1$

c. $\frac{x}{m} < 1$ **d.** $-\frac{x}{m} < 1$

A.


B.


C.


D.


31. MAKING AN ARGUMENT You run for 2 hours at a speed no faster than 6.3 miles per hour.

a. Write and solve an inequality that represents the possible numbers of miles you run.

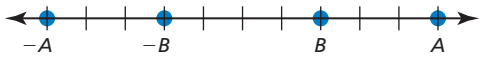
b. A marathon is approximately 26.2 miles. Your friend says that if you continue to run at this speed, you will not be able to complete a marathon in less than 4 hours. Is your friend correct? Explain.

32. THOUGHT PROVOKING The inequality $\frac{x}{4} \leq 5$ has a solution of $x = p$. Write a second inequality that also has a solution of $x = p$.

33. PROBLEM SOLVING The U.S. Mint pays \$0.02 to produce every penny. How many pennies are produced when the U.S. Mint pays more than \$6 million in production costs?

34. REASONING Are $x \leq \frac{2}{3}$ and $-3x \leq -2$ equivalent? Explain your reasoning.

35. ANALYZING RELATIONSHIPS Consider the number line shown.



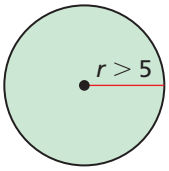
a. Write an inequality relating A and B.

b. Write an inequality relating $-A$ and $-B$.

c. Use the results from parts (a) and (b) to explain why the direction of the inequality symbol must be reversed when multiplying or dividing each side of an inequality by the same negative number.

36. REASONING Why might solving the inequality $\frac{4}{x} \geq 2$ by multiplying each side by x lead to an error? (*Hint:* Consider $x > 0$ and $x < 0$.)

37. MATHEMATICAL CONNECTIONS The radius of a circle is represented by the formula $r = \frac{C}{2\pi}$. Write and solve an inequality that represents the possible circumferences C of the circle.



38. CRITICAL THINKING A water-skiing instructor recommends that a boat pulling a beginning skier has a speed less than 18 miles per hour. Write and solve an inequality that represents the possible distances d (in miles) that a beginner can travel in 45 minutes of practice time.

39. CRITICAL THINKING A local zoo employs 36 people to take care of the animals each day. At most, 24 of the employees work full time. Write and solve an inequality that represents the fraction of employees who work part time. Graph the solution.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Section 1.2 and Section 1.3)*

40. $5x + 3 = 13$ **41.** $\frac{1}{2}y - 8 = -10$

42. $-3n + 2 = 2n - 3$ **43.** $\frac{1}{2}z + 4 = \frac{5}{2}z - 8$

Tell which number is greater. *(Skills Review Handbook)*

44. 0.8, 85% **45.** $\frac{16}{30}$, 50% **46.** 120%, 0.12 **47.** 60%, $\frac{2}{3}$

2.4 Solving Multi-Step Inequalities

COMMON CORE

Learning Standards
HSA-CED.A.1
HSA-REI.B.3

Essential Question

How can you solve a multi-step inequality?

EXPLORATION 1 Solving a Multi-Step Inequality

Work with a partner.

- Use what you already know about solving equations and inequalities to solve each multi-step inequality. Justify each step.
- Match each inequality with its graph. Use a graphing calculator to check your answer.

JUSTIFYING STEPS

To be proficient in math, you need to justify each step in a solution and communicate your justification to others.

a. $2x + 3 \leq x + 5$

b. $-2x + 3 > x + 9$

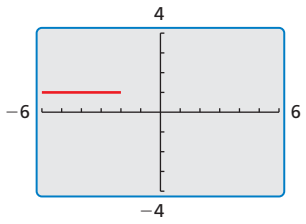
c. $27 \geq 5x + 4x$

d. $-8x + 2x - 16 < -5x + 7x$

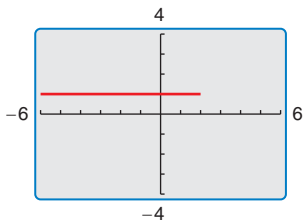
e. $3(x - 3) - 5x > -3x - 6$

f. $-5x - 6x \leq 8 - 8x - x$

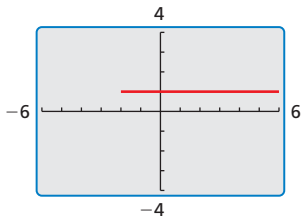
A.



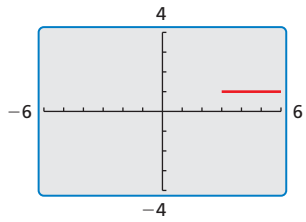
B.



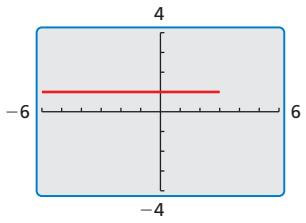
C.



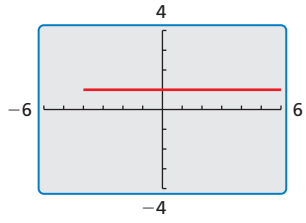
D.



E.



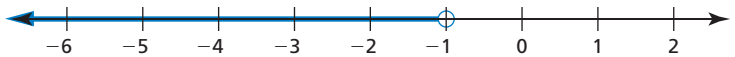
F.



Communicate Your Answer

2. How can you solve a multi-step inequality?

3. Write two different multi-step inequalities whose solutions are represented by the graph.



2.4 Lesson

What You Will Learn

- Solve multi-step inequalities.
- Use multi-step inequalities to solve real-life problems.

Solving Multi-Step Inequalities

To solve a multi-step inequality, simplify each side of the inequality, if necessary. Then use inverse operations to isolate the variable. Be sure to reverse the inequality symbol when multiplying or dividing by a negative number.

EXAMPLE 1 Solving Multi-Step Inequalities

Solve each inequality. Graph each solution.

a. $\frac{y}{-6} + 7 < 9$ **b.** $2v - 4 \geq 8$

SOLUTION

a. $\frac{y}{-6} + 7 < 9$ Write the inequality.

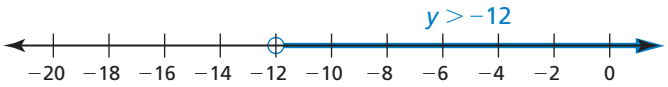
$-7 \quad -7$ Subtract 7 from each side.

$\frac{y}{-6} < 2$ Simplify.

$-6 \cdot \frac{y}{-6} > -6 \cdot 2$ Multiply each side by –6. Reverse the inequality symbol.

$y > -12$ Simplify.

The solution is $y > -12$.



b. $2v - 4 \geq 8$ Write the inequality.

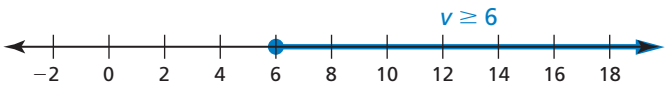
$+4 \quad +4$ Add 4 to each side.

$2v \geq 12$ Simplify.

$\frac{2v}{2} \geq \frac{12}{2}$ Divide each side by 2.

$v \geq 6$ Simplify.

The solution is $v \geq 6$.



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Solve the inequality. Graph the solution.

1. $4b - 1 < 7$ **2.** $8 - 9c \geq -28$

3. $\frac{n}{-2} + 11 > 12$ **4.** $6 \geq 5 - \frac{v}{3}$

EXAMPLE 2 Solving an Inequality with Variables on Both Sides

Solve $6x - 5 < 2x + 11$.

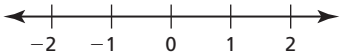
SOLUTION

$6x - 5 < 2x + 11$	Write the inequality.
$+5 \quad +5$	Add 5 to each side.
$6x < 2x + 16$	Simplify.
$-2x \quad -2x$	Subtract $2x$ from each side.
$4x < 16$	Simplify.
$\frac{4x}{4} < \frac{16}{4}$	Divide each by 4.
$x < 4$	Simplify.

▶ The solution is $x < 4$.

When solving an inequality, if you obtain an equivalent inequality that is true, such as $-5 < 0$, the solutions of the inequality are *all real numbers*. If you obtain an equivalent inequality that is false, such as $3 \leq -2$, the inequality has *no solution*.

Graph of an inequality whose solutions are all real numbers



Graph of an inequality that has no solution

EXAMPLE 3 Inequalities with Special Solutions

Solve (a) $8b - 3 > 4(2b + 3)$ and (b) $2(5w - 1) \leq 7 + 10w$.

SOLUTION

a.

$8b - 3 > 4(2b + 3)$	Write the inequality.
$8b - 3 > 8b + 12$	Distributive Property
$-8b \quad -8b$	Subtract $8b$ from each side.
$-3 > 12$ ✗	Simplify.

▶ The inequality $-3 > 12$ is false. So, there is no solution.

b.

$2(5w - 1) \leq 7 + 10w$	Write the inequality.
$10w - 2 \leq 7 + 10w$	Distributive Property
$-10w \quad -10w$	Subtract $10w$ from each side.
$-2 \leq 7$	Simplify.

▶ The inequality $-2 \leq 7$ is true. So, all real numbers are solutions.

LOOKING FOR STRUCTURE

When the variable terms on each side of an inequality are the same, the constant terms will determine whether the inequality is true or false.

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Solve the inequality.

5. $5x - 12 \leq 3x - 4$

6. $2(k - 5) < 2k + 5$

7. $-4(3n - 1) > -12n + 5.2$

8. $3(2a - 1) \geq 10a - 11$

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**

You need a mean score of at least 90 points to advance to the next round of the touch-screen trivia game. What scores in the fifth game will allow you to advance?

SOLUTION

1. Understand the Problem You know the scores of your first four games. You are asked to find the scores in the fifth game that will allow you to advance.

2. Make a Plan Use the definition of the mean of a set of numbers to write an inequality. Then solve the inequality and answer the question.

3. Solve the Problem Let x be your score in the fifth game.

$$\frac{95 + 91 + 77 + 89 + x}{5} \geq 90$$ Write an inequality.

$$\frac{352 + x}{5} \geq 90$$ Simplify.

$$5 \cdot \frac{352 + x}{5} \geq 5 \cdot 90$$ Multiply each side by 5.

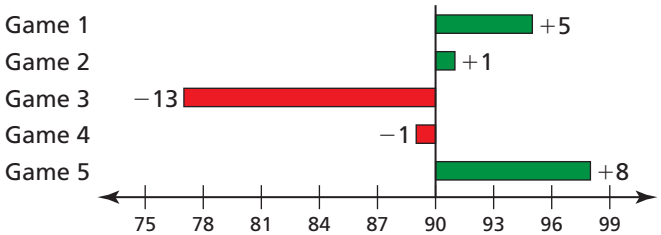
$$352 + x \geq 450$$ Simplify.

$$-352 \quad -352$$ Subtract 352 from each side.

$$x \geq 98$$ Simplify.

▶ A score of at least 98 points will allow you to advance.

4. Look Back You can draw a diagram to check that your answer is reasonable. The horizontal bar graph shows the differences between the game scores and the desired mean of 90.



To have a mean of 90, the sum of the differences must be zero.

$$5 + 1 - 13 - 1 + 8 = 0$$ ✓

REMEMBER

The mean in Example 4 is equal to the sum of the game scores divided by the number of games.

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9. WHAT IF? You need a mean score of at least 85 points to advance to the next round. What scores in the fifth game will allow you to advance?

2.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Compare solving multi-step inequalities and solving multi-step equations.
2. **WRITING** Without solving, how can you tell that the inequality $4x + 8 \leq 4x - 3$ has no solution?

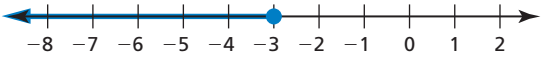
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, match the inequality with its graph.

3. $7b - 4 \leq 10$
4. $4p + 4 \geq 12$
5. $-6g + 2 \geq 20$
6. $3(2 - f) \leq 15$

A.


B.


C.


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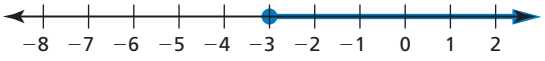


In Exercises 7–16, solve the inequality. Graph the solution. *(See Example 1.)*

7. $2x - 3 > 7$
8. $5y + 9 \leq 4$
9. $-9 \leq 7 - 8v$
10. $2 > -3t - 10$
11. $\frac{w}{2} + 4 > 5$
12. $1 + \frac{m}{3} \leq 6$
13. $\frac{p}{-8} + 9 > 13$
14. $3 + \frac{r}{-4} \leq 6$
15. $6 \geq -6(a + 2)$
16. $18 \leq 3(b - 4)$

In Exercises 17–28, solve the inequality. *(See Examples 2 and 3.)*

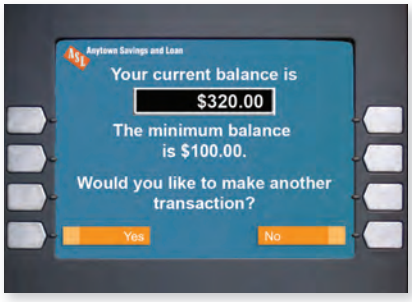
17. $4 - 2m > 7 - 3m$
18. $8n + 2 \leq 8n - 9$
19. $-2d - 2 < 3d + 8$
20. $8 + 10f > 14 - 2f$
21. $8g - 5g - 4 \leq -3 + 3g$
22. $3w - 5 > 2w + w - 7$
23. $6(\ell + 3) < 3(2\ell + 6)$
24. $2(5c - 7) \geq 10(c - 3)$
25. $4\left(\frac{1}{2}t - 2\right) > 2(t - 3)$
26. $15\left(\frac{1}{3}b + 3\right) \leq 6(b + 9)$
27. $9j - 6 + 6j \geq 3(5j - 2)$
28. $6h - 6 + 2h < 2(4h - 3)$

ERROR ANALYSIS **In Exercises 29 and 30, describe and correct the error in solving the inequality.**

29.
$$\frac{x}{4} + 6 \geq 3$$
$$x + 6 \geq 12$$
$$x \geq 6$$

30.
$$-2(1 - x) \leq 2x - 7$$
$$-2 + 2x \leq 2x - 7$$
$$-2 \leq -7$$
All real numbers are solutions.

31. **MODELING WITH MATHEMATICS** Write and solve an inequality that represents how many \$20 bills you can withdraw from the account without going below the minimum balance. *(See Example 4.)*

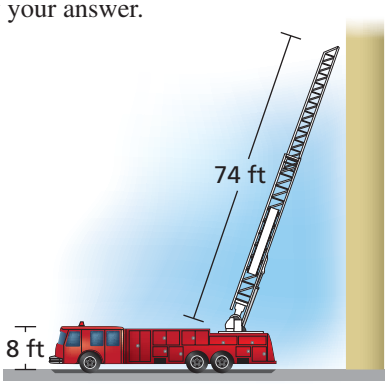


32. MODELING WITH MATHEMATICS A woodworker wants to earn at least $25 an hour making and selling cabinets. He pays $125 for materials. Write and solve an inequality that represents how many hours the woodworker can spend building the cabinet.

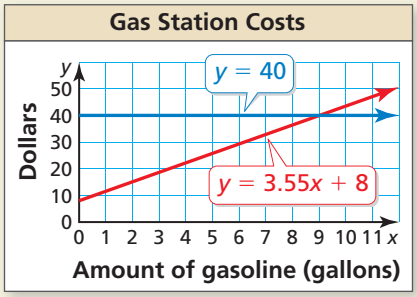
33. MATHEMATICAL CONNECTIONS The area of the rectangle is greater than 60 square feet. Write and solve an inequality to find the possible values of x.

34. MAKING AN ARGUMENT Forest Park Campgrounds charges a $100 membership fee plus $35 per night. Woodland Campgrounds charges a $20 membership fee plus $55 per night. Your friend says that if you plan to camp for four or more nights, then you should choose Woodland Campgrounds. Is your friend correct? Explain.

35. PROBLEM SOLVING The height of one story of a building is about 10 feet. The bottom of the ladder on the fire truck must be at least 24 feet away from the building. How many stories can the ladder reach? Justify your answer.



36. HOW DO YOU SEE IT? The graph shows your budget and the total cost of x gallons of gasoline and a car wash. You want to determine the possible amounts (in gallons) of gasoline you can buy within your budget.



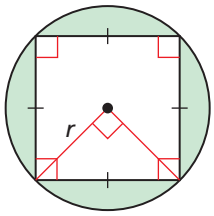
a. What is your budget?

b. How much does a gallon of gasoline cost? How much does a car wash cost?

c. Write an inequality that represents the possible amounts of gasoline you can buy.

d. Use the graph to estimate the solution of your inequality in part (c).

37. PROBLEM SOLVING For what values of r will the area of the shaded region be greater than or equal to $9(\pi - 2)$?



38. THOUGHT PROVOKING A runner's times (in minutes) in the four races he has completed are 25.5, 24.3, 24.8, and 23.5. The runner plans to run at least one more race and wants to have an average time less than 24 minutes. Write and solve an inequality to show how the runner can achieve his goal.

REASONING In Exercises 39 and 40, find the value of a for which the solution of the inequality is all real numbers.

39. $a(x + 3) < 5x + 15 - x$

40. $3x + 8 + 2ax \geq 3ax - 4a$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the sentence as an inequality. *(Section 2.1)*

41. Six times a number y is less than or equal to 10.

42. A number p plus 7 is greater than 24.

43. The quotient of a number r and 7 is no more than 18.

2.1–2.4 What Did You Learn?

Core Vocabulary

inequality, *p. 54*
solution of an inequality, *p. 55*
solution set, *p. 55*
graph of an inequality, *p. 56*
equivalent inequalities, *p. 62*

Core Concepts

Section 2.1

Representing Linear Inequalities, *p. 57*

Section 2.2

Addition Property of Inequality, *p. 62*
Subtraction Property of Inequality, *p. 63*

Section 2.3

Multiplication and Division Properties of Inequality ($c > 0$), *p. 68*
Multiplication and Division Properties of Inequality ($c < 0$), *p. 69*

Section 2.4

Solving Multi-Step Inequalities, *p. 74*
Special Solutions of Linear Inequalities, *p. 75*

Mathematical Practices

1. Explain the meaning of the inequality symbol in your answer to Exercise 47 on page 59. How did you know which symbol to use?
2. In Exercise 30 on page 66, why is it important to check the reasonableness of your answer in part (a) before answering part (b)?
3. Explain how considering the units involved in Exercise 29 on page 71 helped you answer the question.

Study Skills

Analyzing Your Errors

Application Errors

What Happens: You can do numerical problems, but you struggle with problems that have context.

How to Avoid This Error: Do not just mimic the steps of solving an application problem. Explain out loud what the question is asking and why you are doing each step. After solving the problem, ask yourself, "Does my solution make sense?"

2.1–2.4 Quiz

Write the sentence as an inequality. *(Section 2.1)*

1. A number z minus 6 is greater than or equal to 11.

2. Twelve is no more than the sum of -1.5 times a number w and 4.

Write an inequality that represents the graph. *(Section 2.1)*

3.

4.

Solve the inequality. Graph the solution. *(Section 2.2 and Section 2.3)*

5. $9 + q \leq 15$

6. $z - (-7) < 5$

7. $-3 < y - 4$

8. $3p \geq 18$

9. $6 > \frac{w}{-2}$

10. $-20x > 5$

Solve the inequality. *(Section 2.4)*

11. $3y - 7 \geq 17$

12. $8(3g - 2) \leq 12(2g + 1)$

13. $6(2x - 1) \geq 3(4x + 1)$

14. Three requirements for a lifeguard training course are shown. *(Section 2.1)*

a. Write and graph three inequalities that represent the requirements.

b. You can swim 250 feet, tread water for 6 minutes, and swim 35 feet underwater without taking a breath. Do you satisfy the requirements of the course? Explain.

15. The maximum volume of an American white pelican's bill is about 700 cubic inches. A pelican scoops up 100 cubic inches of water. Write and solve an inequality that represents the additional volumes the pelican's bill can contain. *(Section 2.2)*

16. You save \$15 per week to purchase one of the bikes shown. *(Section 2.3 and Section 2.4)*

a. Write and solve an inequality to find the numbers of weeks you need to save to purchase a bike.

b. Your parents give you \$65 to help you buy the new bike. How does this affect you answer in part (a)? Use an inequality to justify your answer.

2.5 Solving Compound Inequalities

COMMON CORE

Learning Standards
HSA-CED.A.1
HSA-REI.B.3

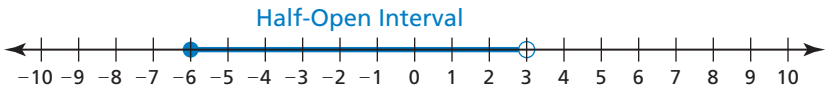
Essential Question How can you use inequalities to describe intervals on the real number line?

EXPLORATION 1 Describing Intervals on the Real Number Line

Work with a partner. In parts (a)–(d), use two inequalities to describe the interval.

REASONING ABSTRACTLY
To be proficient in math, you need to create a clear representation of the problem at hand.

a.



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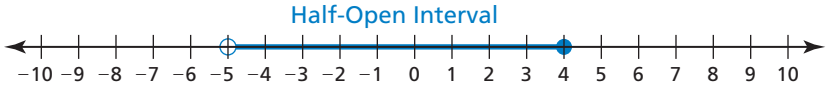
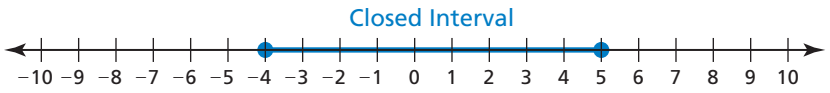
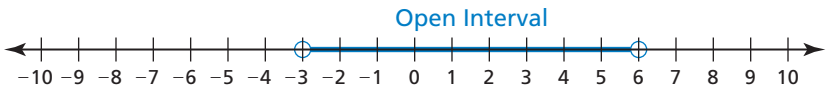


c.



d.

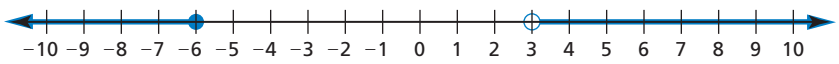


e. Do you use "and" or "or" to connect the two inequalities in parts (a)–(d)? Explain.

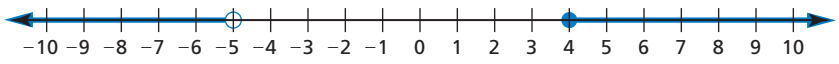
EXPLORATION 2 Describing Two Infinite Intervals

Work with a partner. In parts (a)–(d), use two inequalities to describe the interval.

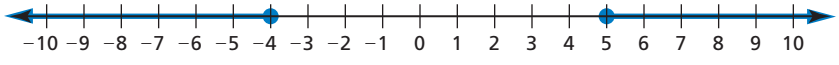
a.



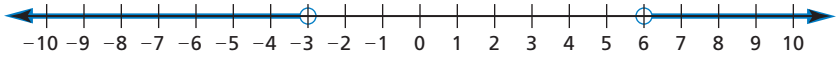
b.



c.



d.



e. Do you use "and" or "or" to connect the two inequalities in parts (a)–(d)? Explain.

Communicate Your Answer

3. How can you use inequalities to describe intervals on the real number line?

2.5 Lesson

Core Vocabulary

compound inequality, *p. 82*

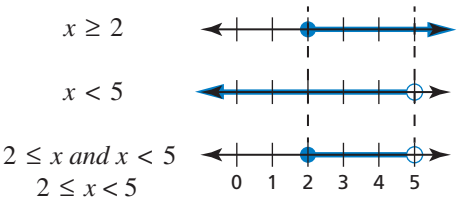
What You Will Learn

- Write and graph compound inequalities.
- Solve compound inequalities.
- Use compound inequalities to solve real-life problems.

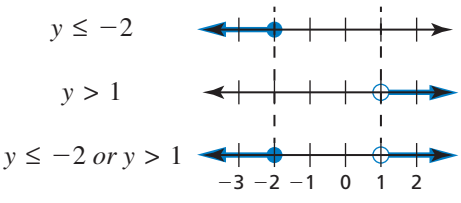
Writing and Graphing Compound Inequalities

A **compound inequality** is an inequality formed by joining two inequalities with the word "and" or the word "or."

The graph of a compound inequality with "and" is the *intersection* of the graphs of the inequalities. The graph shows numbers that are solutions of *both* inequalities.



The graph of a compound inequality with "or" is the *union* of the graphs of the inequalities. The graph shows numbers that are solutions of *either* inequality.



EXAMPLE 1 Writing and Graphing Compound Inequalities

Write each sentence as an inequality. Graph each inequality.

a. A number x is greater than -8 and less than or equal to 4.

b. A number y is at most 0 or at least 2.

REMEMBER

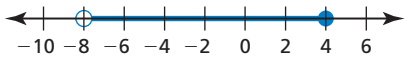
A compound inequality with "and" can be written as a single inequality. For example, you can write $x > -8$ and $x \leq 4$ as $-8 < x \leq 4$.

SOLUTION

a. A number x is greater than -8 and less than or equal to 4.

$x > -8$ *and* $x \leq 4$

▶ An inequality is $-8 < x \leq 4$.

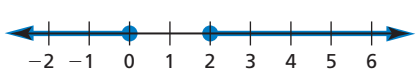


Graph the intersection of the graphs of $x > -8$ and $x \leq 4$.

b. A number y is at most 0 or at least 2.

$y \leq 0$ *or* $y \geq 2$

▶ An inequality is $y \leq 0$ *or* $y \geq 2$.



Graph the union of the graphs of $y \leq 0$ and $y \geq 2$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Write the sentence as an inequality. Graph the inequality.

1. A number d is more than 0 and less than 10.
2. A number a is fewer than -6 or no less than -3.

Solving Compound Inequalities

You can solve a compound inequality by solving two inequalities separately. When a compound inequality with "and" is written as a single inequality, you can solve the inequality by performing the same operation on each expression.

LOOKING FOR STRUCTURE

To be proficient in math, you need to see complicated things as single objects or as being composed of several objects.

EXAMPLE 2 Solving Compound Inequalities with "And"

Solve each inequality. Graph each solution.

a. $-4 < x - 2 < 3$ **b.** $-3 < -2x + 1 \leq 9$

SOLUTION

a. Separate the compound inequality into two inequalities, then solve.

$-4 < x - 2$ *and* $x - 2 < 3$ Write two inequalities.

$+2 \quad +2$ $+2 \quad +2$ Add 2 to each side.

$-2 < x$ *and* $x < 5$ Simplify.

▶ The solution is $-2 < x < 5$.

b. $-3 < -2x + 1 \leq 9$ Write the inequality.

$-1 \quad -1 \quad -1$ Subtract 1 from each expression.

$-4 < -2x \leq 8$ Simplify.

$\frac{-4}{-2} > \frac{-2x}{-2} \geq \frac{8}{-2}$ Divide each expression by -2. Reverse each inequality symbol.

$2 > x \geq -4$ Simplify.

▶ The solution is $-4 \leq x < 2$.

EXAMPLE 3 Solving a Compound Inequality with "Or"

Solve $3y - 5 < -8$ *or* $2y - 1 > 5$. Graph the solution.

SOLUTION

$3y - 5 < -8$ *or* $2y - 1 > 5$ Write the inequality.

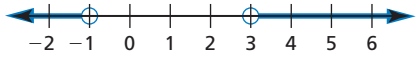
$+5 \quad +5$ $+1 \quad +1$ Addition Property of Inequality

$3y < -3$ $2y > 6$ Simplify.

$\frac{3y}{3} < \frac{-3}{3}$ $\frac{2y}{2} > \frac{6}{2}$ Division Property of Inequality

$y < -1$ *or* $y > 3$ Simplify.

▶ The solution is $y < -1$ *or* $y > 3$.



Monitoring Progress

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Solve the inequality. Graph the solution.

3. $5 \leq m + 4 < 10$ **4.** $-3 < 2k - 5 < 7$

5. $4c + 3 \leq -5$ *or* $c - 8 > -1$ **6.** $2p + 1 < -7$ *or* $3 - 2p \leq -1$

Solving Real-Life Problems

Operating temperature:
0°C to 35°C

EXAMPLE 4 **Modeling with Mathematics**

Electrical devices should operate effectively within a specified temperature range. Outside the operating temperature range, the device may fail.

a. Write and solve a compound inequality that represents the possible operating temperatures (in degrees Fahrenheit) of the smartphone.

b. Describe one situation in which the surrounding temperature could be below the operating range and one in which it could be above.

SOLUTION

1. Understand the Problem You know the operating temperature range in degrees Celsius. You are asked to write and solve a compound inequality that represents the possible operating temperatures (in degrees Fahrenheit) of the smartphone. Then you are asked to describe situations outside this range.

2. Make a Plan Write a compound inequality in degrees Celsius. Use the formula $C = \frac{5}{9}(F - 32)$ to rewrite the inequality in degrees Fahrenheit. Then solve the inequality and describe the situations.

3. Solve the Problem Let C be the temperature in degrees Celsius, and let F be the temperature in degrees Fahrenheit.

STUDY TIP

You can also solve the inequality by multiplying each expression by $\frac{9}{5}$.

$$0 \le C \le 35 \qquad \text{Write the inequality using } C.$$

$$0 \le \tfrac{5}{9}(F - 32) \le 35 \qquad \text{Substitute } \tfrac{5}{9}(F - 32) \text{ for } C.$$

$$9 \cdot 0 \le 9 \cdot \tfrac{5}{9}(F - 32) \le 9 \cdot 35 \qquad \text{Multiply each expression by 9.}$$

$$0 \le 5(F - 32) \le 315 \qquad \text{Simplify.}$$

$$0 \le 5F - 160 \le 315 \qquad \text{Distributive Property}$$

$$+\,160 \qquad +\,160 \qquad +\,160 \qquad \text{Add 160 to each expression.}$$

$$160 \le 5F \le 475 \qquad \text{Simplify.}$$

$$\frac{160}{5} \le \frac{5F}{5} \le \frac{475}{5} \qquad \text{Divide each expression by 5.}$$

$$32 \le F \le 95 \qquad \text{Simplify.}$$

▶ The solution is $32 \le F \le 95$. So, the operating temperature range of the smartphone is 32°F to 95°F. One situation when the surrounding temperature could be below this range is winter in Alaska. One situation when the surrounding temperature could be above this range is daytime in the Mojave Desert of the American Southwest.

4. Look Back You can use the formula $C = \frac{5}{9}(F - 32)$ to check that your answer is correct. Substitute 32 and 95 for F in the formula to verify that 0°C and 35°C are the minimum and maximum operating temperatures in degrees Celsius.

−40°C to 15°C

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. Write and solve a compound inequality that represents the temperature rating (in degrees Fahrenheit) of the winter boots.

2.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. WRITING Compare the graph of $-6 \le x \le -4$ with the graph of $x \le -6$ *or* $x \ge -4$.

2. WHICH ONE DOESN'T BELONG? Which compound inequality does *not* belong with the other three? Explain your reasoning.

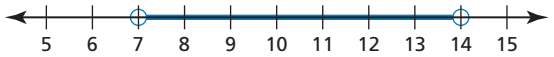
$a > 4$ *or* $a < -3$ | $a < -2$ *or* $a > 8$ | $a > 7$ *or* $a < -5$ | $a < 6$ *or* $a > -9$

Monitoring Progress and Modeling with Mathematics

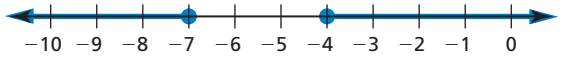
In Exercises 3–6, write a compound inequality that is represented by the graph.

3.

4.



5.



6.

In Exercises 7–10, write the sentence as an inequality. Graph the inequality. *(See Example 1.)*

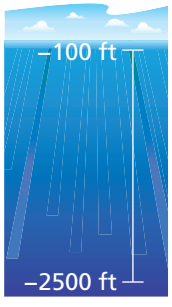
7. A number p is less than 6 and greater than 2.

8. A number n is less than or equal to -7 or greater than 12.

9. A number m is more than $-7\frac{2}{3}$ or at most -10.

10. A number r is no less than -1.5 and fewer than 9.5.

11. MODELING WITH MATHEMATICS Slitsnails are large mollusks that live in deep waters. They have been found in the range of elevations shown. Write and graph a compound inequality that represents this range.



12. MODELING WITH MATHEMATICS The life zones on Mount Rainier, a mountain in Washington, can be approximately classified by elevation, as follows.

Low-elevation forest: above 1700 feet to 2500 feet
Mid-elevation forest: above 2500 feet to 4000 feet
Subalpine: above 4000 feet to 6500 feet
Alpine: above 6500 feet to the summit

Elevation of Mount Rainier: 14,410 ft

Write a compound inequality that represents the elevation range for each type of plant life.

a. trees in the low-elevation forest zone

b. flowers in the subalpine and alpine zones

In Exercises 13–20, solve the inequality. Graph the solution. *(See Examples 2 and 3.)*

13. $6 < x + 5 \le 11$

14. $24 > -3r \ge -9$

15. $v + 8 < 3$ *or* $-8v < -40$

16. $-14 > w + 3$ *or* $3w \ge -27$

17. $2r + 3 < 7$ *or* $-r + 9 \le 2$

18. $-6 < 3n + 9 < 21$

19. $-12 < \frac{1}{2}(4x + 16) < 18$

20. $35 < 7(2 - b)$ *or* $\frac{1}{3}(15b - 12) \ge 21$

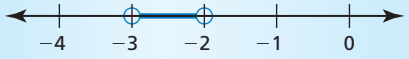
ERROR ANALYSIS **In Exercises 21 and 22, describe and correct the error in solving the inequality or graphing the solution.**

21.

$4 < -2x + 3 < 9$

$4 < -2x < 6$

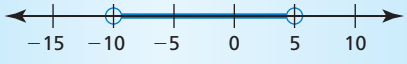
$-2 > x > -3$



22.

$x - 2 > 3$ or $x + 8 < -2$

$x > 5$ or $x < -10$



23. **MODELING WITH MATHEMATICS** Write and solve a compound inequality that represents the possible temperatures (in degrees Fahrenheit) of the interior of the iceberg. *(See Example 4.)*

24. **PROBLEM SOLVING** A ski shop sells skis with lengths ranging from 150 centimeters to 220 centimeters. The shop says the length of the skis should be about 1.16 times a skier's height (in centimeters). Write and solve a compound inequality that represents the heights of skiers the shop does *not* provide skis for.

In Exercises 25–30, solve the inequality. Graph the solution, if possible.

25. $22 < -3c + 4 < 14$

26. $2m - 1 \geq 5$ *or* $5m > -25$

27. $-y + 3 \leq 8$ *and* $y + 2 > 9$

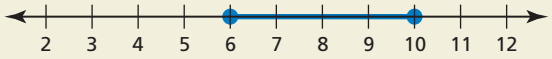
28. $x - 8 \leq 4$ *or* $2x + 3 > 9$

29. $2n + 19 \leq 10 + n$ *or* $-3n + 3 < -2n + 33$

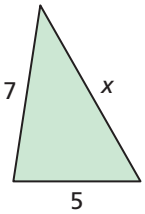
30. $3x - 18 < 4x - 23$ *and* $x - 16 < -22$

31. **REASONING** Fill in the compound inequality $4(x - 6)$ ☐ $2(x - 10)$ *and* $5(x + 2) \geq 2(x + 8)$ with $<$, $\leq$, $>$, or $\geq$ so that the solution is only one value.

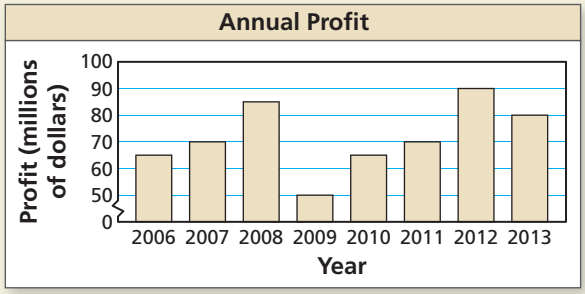
32. **THOUGHT PROVOKING** Write a real-life story that can be modeled by the graph.



33. **MAKING AN ARGUMENT** The sum of the lengths of any two sides of a triangle is greater than the length of the third side. Use the triangle shown to write and solve three inequalities. Your friend claims the value of x can be 1. Is your friend correct? Explain.



34. **HOW DO YOU SEE IT?** The graph shows the annual profits of a company from 2006 to 2013.



a. Write a compound inequality that represents the annual profits from 2006 to 2013.

b. You can use the formula $P = R - C$ to find the profit P, where R is the revenue and C is the cost. From 2006 to 2013, the company's annual cost was about \$125 million. Is it possible the company had an annual revenue of \$160 million from 2006 to 2013? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Graph the solutions, if possible. *(Section 1.4)*

35. $\left|\frac{d}{9}\right| = 6$ **36.** $7|5p - 7| = -21$ **37.** $|r + 2| = |3r - 4|$ **38.** $\left|\frac{1}{2}w - 6\right| = |w + 7|$

Find and interpret the mean absolute deviation of the data. *(Skills Review Handbook)*

39. 1, 1, 2, 5, 6, 8, 10, 12, 12, 13

40. 24, 26, 28, 28, 30, 30, 32, 32, 34, 36

2.6 Solving Absolute Value Inequalities

Common Core

Learning Standards
HSA-CED.A.1
HSA-REI.B.3

Essential Question

How can you solve an absolute value inequality?

EXPLORATION 1 Solving an Absolute Value Inequality Algebraically

Work with a partner. Consider the absolute value inequality

$$|x + 2| \leq 3.$$

a. Describe the values of $x + 2$ that make the inequality true. Use your description to write two linear inequalities that represent the solutions of the absolute value inequality.

b. Use the linear inequalities you wrote in part (a) to find the solutions of the absolute value inequality.

c. How can you use linear inequalities to solve an absolute value inequality?

MAKING SENSE OF PROBLEMS

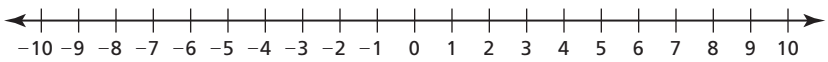
To be proficient in math, you need to explain to yourself the meaning of a problem and look for entry points to its solution.

EXPLORATION 2 Solving an Absolute Value Inequality Graphically

Work with a partner. Consider the absolute value inequality

$$|x + 2| \leq 3.$$

a. On a real number line, locate the point for which $x + 2 = 0$.



b. Locate the points that are within 3 units from the point you found in part (a). What do you notice about these points?

c. How can you use a number line to solve an absolute value inequality?

EXPLORATION 3 Solving an Absolute Value Inequality Numerically

Work with a partner. Consider the absolute value inequality

$$|x + 2| \leq 3.$$

a. Use a spreadsheet, as shown, to solve the absolute value inequality.

b. Compare the solutions you found using the spreadsheet with those you found in Explorations 1 and 2. What do you notice?

c. How can you use a spreadsheet to solve an absolute value inequality?

	A	B
1	*x*	|*x* + 2|
2	-6	4
3	-5	
4	-4	
5	-3	
6	-2	
7	-1	
8	0	
9	1	
10	2	

abs(A2 + 2)

Communicate Your Answer

4. How can you solve an absolute value inequality?

5. What do you like or dislike about the algebraic, graphical, and numerical methods for solving an absolute value inequality? Give reasons for your answers.

2.6 Lesson

What You Will Learn

- Solve absolute value inequalities.
- Use absolute value inequalities to solve real-life problems.

Core Vocabulary

absolute value inequality, *p. 88*
absolute deviation, *p. 90*

Previous
compound inequality
mean

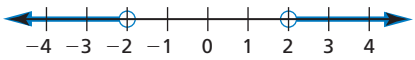
Solving Absolute Value Inequalities

An **absolute value inequality** is an inequality that contains an absolute value expression. For example, $|x| < 2$ and $|x| > 2$ are absolute value inequalities. Recall that $|x| = 2$ means the distance between x and 0 is 2.

The inequality $|x| < 2$ means the distance between x and 0 is *less than* 2.

The graph of $|x| < 2$ is the graph of $x > -2$ *and* $x < 2$.

The inequality $|x| > 2$ means the distance between x and 0 is *greater than* 2.



The graph of $|x| > 2$ is the graph of $x < -2$ *or* $x > 2$.

You can solve these types of inequalities by solving a compound inequality.

Core Concept

Solving Absolute Value Inequalities

To solve $|ax + b| < c$ for $c > 0$, solve the compound inequality

$$ax + b > -c \quad \textit{and} \quad ax + b < c.$$

To solve $|ax + b| > c$ for $c > 0$, solve the compound inequality

$$ax + b < -c \quad \textit{or} \quad ax + b > c.$$

In the inequalities above, you can replace $<$ with $\leq$ and $>$ with $\geq$.

EXAMPLE 1 Solving Absolute Value Inequalities

Solve each inequality. Graph each solution, if possible.

a. $|x + 7| \leq 2$ **b.** $|8x - 11| < 0$

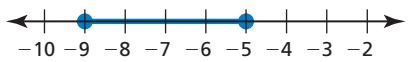
SOLUTION

a. Use $|x + 7| \leq 2$ to write a compound inequality. Then solve.

$$x + 7 \geq -2 \quad \textit{and} \quad x + 7 \leq 2 \qquad \text{Write a compound inequality.}$$
$$-7 \quad -7 \qquad\qquad -7 \quad -7 \qquad \text{Subtract 7 from each side.}$$
$$x \geq -9 \quad \textit{and} \quad x \leq -5 \qquad \text{Simplify.}$$

REMEMBER
A compound inequality with "and" can be written as a single inequality.

▶ The solution is $-9 \leq x \leq -5$.



b. By definition, the absolute value of an expression must be greater than or equal to 0. The expression $|8x - 11|$ cannot be less than 0.

▶ So, the inequality has no solution.

Monitoring Progress

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Solve the inequality. Graph the solution, if possible.

1. $|x| \leq 3.5$ **2.** $|k - 3| < -1$ **3.** $|2w - 1| < 11$

EXAMPLE 2 Solving Absolute Value Inequalities

Solve each inequality. Graph each solution.

a. $|c - 1| \geq 5$ **b.** $|10 - m| \geq -2$ **c.** $4|2x - 5| + 1 > 21$

SOLUTION

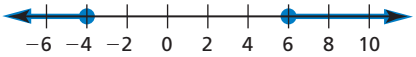
a. Use $|c - 1| \geq 5$ to write a compound inequality. Then solve.

$c - 1 \leq -5$ *or* $c - 1 \geq 5$ Write a compound inequality.

$+1 \quad +1$ $+1 \quad +1$ Add 1 to each side.

$c \leq -4$ *or* $c \geq 6$ Simplify.

▶ The solution is $c \leq -4$ *or* $c \geq 6$.



b. By definition, the absolute value of an expression must be greater than or equal to 0. The expression $|10 - m|$ will always be greater than -2.

▶ So, all real numbers are solutions.

c. First isolate the absolute value expression on one side of the inequality.

$4|2x - 5| + 1 > 21$ Write the inequality.

$-1 \quad -1$ Subtract 1 from each side.

$4|2x - 5| > 20$ Simplify.

$\frac{4|2x - 5|}{4} > \frac{20}{4}$ Divide each side by 4.

$|2x - 5| > 5$ Simplify.

Then use $|2x - 5| > 5$ to write a compound inequality. Then solve.

$2x - 5 < -5$ *or* $2x - 5 > 5$ Write a compound inequality.

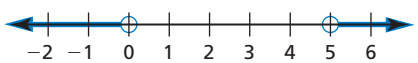
$+5 \quad +5$ $+5 \quad +5$ Add 5 to each side.

$2x < 0$ $2x > 10$ Simplify.

$\frac{2x}{2} < \frac{0}{2}$ $\frac{2x}{2} > \frac{10}{2}$ Divide each side by 2.

$x < 0$ *or* $x > 5$ Simplify.

▶ The solution is $x < 0$ *or* $x > 5$.



Monitoring Progress

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Solve the inequality. Graph the solution.

4. $|x + 3| > 8$ **5.** $|n + 2| - 3 \geq -6$ **6.** $3|d + 1| - 7 \geq -1$

Solving Real-Life Problems

The **absolute deviation** of a number x from a given value is the absolute value of the difference of x and the given value.

$$\text{absolute deviation} = |x - \text{given value}|$$

EXAMPLE 3 Modeling with Mathematics

Computer prices	
$890	$750
$650	$370
$660	$670
$450	$650
$725	$825

You are buying a new computer. The table shows the prices of computers in a store advertisement. You are willing to pay the mean price with an absolute deviation of at most $100. How many of the computer prices meet your condition?

SOLUTION

1. **Understand the Problem** You know the prices of 10 computers. You are asked to find how many computers are at most $100 from the mean price.

2. **Make a Plan** Calculate the mean price by dividing the sum of the prices by the number of prices, 10. Use the absolute deviation and the mean price to write an absolute value inequality. Then solve the inequality and use it to answer the question.

3. **Solve the Problem**

The mean price is $\frac{6640}{10} = \$664$. Let x represent a price you are willing to pay.

$$|x - 664| \leq 100 \qquad \text{Write the absolute value inequality.}$$

$$-100 \leq x - 664 \leq 100 \qquad \text{Write a compound inequality.}$$

$$564 \leq x \leq 764 \qquad \text{Add 664 to each expression and simplify.}$$

▶ The prices you will consider must be at least $564 and at most $764. Six prices meet your condition: $750, $650, $660, $670, $650, and $725.

4. **Look Back** You can check that your answer is correct by graphing the computer prices and the mean on a number line. Any point within 100 of 664 represents a price that you will consider.

STUDY TIP

The absolute deviation of at most $100 from the mean, $664, is given by the inequality $|x - 664| \leq 100$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. **WHAT IF?** You are willing to pay the mean price with an absolute deviation of at most $75. How many of the computer prices meet your condition?

Concept Summary

Solving Inequalities

One-Step and Multi-Step Inequalities

- Follow the steps for solving an equation. Reverse the inequality symbol when multiplying or dividing by a negative number.

Compound Inequalities

- If necessary, write the inequality as two separate inequalities. Then solve each inequality separately. Include *and* or *or* in the solution.

Absolute Value Inequalities

- If necessary, isolate the absolute value expression on one side of the inequality. Write the absolute value inequality as a compound inequality. Then solve the compound inequality.

2.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **REASONING** Can you determine the solution of $|4x - 2| \geq -6$ without solving? Explain.
2. **WRITING** Describe how solving $|w - 9| \leq 2$ is different from solving $|w - 9| \geq 2$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–18, solve the inequality. Graph the solution, if possible. *(See Examples 1 and 2.)*

3. $|x| < 3$
4. $|y| \geq 4.5$
5. $|d + 9| > 3$
6. $|h - 5| \leq 10$
7. $|2s - 7| \geq -1$
8. $|4c + 5| > 7$
9. $|5p + 2| < -4$
10. $|9 - 4n| < 5$
11. $|6t - 7| - 8 \geq 3$
12. $|3j - 1| + 6 > 0$
13. $3|14 - m| > 18$
14. $-4|6b - 8| \leq 12$
15. $2|3w + 8| - 13 \leq -5$
16. $-3|2 - 4u| + 5 < -13$
17. $6|-f + 3| + 7 > 7$
18. $\frac{2}{3}|4v + 6| - 2 \leq 10$

19. **MODELING WITH MATHEMATICS** The rules for an essay contest say that entries can have 500 words with an absolute deviation of at most 30 words. Write and solve an absolute value inequality that represents the acceptable numbers of words. *(See Example 3.)*

20. **MODELING WITH MATHEMATICS** The normal body temperature of a camel is 37°C. This temperature varies by up to 3°C throughout the day. Write and solve an absolute value inequality that represents the range of normal body temperatures (in degrees Celsius) of a camel throughout the day.

ERROR ANALYSIS **In Exercises 21 and 22, describe and correct the error in solving the absolute value inequality.**

21. ✗
$$|x - 5| < 20$$
$$x - 5 < 20$$
$$x < 25$$

22. ✗
$$|x + 4| > 13$$
$$x + 4 > -13 \quad \text{and} \quad x + 4 < 13$$
$$x > -17 \quad \text{and} \quad x < 9$$
$$-17 < x < 9$$

In Exercises 23–26, write the sentence as an absolute value inequality. Then solve the inequality.

23. A number is less than 6 units from 0.
24. A number is more than 9 units from 3.
25. Half of a number is at most 5 units from 14.
26. Twice a number is no less than 10 units from -1.

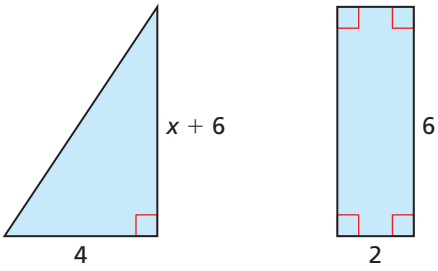
27. **PROBLEM SOLVING** An auto parts manufacturer throws out gaskets with weights that are not within 0.06 pound of the mean weight of the batch. The weights (in pounds) of the gaskets in a batch are 0.58, 0.63, 0.65, 0.53, and 0.61. Which gasket(s) should be thrown out?

28. **PROBLEM SOLVING** Six students measure the acceleration (in meters per second per second) of an object in free fall. The measured values are shown. The students want to state that the absolute deviation of each measured value x from the mean is at most d. Find the value of d.

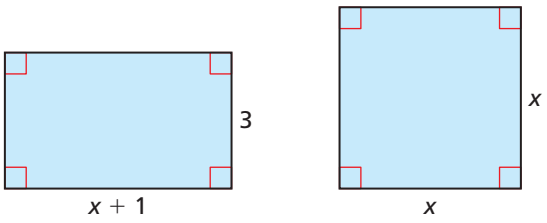
10.56, 9.52, 9.73, 9.80, 9.78, 10.91

MATHEMATICAL CONNECTIONS **In Exercises 29 and 30, write an absolute value inequality that represents the situation. Then solve the inequality.**

29. The difference between the areas of the figures is less than 2.



30. The difference between the perimeters of the figures is less than or equal to 3.



REASONING **In Exercises 31–34, tell whether the statement is true or false. If it is false, explain why.**

31. If a is a solution of $|x + 3| \leq 8$, then a is also a solution of $x + 3 \geq -8$.

32. If a is a solution of $|x + 3| > 8$, then a is also a solution of $x + 3 > 8$.

33. If a is a solution of $|x + 3| \geq 8$, then a is also a solution of $x + 3 \geq -8$.

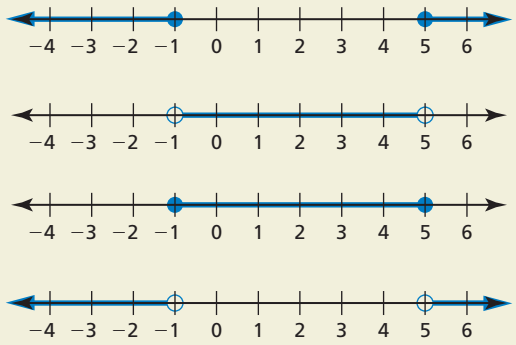
34. If a is a solution of $x + 3 \leq -8$, then a is also a solution of $|x + 3| \geq 8$.

35. MAKING AN ARGUMENT One of your classmates claims that the solution of $|n| > 0$ is all real numbers. Is your classmate correct? Explain your reasoning.

36. THOUGHT PROVOKING Draw and label a geometric figure so that the perimeter P of the figure is a solution of the inequality $|P - 60| \leq 12$.

37. REASONING What is the solution of the inequality $|ax + b| < c$, where $c < 0$? What is the solution of the inequality $|ax + b| > c$, where $c < 0$? Explain.

38. HOW DO YOU SEE IT? Write an absolute value inequality for each graph.



How did you decide which inequality symbol to use for each inequality?

39. WRITING Explain why the solution set of the inequality $|x| < 5$ is the *intersection* of two sets, while the solution set of the inequality $|x| > 5$ is the *union* of two sets.

40. PROBLEM SOLVING Solve the compound inequality below. Describe your steps.

$$|x - 3| < 4 \text{ and } |x + 2| > 8$$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Plot the ordered pair in a coordinate plane. Describe the location of the point. *(Skills Review Handbook)*

41. $A(1, 3)$ **42.** $B(0, -3)$ **43.** $C(-4, -2)$ **44.** $D(-1, 2)$

Copy and complete the table. *(Skills Review Handbook)*

45.

x	0	1	2	3	4
$5x + 1$					

46.

x	−2	−1	0	1	2
$-2x - 3$					

2.5–2.6 What Did You Learn?

Core Vocabulary

compound inequality, *p. 82*
absolute value inequality, *p. 88*
absolute deviation, *p. 90*

Core Concepts

Section 2.5

Writing and Graphing Compound Inequalities, *p. 82*
Solving Compound Inequalities, *p. 83*

Section 2.6

Solving Absolute Value Inequalities, *p. 88*

Mathematical Practices

1. How can you use a diagram to help you solve Exercise 12 on page 85?
2. In Exercises 13 and 14 on page 85, how can you use structure to break down the compound inequality into two inequalities?
3. Describe the given information and the overall goal of Exercise 27 on page 91.
4. For false statements in Exercises 31–34 on page 92, use examples to show the statements are false.

Performance Task

Grading Calculations

You are not doing as well as you had hoped in one of your classes. So, you want to figure out the minimum grade you need on the final exam to receive the semester grade that you want. Is it still possible to get an A? How would you explain your calculations to a classmate?

To explore the answers to this question and more, go to ***BigIdeasMath.com***.

2 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

2.1 Writing and Graphing Inequalities *(pp. 53–60)*

a. A number x plus 36 is no more than 40. Write this sentence as an inequality.

A number x plus 36 is no more than 40.

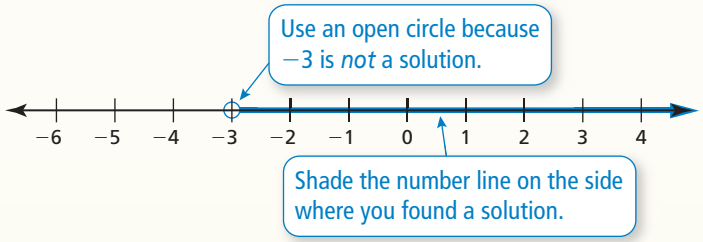
$x + 36 \quad \leq \quad 40$

▶ An inequality is $x + 36 \leq 40$.

b. Graph $w > -3$.

Test a number to the left of -3. $w = -4$ is *not* a solution.

Test a number to the right of -3. $w = 0$ is a solution.



Write the sentence as an inequality.

1. A number d minus 2 is less than -1.
2. Ten is at least the product of a number h and 5.

Graph the inequality.

3. $x > 4$
4. $y \leq 2$
5. $-1 \geq z$

2.2 Solving Inequalities Using Addition or Subtraction *(pp. 61–66)*

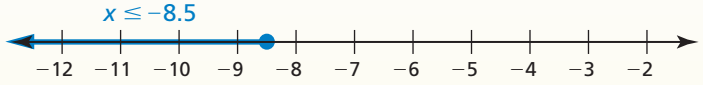
Solve $x + 2.5 \leq -6$. Graph the solution.

$x + 2.5 \leq -6$ Write the inequality.

Subtraction Property of Inequality → $-2.5 \quad -2.5$ Subtract 2.5 from each side.

$x \leq -8.5$ Simplify.

▶ The solution is $x \leq -8.5$.



Solve the inequality. Graph the solution.

6. $p + 4 < 10$
7. $r - 4 < -6$
8. $2.1 \geq m - 6.7$

2.3 Solving Inequalities Using Multiplication or Division *(pp. 67–72)*

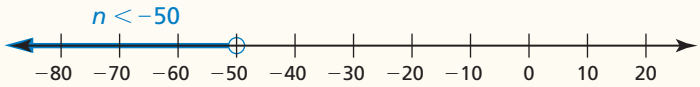
Solve $\frac{n}{-10} > 5$. Graph the solution.

$$\frac{n}{-10} > 5$$ Write the inequality.

Multiplication Property of Inequality → $-10 \cdot \frac{n}{-10} < -10 \cdot 5$ Multiply each side by -10. Reverse the inequality symbol.

$$n < -50$$ Simplify.

▶ The solution is $n < -50$.



Solve the inequality. Graph the solution.

9. $3x > -21$

10. $-4 \leq \frac{g}{5}$

11. $-\frac{3}{4}n \leq 3$

12. $\frac{s}{-8} \geq 11$

13. $36 < 2q$

14. $-1.2k > 6$

2.4 Solving Multi-Step Inequalities *(pp. 73–78)*

Solve $22 + 3y \geq 4$. Graph the solution.

$22 + 3y \geq 4$ Write the inequality.

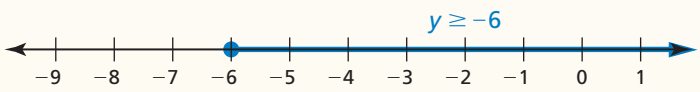
$-22 \quad -22$ Subtract 22 from each side.

$3y \geq -18$ Simplify.

$\frac{3y}{3} \geq \frac{-18}{3}$ Divide each side by 3.

$y \geq -6$ Simplify.

▶ The solution is $y \geq -6$.



Solve the inequality. Graph the solution, if possible.

15. $3x - 4 > 11$

16. $-4 < \frac{b}{2} + 9$

17. $7 - 3n \leq n + 3$

18. $2(-4s + 2) \geq -5s - 10$

19. $6(2t + 9) \leq 12t - 1$

20. $3r - 8 > 3(r - 6)$

2.5 Solving Compound Inequalities *(pp. 81–86)*

Solve $-1 \le -2d + 7 \le 9$. Graph the solution.

$-1 \le -2d + 7 \le 9$ — Write the inequality.

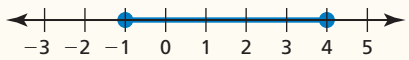
$\underline{-7} \quad \underline{-7} \quad \underline{-7}$ — Subtract 7 from each expression.

$-8 \le -2d \le 2$ — Simplify.

$\frac{-8}{-2} \ge \frac{-2d}{-2} \ge \frac{2}{-2}$ — Divide each expression by -2. Reverse each inequality symbol.

$4 \ge d \ge -1$ — Simplify.

▶ The solution is $-1 \le d \le 4$.



21. A number x is more than -6 and at most 8. Write this sentence as an inequality. Graph the inequality.

Solve the inequality. Graph the solution.

22. $19 \ge 3z + 1 \ge -5$

23. $\frac{r}{4} < -5$ *or* $-2r - 7 \le 3$

2.6 Solving Absolute Value Inequalities *(pp. 87–92)*

Solve $|2x + 11| + 3 > 8$. Graph the solution.

$|2x + 11| + 3 > 8$ — Write the inequality.

$\underline{-3} \quad \underline{-3}$ — Subtract 3 from each side.

$|2x + 11| > 5$ — Simplify.

$2x + 11 < -5$ *or* $2x + 11 > 5$ — Write a compound inequality.

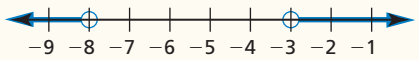
$\underline{-11} \quad \underline{-11} \qquad \underline{-11} \quad \underline{-11}$ — Subtract 11 from each side.

$2x < -16 \qquad 2x > -6$ — Simplify.

$\frac{2x}{2} < \frac{-16}{2} \qquad \frac{2x}{2} > \frac{-6}{2}$ — Divide each side by 2.

$x < -8$ *or* $x > -3$ — Simplify.

▶ The solution is $x < -8$ *or* $x > -3$.



Solve the inequality. Graph the solution, if possible.

24. $|m| \ge 10$

25. $|k - 9| < -4$

26. $4|f - 6| \le 12$

27. $5|b + 8| - 7 > 13$

28. $|-3g - 2| + 1 < 6$

29. $|9 - 2j| + 10 \ge 2$

30. A safety regulation states that the height of a guardrail should be 106 centimeters with an absolute deviation of no more than 7 centimeters. Write and solve an absolute value inequality that represents the acceptable heights of a guardrail.

2 Chapter Test

Write the sentence as an inequality.

1. The sum of a number y and 9 is at least -1.
2. A number r is more than 0 or less than or equal to -8.
3. A number k is less than 3 units from 10.

Solve the inequality. Graph the solution, if possible.

4. $\frac{x}{2} - 5 \geq -9$
5. $-4s < 6s + 1$
6. $4p + 3 \geq 2(2p + 1)$
7. $-7 < 2c - 1 < 10$
8. $-2 \leq 4 - 3a \leq 13$
9. $-5 < 2 - h$ *or* $6h + 5 > 71$
10. $|2q + 8| > 4$
11. $-2|y - 3| - 5 \geq -4$
12. $4|-3b + 5| - 9 < 7$

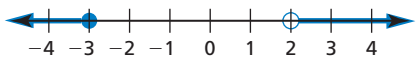
13. You start a small baking business, and you want to earn a profit of at least \$250 in the first month. The expenses in the first month are \$155. What are the possible revenues that you need to earn to meet the profit goal?

14. A manufacturer of bicycle parts requires that a bicycle chain have a width of 0.3 inch with an absolute deviation of at most 0.0003 inch. Write and solve an absolute value inequality that represents the acceptable widths.

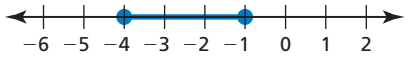
15. Let a, b, c, and d be constants. Describe the possible solution sets of the inequality $ax + b < cx + d$.

Write and graph a compound inequality that represents the numbers that are *not* solutions of the inequality represented by the graph shown. Explain your reasoning.

16.



17.

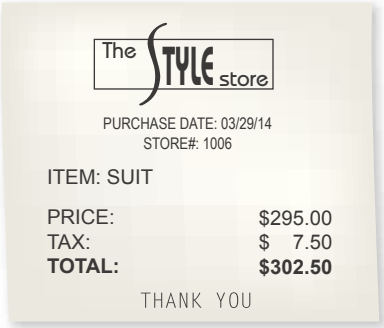


18. A state imposes a sales tax on items of clothing that cost more than \$175. The tax applies only to the difference of the price of the item and \$175.

   a. Use the receipt shown to find the tax rate (as a percent).

   b. A shopper has \$430 to spend on a winter coat. Write and solve an inequality to find the prices p of coats that the shopper can afford. Assume that $p \geq 175$.

   c. Another state imposes a 5% sales tax on the entire price of an item of clothing. For which prices would paying the 5% tax be cheaper than paying the tax described above? Write and solve an inequality to find your answer and list three prices that are solutions.



2 Standards Assessment

1. The expected attendance at a school event is 65 people. The actual attendance can vary by up to 30 people. Which equation can you use to find the minimum and maximum attendances? *(HSA-CED.A.1)*

Ⓐ $|x - 65| = 30$ Ⓑ $|x + 65| = 30$

Ⓒ $|x - 30| = 65$ Ⓓ $|x + 30| = 65$

2. Fill in values for a and b so that each statement is true for the inequality $ax + 4 \leq 3x + b$. *(HSA-REI.B.3)*

a. When $a = 5$ and $b =$ _____, $x \leq -3$.

b. When $a =$ _____ and $b =$ _____, the solution of the inequality is all real numbers.

c. When $a =$ _____ and $b =$ _____, the inequality has no solution.

3. Place each inequality into one of the two categories. *(HSA-REI.B.3)*

At least one integer solution	No integer solutions

$5x - 6 + x \geq 2x - 8$

$2(3x + 8) > 3(2x + 6)$

$17 < 4x + 5 < 21$

$x - 8 + 4x \leq 3(x - 3) + 2x$

$9x - 3 < 12$ *or* $6x + 2 > -10$

$5(x - 1) \leq 5x - 3$

4. Admission to a play costs $25. A season pass costs $180. *(HSA-CED.A.1)*

a. Write an inequality that represents the numbers x of plays you must attend for the season pass to be a better deal.

b. Select the numbers of plays for which the season pass is *not* a better deal.

0	1	2	3	4
5	6	7	8	9
10	11	12	13	14

5. Select the values of a that make the solution of the equation $3(2x - 4) = 4(ax - 2)$ positive. *(HSA-REI.B.3)*

-2	-1	0	1	2	3	4	5

6. Fill in the compound inequality with $<$, $\leq$, $=$, $\geq$, or $>$ so the solution is shown in the graph. *(HSA-CED.A.1)*

$4x - 18$ ☐ $-x - 3$ *and* $-3x - 9$ ☐ -3

7. You have a \$250 gift card to use at a sporting goods store. *(HSA-CED.A.1)*

a. Write an inequality that represents the possible numbers x of pairs of socks you can buy when you buy 2 pairs of sneakers. Can you buy 8 pairs of socks? Explain.

b. Describe what the inequality $60 + 80x \leq 250$ represents in this context.

8. Consider the equation shown, where a, b, c, and d are integers. *(HSA-REI.B.3)*

$$ax + b = cx + d$$

Student A claims the equation will always have one solution. Student B claims the equation will always have no solution. Use the numbers shown to answer parts (a)–(c).

-1	0	1	2	3	4	5	6

a. Select values for a, b, c, and d to create an equation that supports Student A's claim.

b. Select values for a, b, c, and d to create an equation that supports Student B's claim.

c. Select values for a, b, c, and d to create an equation that shows both Student A and Student B are incorrect.