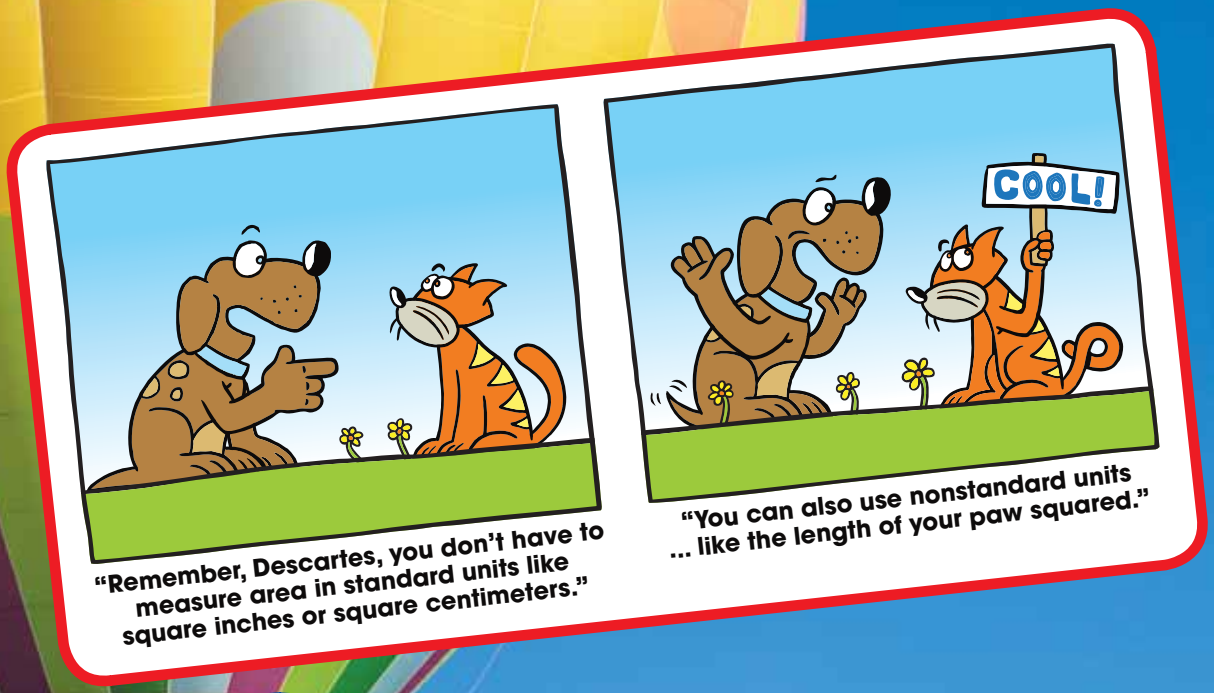
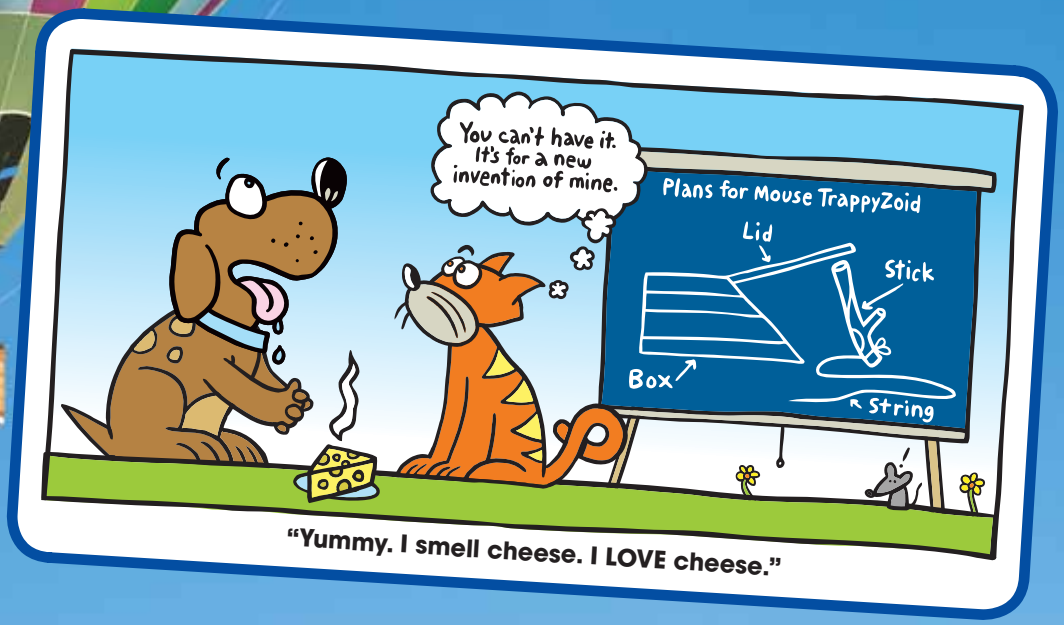
4 Areas of Polygons



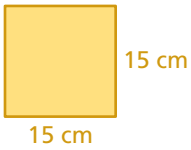


What You Learned Before

Finding Areas of Squares and Rectangles (4.MD.3)

Example 1 **Find the area of the square or rectangle.**

a.



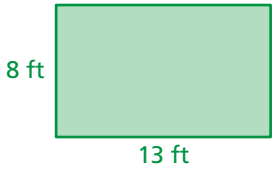
$A = s^2$ Write formula.

$= 15^2$ Substitute.

$= 225$ Simplify.

∴ The area of the square is 225 square centimeters.

b.



$A = \ell w$

$= 13(8)$

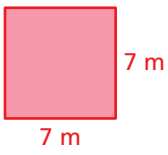
$= 104$

∴ The area of the rectangle is 104 square feet.

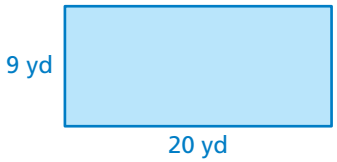
Try It Yourself

Find the area of the square or rectangle.

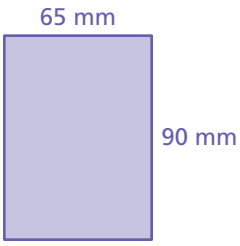
1.



2.



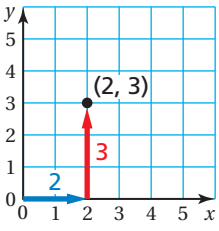
3.



Plotting Ordered Pairs (5.G.1)

Example 2 **Plot (2, 3) in a coordinate plane.**

Start at the origin. Move 2 units right and 3 units up. Then plot the point.



Try It Yourself

Plot the ordered pair in a coordinate plane.

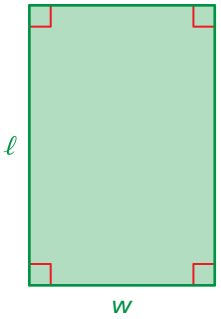
4. (1, 4) **5.** (3, 2) **6.** (5, 1)

4.1 Areas of Parallelograms

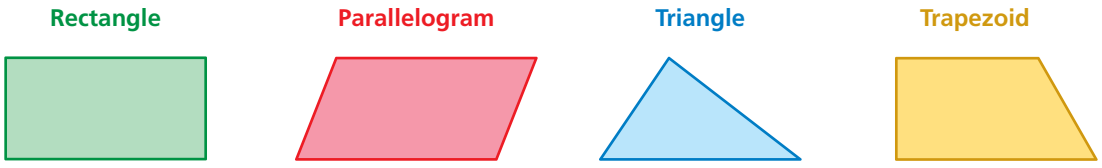
Essential Question How can you derive a formula for the area of a parallelogram?

A **polygon** is a closed figure in a plane that is made up of three or more line segments that intersect only at their endpoints. Several examples of polygons are parallelograms, triangles, and trapezoids.

The formulas for the areas of polygons can be derived from one area formula, the area of a rectangle. Recall that the area of a rectangle is the product of its length ℓ and its width w. The process you use to derive these other formulas is called *deductive reasoning*.



Area $= \ell w$



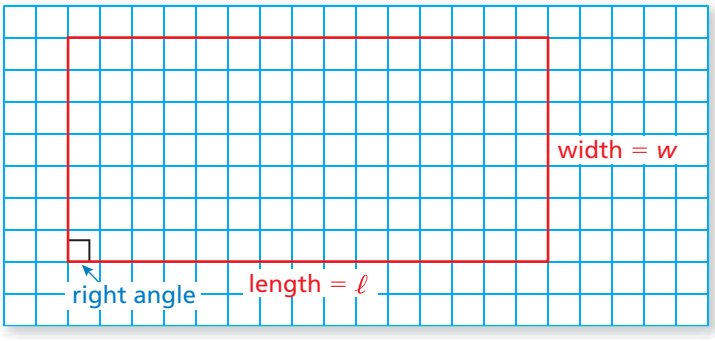
Defined → **Derive formula.** → **Derive formula.** → **Derive formula.**

Grades 4 and 5 | *Lesson 4.1* | *Lesson 4.2* | *Lesson 4.3*

1 ACTIVITY: Deriving the Area Formula of a Parallelogram

Work with a partner.

a. Draw *any* rectangle on a piece of grid paper. An example is shown below. Label the length and width. Then find the area of your rectangle.



b. Cut your rectangle into two pieces to form a parallelogram. Compare the area of the rectangle with the area of the parallelogram. What do you notice? Use your results to write a formula for the area A of a parallelogram.

$A =$ _____ Formula

Common Core

Geometry

In this lesson, you will
- find areas of parallelograms.
- solve real-life problems.

Learning Standard 6.G.1

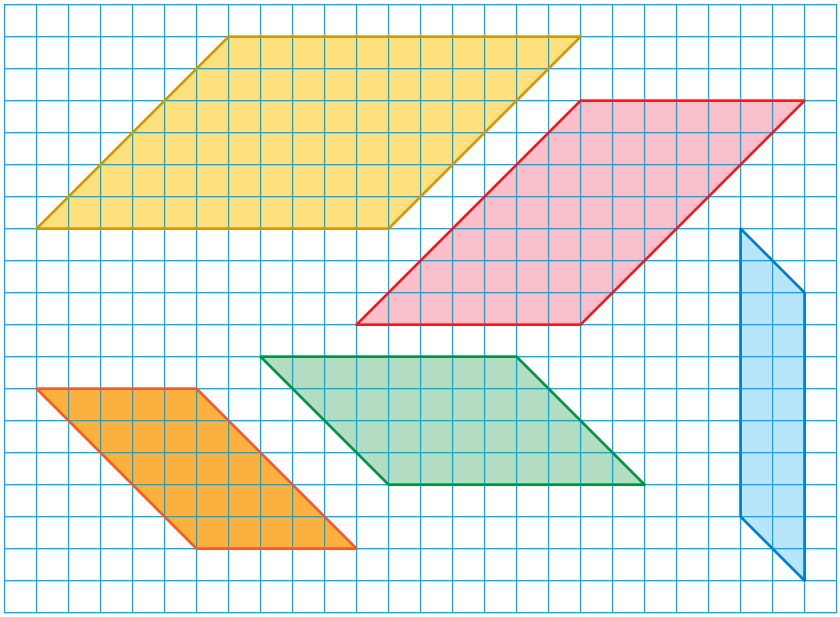
2 ACTIVITY: Finding Areas of Parallelograms

Math Practice 3

Use Assumptions

How are rectangles and parallelograms similar? How can you use this information to solve the problem?

Work with a partner.



a. Find the area of each parallelogram by cutting it into two pieces to form a rectangle.

b. Use the formula you wrote in Activity 1 to find the area of each parallelogram. Compare your answers to those in part (a).

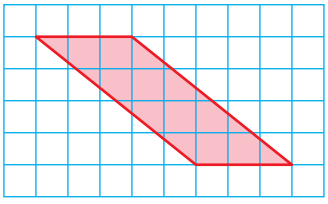
c. Count unit squares for each parallelogram to check your results.

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you derive a formula for the area of a parallelogram?

4. **REASONING** The areas of a rectangle and a parallelogram are equal. The length of a rectangle is equal to the base of the parallelogram. What can you say about the width of the rectangle and the height of the parallelogram? Draw a diagram to support your answer.

5. What is the height of the parallelogram shown? How do you know?



Practice Use what you learned about the areas of parallelograms to complete Exercises 3–5 on page 156.

4.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
polygon, *p. 152*

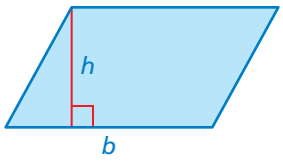
The *area* of a polygon is the amount of surface it covers. You can find the area of a parallelogram in much the same way as you can find the area of a rectangle.

Key Idea

Area of a Parallelogram

Words The area A of a parallelogram is the product of its base b and its height h.

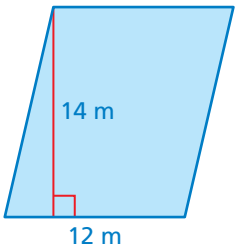
Algebra $A = bh$



EXAMPLE 1 Finding Areas of Parallelograms

Find the area of each parallelogram.

a.



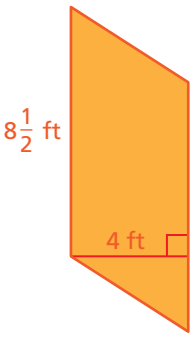
$A = bh$ Write formula.

$= 12(14)$ Substitute values.

$= 168$ Multiply.

∴ The area of the parallelogram is 168 square meters.

b.



$A = bh$

$= 8\frac{1}{2}(4)$

$= 34$

∴ The area of the parallelogram is 34 square feet.

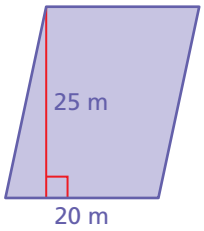
Remember
Area is measured in square units.

On Your Own

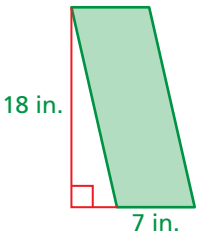
Now You're Ready
Exercises 3–8

Find the area of the parallelogram.

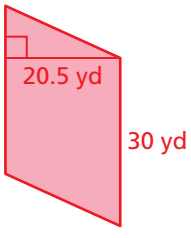
1.



2.

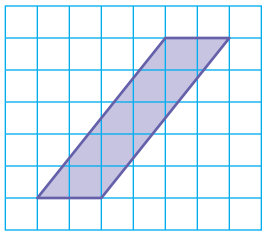


3.



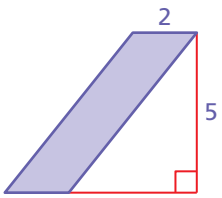
Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Finding the Area of a Parallelogram on a Grid



Find the area of the parallelogram.

Count grid lines to find the dimensions.
The base b is 2 units, and the height h is 5 units.



$A = bh$ Write formula.

$= 2(5)$ Substitute values.

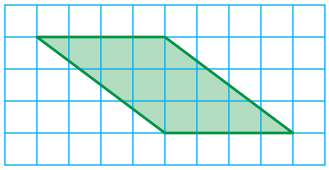
$= 10$ Multiply.

∴ The area of the parallelogram is 10 square units.

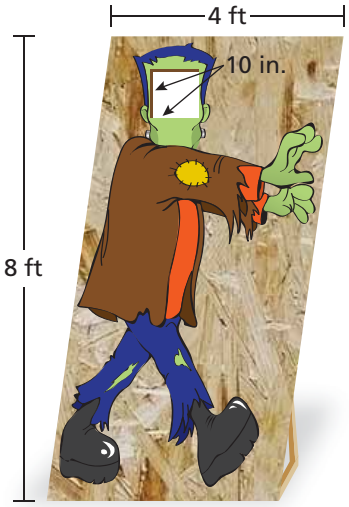
On Your Own

Now You're Ready
Exercises 11–13

4. Find the area of the parallelogram.



EXAMPLE 3 Real-Life Application



You make a photo prop for a school fair. You cut a 10-inch square out of a parallelogram-shaped piece of wood. What is the area of the photo prop?

Convert the dimensions of the piece of wood to inches.

There are 12 inches in 1 foot, so the base is $4 \cdot 12 = 48$ inches and the height is $8 \cdot 12 = 96$ inches.

Use a verbal model to solve the problem.

area of photo prop = area of wood − area of square

$= 96(48) - 10^2$ Substitute.

$= 96(48) - 100$ Evaluate 10^2.

$= 4608 - 100$ Multiply 96 and 48.

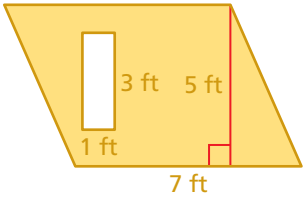
$= 4508$ Subtract 100 from 4608.

∴ The area of the photo prop is 4508 square inches.

On Your Own

Now You're Ready
Exercises 14–16

5. Find the area of the shaded region.



6. **WHAT IF?** In Example 3, you cut a 12-inch square out of the piece of wood. What is the area of the photo prop?

4.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** What is the area of a polygon? Explain how the perimeter and the area of the polygon are different.
2. **CHOOSE TOOLS** Construct a parallelogram that has an area of 24 square inches. Explain your method.

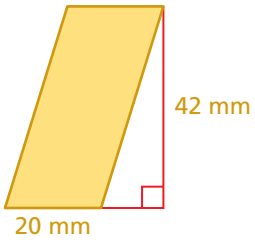
Practice and Problem Solving

Find the area of the parallelogram.

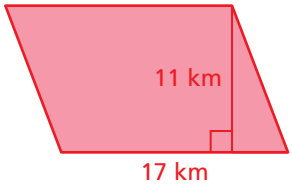
1

3.

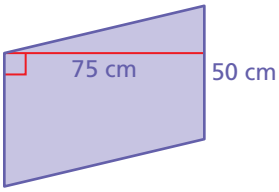
4.



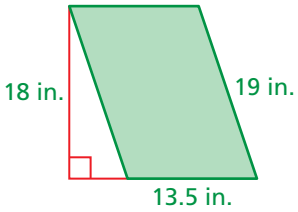
5.



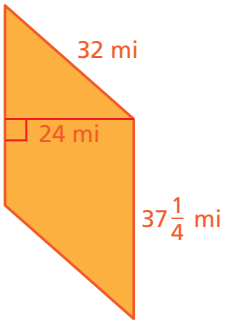
6.



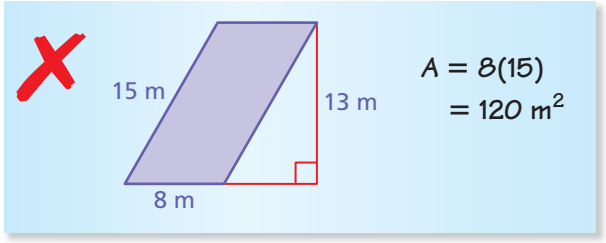
7.



8.



9. **ERROR ANALYSIS** Describe and correct the error in finding the area of the parallelogram.

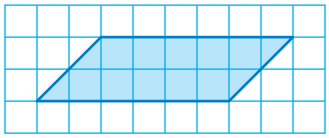


10. **CERAMIC TILE** A ceramic tile in the shape of a parallelogram has a base of 4 inches and a height of 1.5 inches. What is the area of the tile?

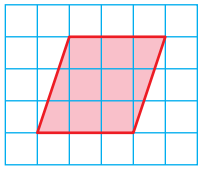
Find the area of the parallelogram.

2

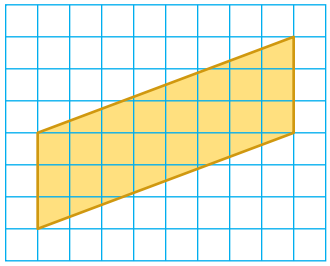
11.



12.

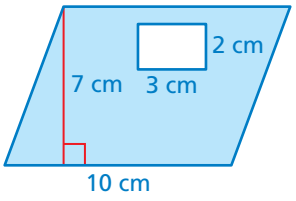


13.

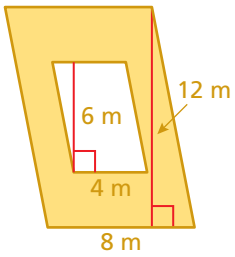


Find the area of the shaded region.

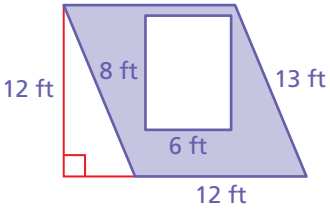
3 **14.**



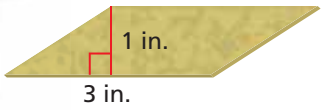
15.



16.

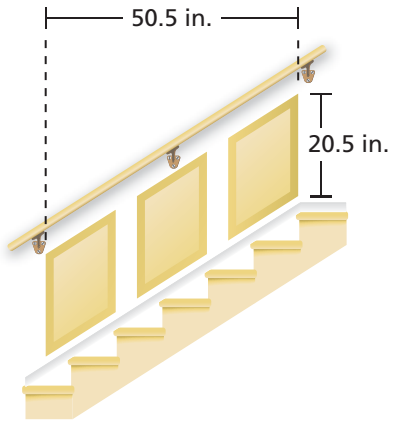


17. **DECK** Your deck has an area of 128 square feet. After adding a section, the area will be $s^2 + 128$ square feet. Draw a diagram of how this can happen.



18. **T-SHIRT DESIGN** You use the parallelogram-shaped sponge to create the T-shirt design. The area of the design is 66 square inches. How many times do you use the sponge to create the design? Draw a diagram to support your answer.

19. **STAIRCASE** The staircase has three parallelogram-shaped panels that are the same size. The horizontal distance between each panel is 4.25 inches. What is the area of one panel?



20. **REASONING** Find the missing dimensions in the table.

Parallelogram	Base	Height	Area
A	$x + 4$		$5x + 20$
B		8	$8x - 24$
C	6		$12x + 6y$

21. **Logic** Each dimension of a parallelogram is multiplied by a positive number n. Write an expression for the area of the new parallelogram.

Fair Game Review What you learned in previous grades & lessons

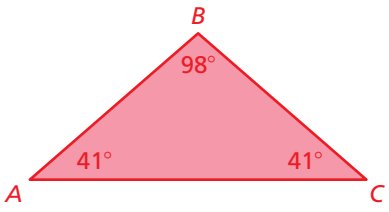
Use mental math to multiply. *(Skills Review Handbook)*

22. $\frac{1}{2} \times 26$ **23.** 82×20 **24.** 16×30 **25.** $\frac{1}{2} \times 236$

26. **MULTIPLE CHOICE** Which of the following describes angle B? *(Skills Review Handbook)*

Ⓐ acute Ⓑ obtuse

Ⓒ right Ⓓ isosceles



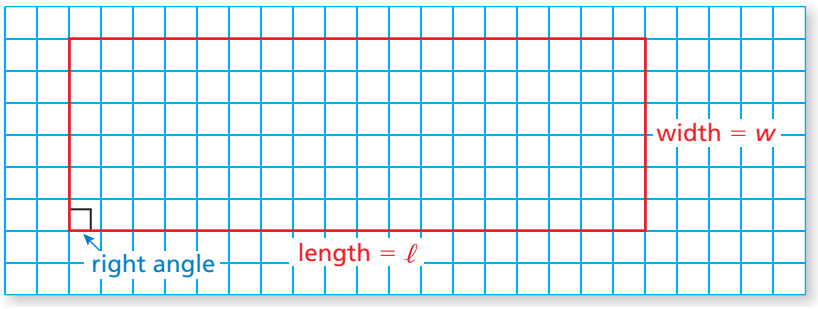
4.2 Areas of Triangles

Essential Question How can you derive a formula for the area of a triangle?

1 ACTIVITY: Deriving the Area Formula of a Triangle

Work with a partner.

a. Draw *any* rectangle on a piece of grid paper. An example is shown below. Label the length and width. Then find the area of your rectangle.



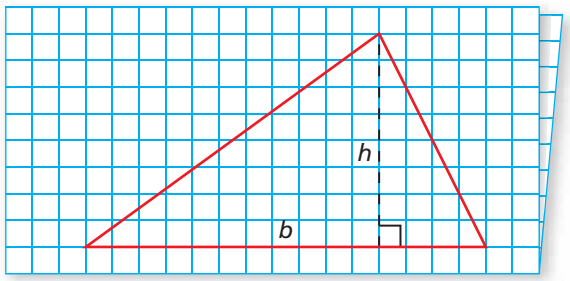
b. Draw a diagonal from one corner of your rectangle to the opposite corner. Cut along the diagonal. Compare the area of the rectangle with the area of the two pieces you cut. What do you notice? Use your results to write a formula for the area A of a triangle.

$A =$ _____ Formula

2 ACTIVITY: Deriving the Area Formula of a Triangle

Work with a partner.

a. Fold a piece of grid paper in half. Draw a triangle so that its base lies on one of the horizontal lines of the paper. Do not use a right triangle. Label the height and the base *inside* the triangle.



fold

b. Estimate the area of your triangle by counting unit squares.

Area ≈ _____ Estimate

c. Cut out the triangle so that you end up with two identical triangles. Form a quadrilateral whose area you know. What type of quadrilateral is it? Explain how you *know* it is this type.

d. Use your results to write a formula for the area of a triangle. Then use your formula to find the exact area of your triangle. Compare this area with your estimate in part (b).

Common Core

Geometry

In this lesson, you will
- find areas of triangles.
- solve real-life problems.

Learning Standard 6.G.1

Math Practice 6

Calculate Accurately

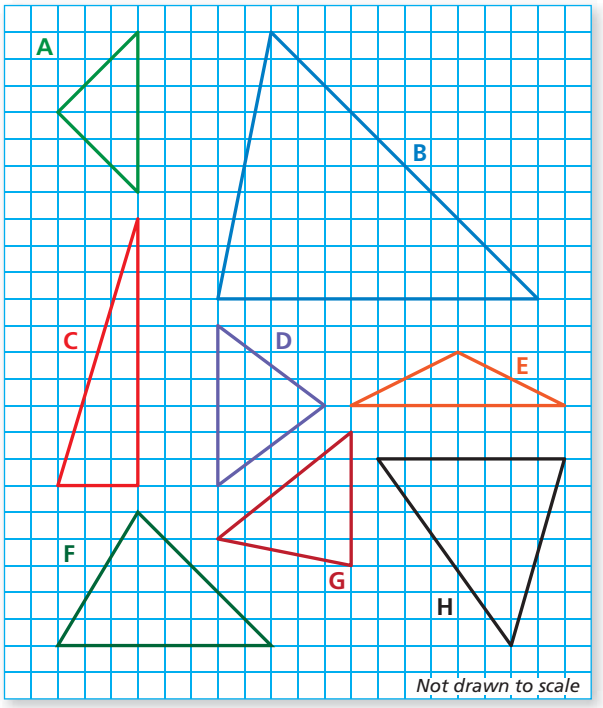
How can you estimate the area of each triangle so that the answer is close to the exact area?

3 ACTIVITY: Estimating and Finding the Area of a Triangle

Work with a partner. Each grid square represents 1 square centimeter.

- **Use estimation to match each triangle with its area.**
- **Then check your work by finding the exact area of each triangle.**

	Area	*Estimate Match*	*Exact Match*
a.	$15\ \text{cm}^2$		
b.	$20\ \text{cm}^2$		
c.	$9\ \text{cm}^2$		
d.	$12\ \text{cm}^2$		
e.	$60\ \text{cm}^2$		
f.	$12\frac{1}{2}\ \text{cm}^2$		
g.	$24\frac{1}{2}\ \text{cm}^2$		
h.	$8\ \text{cm}^2$		



What Is Your Answer?

4. **PARTNER ACTIVITY** Use a piece of centimeter grid paper to create your own "triangle matching activity." Trade with your partner and solve each other's matching activity.

5. **IN YOUR OWN WORDS** How can you derive a formula for the area of a triangle?

Practice Use what you learned about the areas of triangles to complete Exercises 3–5 on page 162.

4.2 Lesson

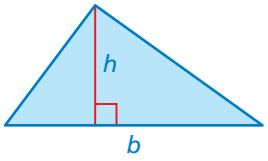
Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Idea

Area of a Triangle

Words The area A of a triangle is one-half the product of its base b and its height h.

Algebra $A = \frac{1}{2}bh$



EXAMPLE 1 Finding the Area of a Triangle

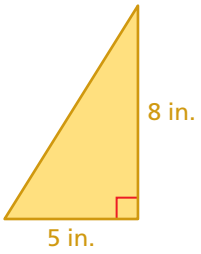
Find the area of the triangle.

$A = \frac{1}{2}bh$ Write formula.

$= \frac{1}{2}(5)(8)$ Substitute 5 for b and 8 for h.

$= \frac{1}{2}(40)$ Multiply 5 and 8.

$= 20$ Multiply $\frac{1}{2}$ and 40.

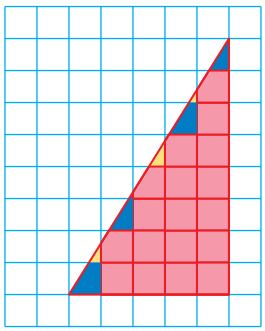


Remember

In Example 1, use the Associative Property of Multiplication to multiply 5 and 8 first.

∴ The area of the triangle is 20 square inches.

Reasonable? Draw the triangle on grid paper and count unit squares. Each square in the grid represents 1 square inch.



Squares full or nearly full: 18

Squares about half full: 4

The area is $18(1) + 4\left(\frac{1}{2}\right) = 20$ square inches.

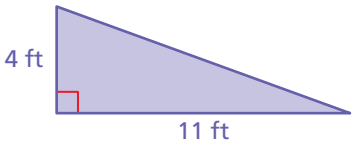
So, the answer is reasonable. ✓

On Your Own

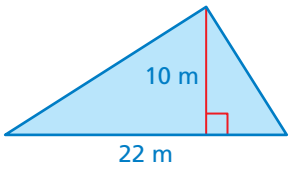
Now You're Ready
Exercises 3–8

Find the area of the triangle.

1.

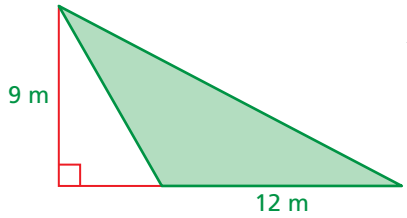


2.



EXAMPLE 2 Finding the Area of a Triangle

Find the area of the triangle.



$A = \frac{1}{2}bh$ Write formula.

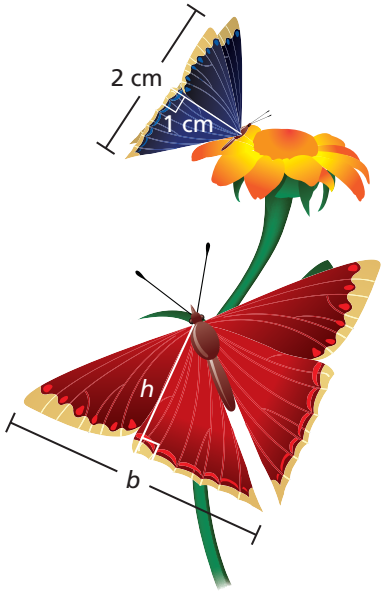
$= \frac{1}{2}(12)(9)$ Substitute 12 for b and 9 for h.

$= 54$ Multiply.

∴ The area of the triangle is 54 square meters.

EXAMPLE 3 Real-Life Application

The base and height of the red butterfly wing are two times greater than the base and height of the blue butterfly wing. How many times greater is the area of the red wing than the area of the blue wing?



Find the area of the blue wing.

$A = \frac{1}{2}bh$ Write formula.

$= \frac{1}{2}(2)(1)$ Substitute 2 for b and 1 for h.

$= 1 \text{ cm}^2$ Multiply.

The red wing dimensions are 2 times greater, so the base is $2 \times 2 = 4$ cm and the height is $2 \times 1 = 2$ cm. Find the area of the red wing.

$A = \frac{1}{2}bh$ Write formula.

$= \frac{1}{2}(4)(2)$ Substitute 4 for b and 2 for h.

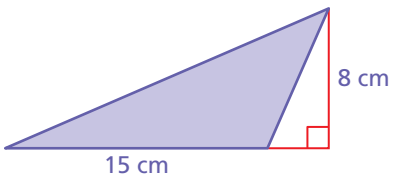
$= 4 \text{ cm}^2$ Multiply.

∴ Because $\frac{4 \text{ cm}^2}{1 \text{ cm}^2} = 4$, the area of the red wing is 4 times greater.

On Your Own

Now You're Ready
Exercises 12–14

3. Find the area of the triangle.



4. WHAT IF? In Example 3, the base and the height of the red butterfly wing are three times greater than those of the blue wing. How many times greater is the area of the red wing?

4.2 Exercises

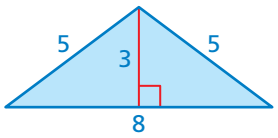
Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **CRITICAL THINKING** Can *any* side of a triangle be labeled as its base? Explain.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What is the area of the triangle?	What is the distance around the triangle?
How many unit squares fit in the triangle?	What is one-half the product of the base and the height?

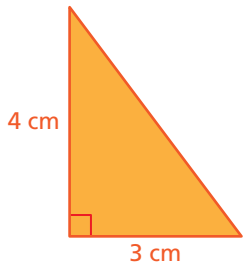


Practice and Problem Solving

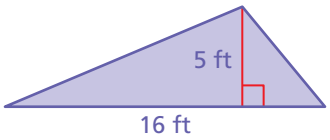
Find the area of the triangle.

1

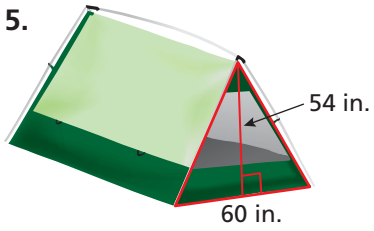
3.



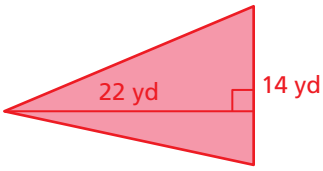
4.



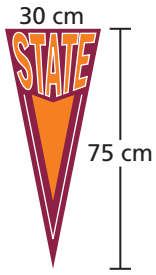
5.



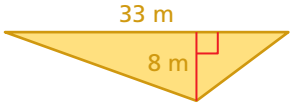
6.



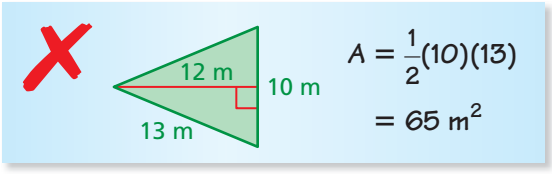
7.



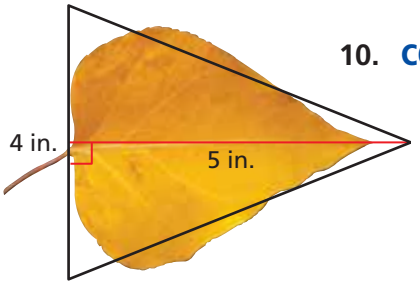
8.



9. **ERROR ANALYSIS** Describe and correct the error in finding the area of the triangle.



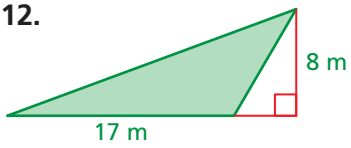
10. **COTTONWOOD LEAF** Estimate the area of the cottonwood leaf.



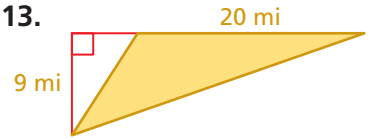
11. **CORNER SHELF** A shelf has the shape of a triangle. The base of the shelf is 36 centimeters, and the height is 18 centimeters. Find the area of the shelf.

Find the area of the triangle.

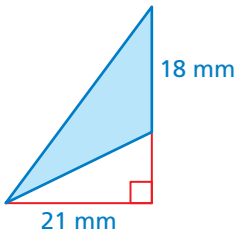
12.



13.

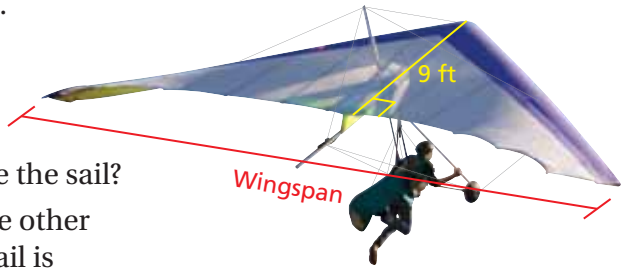


14.



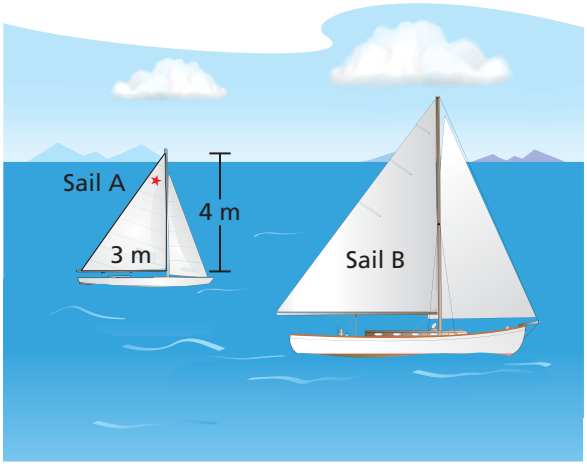
15. OPEN-ENDED Draw and label two triangles that each have an area of 24 square feet.

16. HANG GLIDING The wingspan of the triangular hang glider is 30 feet.



a. How much fabric is needed to make the sail?

b. RESEARCH Use the Internet or some other source to find how the area of the sail is related to the weight limit of the pilot.



17. SAILBOATS The base and the height of Sail B are x times greater than the base and the height of Sail A. How many times greater is the area of Sail B? Write your answer as a power.

18. WRITING You know the height and the perimeter of an equilateral triangle. Explain how to find the area of the triangle. Draw a diagram to support your reasoning.

19. REASONING The base and the height of Triangle A are half the base and the height of Triangle B. How many times greater is the area of Triangle B?

20. Critical Thinking The total area of the polygon is 176 square feet. Find the value of x.

Fair Game Review What you learned in previous grades & lessons

Tell which property is illustrated by the statement. *(Section 3.3)*

21. $n \cdot 1 = n$ **22.** $4 \cdot m = m \cdot 4$ **23.** $(x + 2) + 5 = x + (2 + 5)$

24. MULTIPLE CHOICE What is the first step when using order of operations? *(Section 1.3)*

(A) Multiply or divide from left to right. (B) Add or subtract from left to right.

(C) Perform operations in parentheses. (D) Evaluate numbers with exponents.

4 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

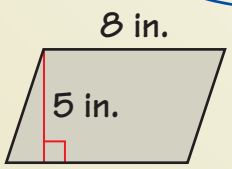
You can use a **four square** to organize information about a topic. Each of the four squares can be a category, such as *definition, vocabulary, example, non-example, words, algebra, table, numbers, visual, graph,* or *equation*. Here is an example of a four square for the area of a parallelogram.

Area of a parallelogram

Words
The area A of a parallelogram is the product of its base b and its height h.

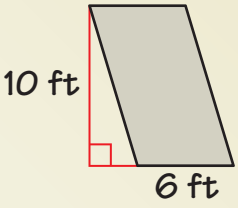
Algebra
$A = bh$

Example
$A = bh$
$= 8(5)$
$= 40$



The area of the parallelogram is 40 square inches.

Example
$A = bh$
$= 6(10)$
$= 60$



The area of the parallelogram is 60 square feet.

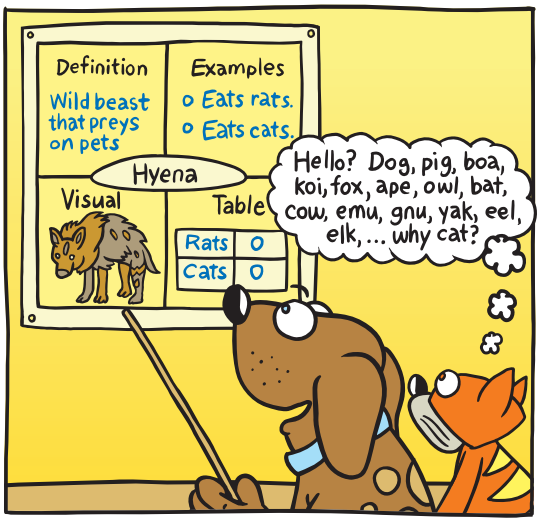
On Your Own

Make a four square to help you study the topic.

1. area of a triangle

After you complete this chapter, make four squares for the following topics.

2. area of a trapezoid
3. area of a composite figure
4. drawing a polygon in a coordinate plane
5. finding distances in the first quadrant



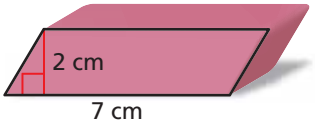
"Sorry, but I have limited space in my four square. I needed pet names with only three letters."

4.1–4.2 Quiz

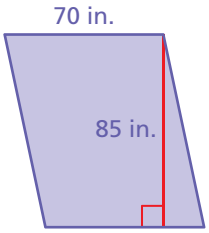
Check It Out
Progress Check
BigIdeasMath.com

Find the area of the parallelogram. *(Section 4.1)*

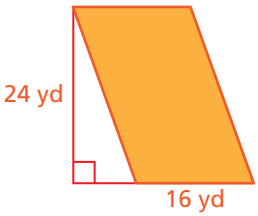
1.



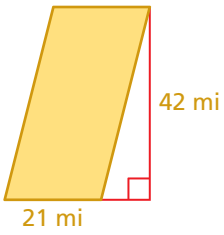
2.



3.

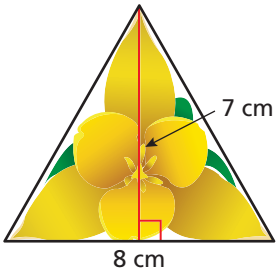


4.

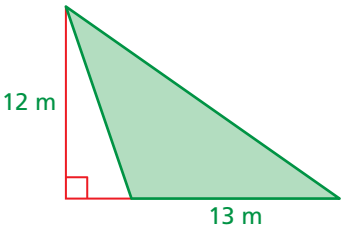


Find the area of the triangle. *(Section 4.2)*

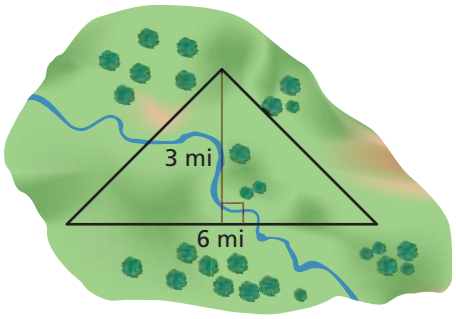
5.



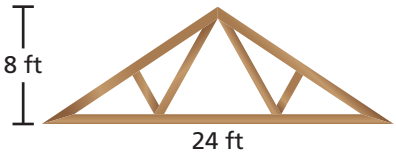
6.



7. LAND A wildlife conservation group buys a plot of land. How much land does it buy? *(Section 4.2)*



8. FRAMING A sheet of plywood is 4 feet wide by 8 feet long. What is the minimum number of sheets of plywood needed to cover the frame? Justify your answer. *(Section 4.2)*



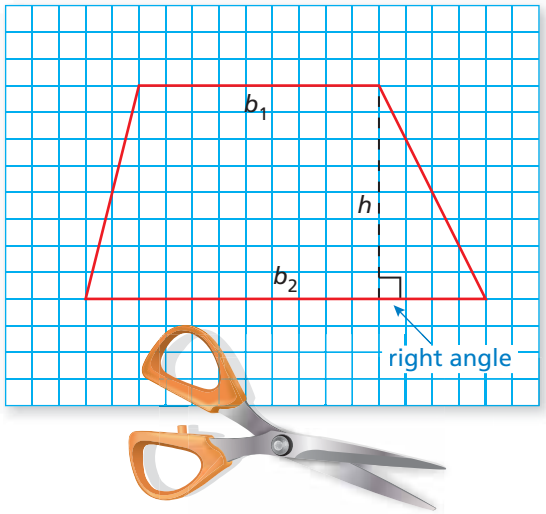
4.3 Areas of Trapezoids

Essential Question How can you derive a formula for the area of a trapezoid?

1 ACTIVITY: Deriving the Area Formula of a Trapezoid

Work with a partner. Use a piece of centimeter grid paper.

a. Draw *any* trapezoid so that its base lies on one of the horizontal lines of the paper.

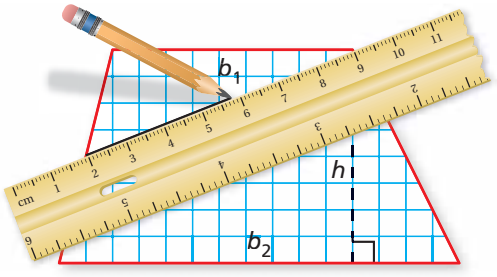


b. Estimate the area of your trapezoid (in square centimeters) by counting unit squares.

Area ≈ _____ Estimate

c. Label the height and the bases *inside* the trapezoid.

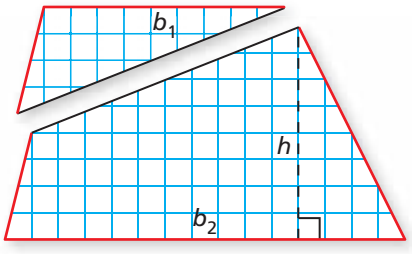
d. Cut out the trapezoid. Mark the midpoint of the side opposite the height. Draw a line from the midpoint to the opposite upper vertex.



e. Cut along the line. You will end up with a triangle and a quadrilateral. Arrange these two figures to form a figure whose area you know.

f. Use your result to write a *formula* for the area of a trapezoid.

Area = _____ Formula



g. Use your formula to find the area of your trapezoid (in square centimeters).

Area = _____ Exact Area

h. Compare this area with your estimate in part (b).

Common Core

Geometry

In this lesson, you will
- find areas of trapezoids.
- solve real-life problems.

Learning Standard 6.G.1

2 ACTIVITY: Writing a Math Lesson

Work with a partner. Use your results from Activity 1 to write a lesson on finding the area of a trapezoid.

Math Practice 6

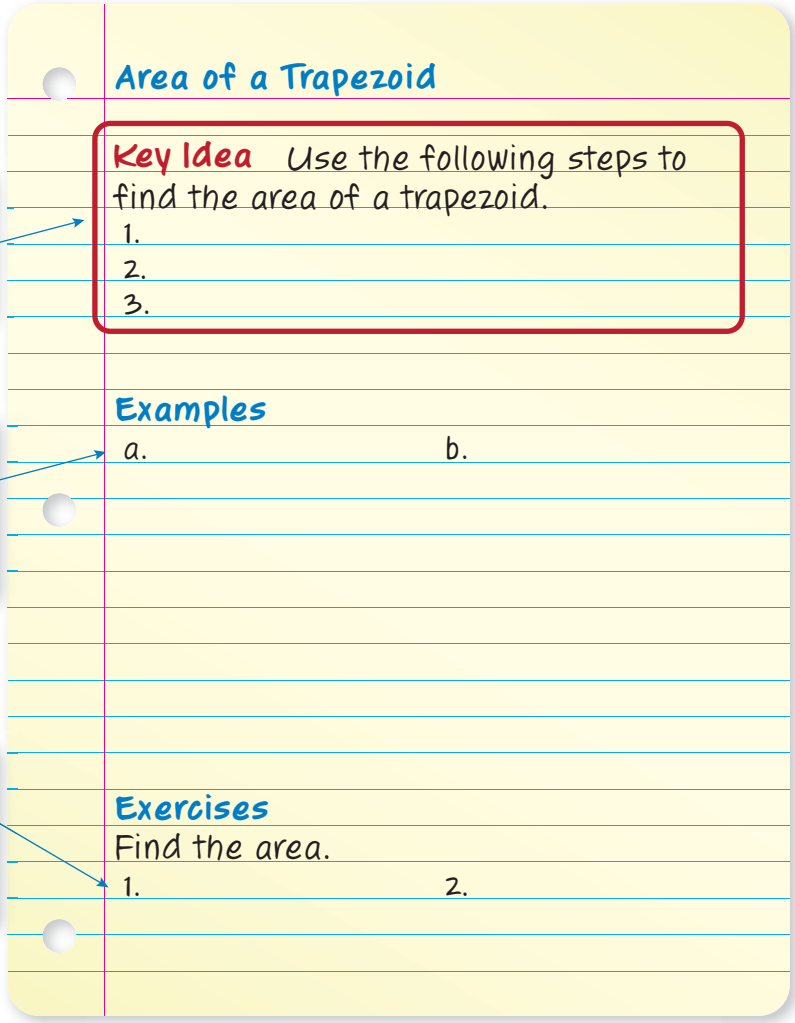
Use Clear Definitions

Do your steps for the *Key Idea* help another person understand how to solve the problem? Do the examples follow your steps?

Describe steps you can use to find the area of a trapezoid.

Write two examples for finding the area of a trapezoid. Include a drawing for each.

Write two exercises for finding the area of a trapezoid. Include an answer sheet.



What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you derive a formula for the area of a trapezoid?

4. In this chapter, you used deductive reasoning to derive new area formulas from area formulas you have already learned. Describe a real-life career in which deductive reasoning is important.

Practice Use what you learned about the areas of trapezoids to complete Exercises 4–6 on page 170.

4.3 Lesson

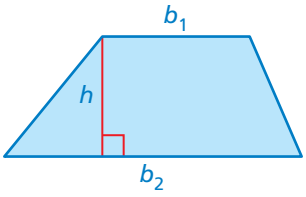
Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Idea

Area of a Trapezoid

Words The area A of a trapezoid is one-half the product of its height h and the sum of its bases b_1 and b_2.

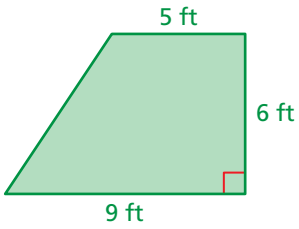
Algebra $A = \frac{1}{2}h(b_1 + b_2)$



EXAMPLE 1 Finding Areas of Trapezoids

Find the area of each trapezoid.

a.



$A = \frac{1}{2}h(b_1 + b_2)$ Write formula.

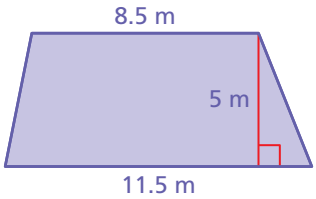
$= \frac{1}{2}(6)(5 + 9)$ Substitute.

$= \frac{1}{2}(6)(14)$ Add.

$= 42$ Multiply.

∴ The area of the trapezoid is 42 square feet.

b.



$A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(5)(8.5 + 11.5)$

$= \frac{1}{2}(5)(20)$

$= 50$

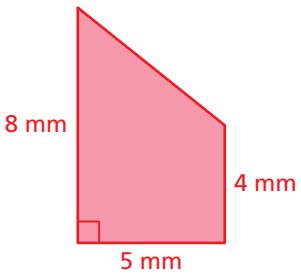
∴ The area of the trapezoid is 50 square meters.

On Your Own

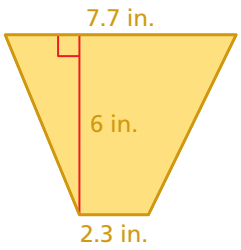
Now You're Ready
Exercises 7–9

Find the area of the trapezoid.

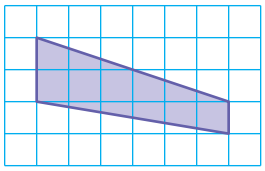
1.



2.



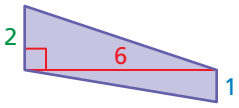
EXAMPLE 2 Finding the Area of a Trapezoid on a Grid



What is the area of the trapezoid?

Ⓐ 6 units^2 Ⓑ 7 units^2 Ⓒ 9 units^2 Ⓓ 12 units^2

Count grid lines to find the dimensions. The height h is 6 units, base b_1 is 1 unit, and base b_2 is 2 units.



$$A = \frac{1}{2}h(b_1 + b_2) \quad \text{Write formula.}$$

$$= \frac{1}{2}(6)(1 + 2) \quad \text{Substitute values.}$$

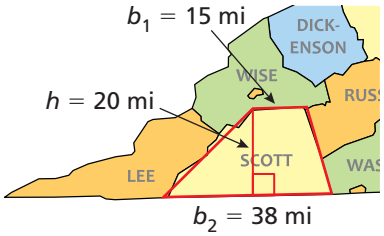
$$= \frac{1}{2}(6)(3) \quad \text{Add.}$$

$$= 9 \quad \text{Multiply.}$$

∴ The area of the trapezoid is 9 square units. The correct answer is Ⓒ.

EXAMPLE 3 Real-Life Application

You can use a trapezoid to approximate the shape of Scott County, Virginia. The population is about 23,200. About how many people are there per square mile?



Find the area of Scott County.

$$A = \frac{1}{2}h(b_1 + b_2) \quad \text{Write formula for area of a trapezoid.}$$

$$= \frac{1}{2}(20)(15 + 38) \quad \text{Substitute 20 for } h\text{, 15 for } b_1\text{, and 38 for } b_2.$$

$$= \frac{1}{2}(20)(53) = 530 \quad \text{Simplify.}$$

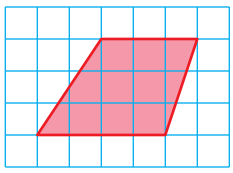
The area of Scott County is about 530 square miles. Divide the population by the area to find the number of people per square mile.

∴ So, there are about $\frac{23{,}200 \text{ people}}{530 \text{ mi}^2} \approx 44$ people per square mile.

On Your Own

Now You're Ready
Exercises 11–13

3. Find the area of the trapezoid.



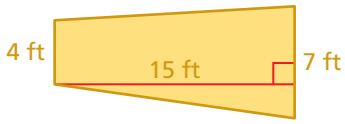
4. **WHAT IF?** In Example 3, the population of Scott County decreases by 550. By how much does the number of people per square mile change? Explain.

4.3 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Identify the bases and the height of the trapezoid.



2. **REASONING** What measures do you need to find the area of a trapezoid?

3. **WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.

$\frac{1}{2}bh$ $\quad \ell w \quad$ $2\ell + 2w$ $\quad \frac{1}{2}h(b_1 + b_2)$

Practice and Problem Solving

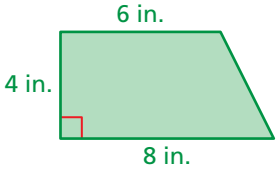
Find the area of the trapezoid.

4. $b_1 = 4, b_2 = 8, h = 2$

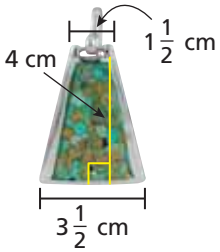
5. $b_1 = 5, b_2 = 7, h = 4$

6. $b_1 = 12, b_2 = 6, h = 3$

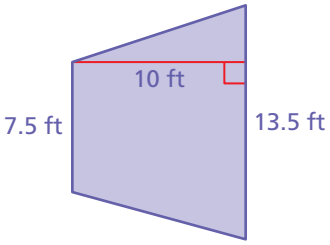
1 7.



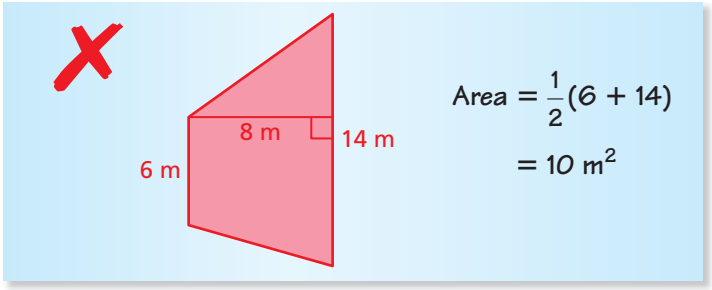
8.



9.

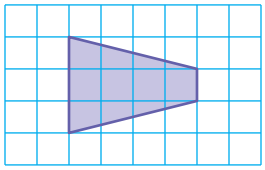


10. **ERROR ANALYSIS** Describe and correct the error in finding the area of the trapezoid.

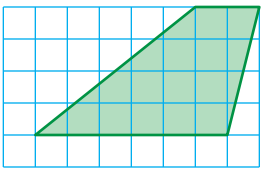


Find the area of the trapezoid.

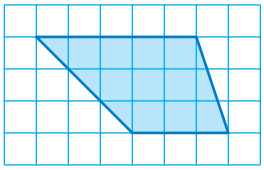
2 11.



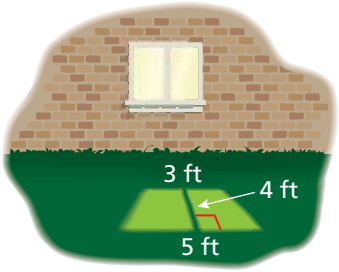
12.



13.



14. **LIGHT** Light shines through a window. What is the area of the trapezoid-shaped region created by the light?



Find the area of a trapezoid with height h and bases b_1 and b_2.

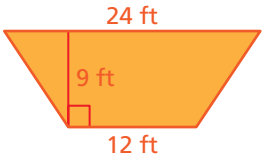
15. $h = 6$ in.
$b_1 = 9$ in.
$b_2 = 11$ in.

16. $h = 22$ cm
$b_1 = 10.5$ cm
$b_2 = 12.5$ cm

17. $h = 12$ mi
$b_1 = 5.6$ mi
$b_2 = 7.4$ mi

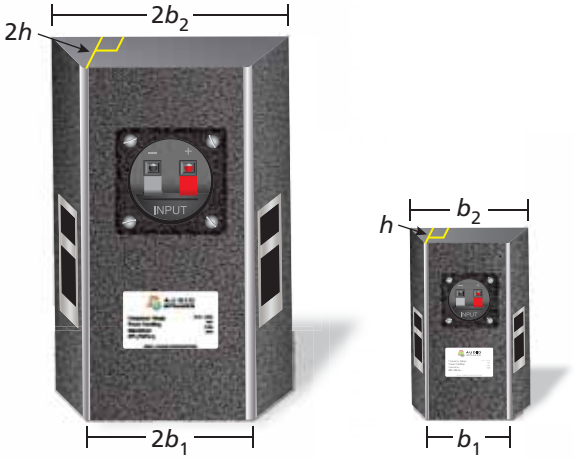
18. $h = 14$ m
$b_1 = 21$ m
$b_2 = 22$ m

19. REASONING The rectangle and the trapezoid have the same area. What is the length ℓ of the rectangle?



20. OPEN-ENDED The area of the trapezoidal student election sign is 5 square feet. Find two possible values for each base length.

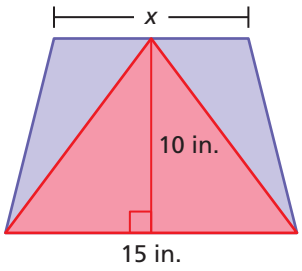
21. AUDIO How many times greater is the area of the floor covered by the larger speaker than by the smaller speaker?



22. Critical Thinking The triangle and the trapezoid share a 15-inch base and a height of 10 inches.

a. The area of the trapezoid is less than twice the area of the triangle. Find the values of x. Explain your reasoning.

b. Can the area of the *trapezoid* be exactly twice the area of the triangle? Explain your reasoning.



Fair Game Review What you learned in previous grades & lessons

Plot the ordered pair in a coordinate plane. *(Skills Review Handbook)*

23. (5, 0)

24. (2, 4)

25. (0, 3)

26. (6, 1)

27. MULTIPLE CHOICE Which expression represents "6 more than x"? *(Section 3.2)*

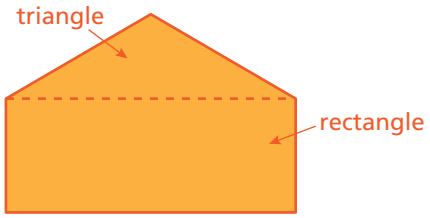
(A) $6 - x$ (B) $6x$ (C) $x + 6$ (D) $\frac{6}{x}$

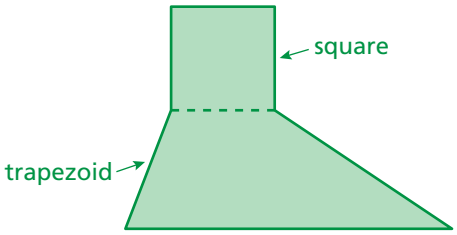
Extension 4.3 Areas of Composite Figures

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
composite figure, *p. 172*

A **composite figure** is made up of triangles, squares, rectangles, and other two-dimensional figures. Here are two examples.

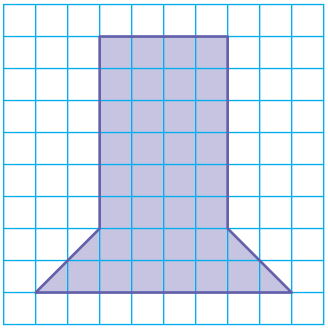




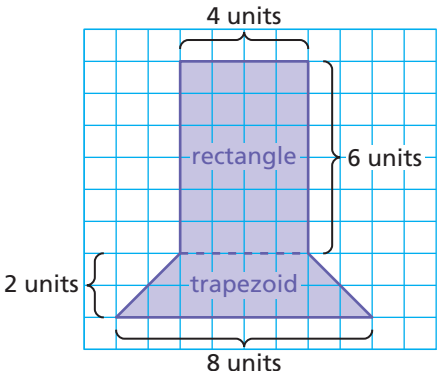
To find the area of a composite figure, separate it into figures with areas you know how to find. Then find the sum of the areas of those figures.

EXAMPLE 1 Finding the Area of a Composite Figure

Find the area of the purple figure.



You can separate the figure into a rectangle and a trapezoid. Count grid lines to find the dimensions of each figure. Then find the area of each figure.



Area of Rectangle

$A = \ell w$

$= 6(4)$

$= 24$

Area of Trapezoid

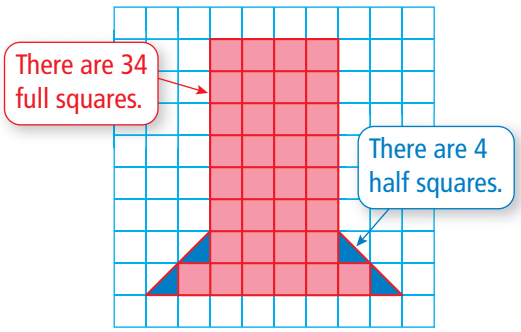
$A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(2)(4 + 8)$

$= 12$

∴ So, the area of the purple figure is $24 + 12 = 36$ square units.

Reasonable? You can check your result by counting unit squares.



Full squares: 34

Half squares: 4

The area is

$$34(1) + 4\left(\frac{1}{2}\right) = 36 \text{ square units.}$$

So, the answer is reasonable. ✓

Study Tip

There is often more than one way to separate composite figures. In Example 1, you can separate the figure into one rectangle and two triangles.

Common Core

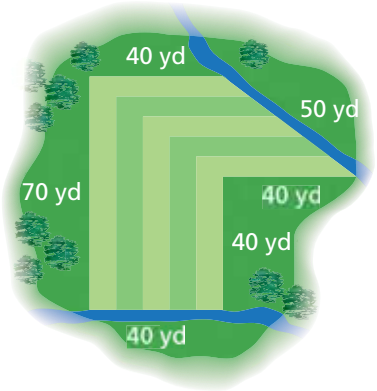
Geometry

In this extension, you will
- find areas of composite figures.
- solve real-life problems.

Applying Standard 6.G.1

Multi-Language Glossary at BigIdeasMath.com

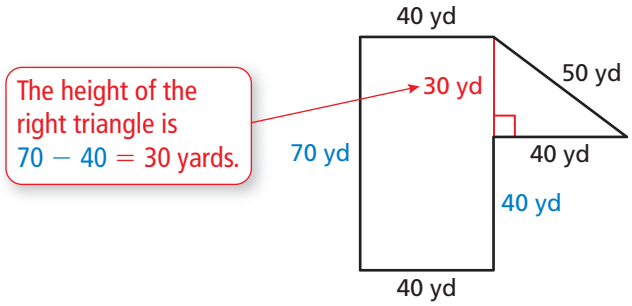
EXAMPLE 2 Real-Life Application



Find the area of the fairway between two streams on a golf course.

There are several ways to separate the fairway into figures whose areas you can find using formulas. It appears that one way is to separate it into a right triangle and a rectangle.

Identify each shape and find any missing dimensions.



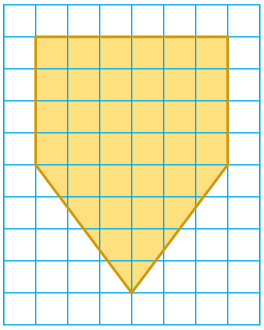
Area of Rectangle	*Area of Right Triangle*
$A = \ell w$	$A = \frac{1}{2}bh$
$= 70(40)$	$= \frac{1}{2}(40)(30)$
$= 2800$	$= 600$

∴ So, the area of the fairway is $2800 + 600 = 3400$ square yards.

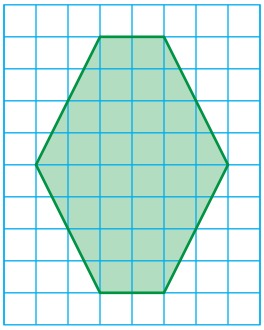
Practice

Find the area of the shaded figure.

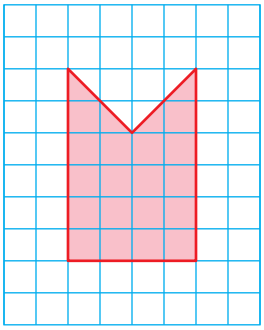
1.



2.

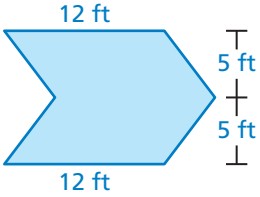


3.

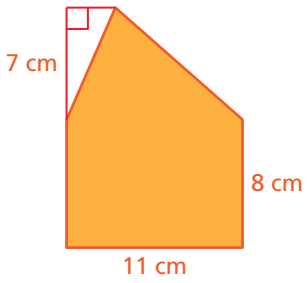


Find the area of the figure.

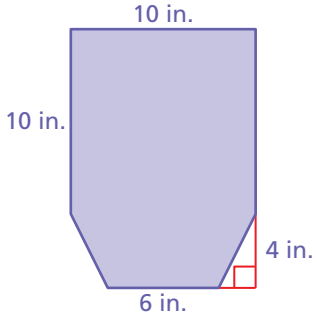
4.



5.



6.



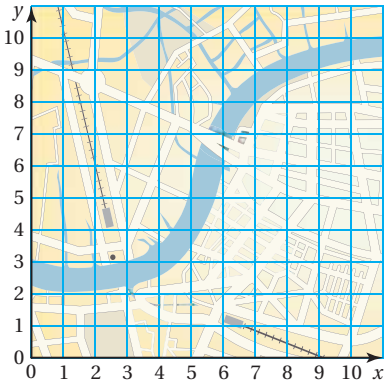
7. ANOTHER METHOD Find the area in Example 2 using a different method.

4.4 Polygons in the Coordinate Plane

Essential Question How can you find the lengths of line segments in a coordinate plane?

1 ACTIVITY: Finding Distances on a Map

Work with a partner. The coordinate grid shows a portion of a city. Each square on the grid represents one square mile.



a. A public library is located at (4, 5). City Hall is located at (7, 5). Plot and label these points.

b. How far is the public library from City Hall?

c. A stadium is located 4 miles from the public library. Give the coordinates of several possible locations of the stadium. Justify your answers by graphing.

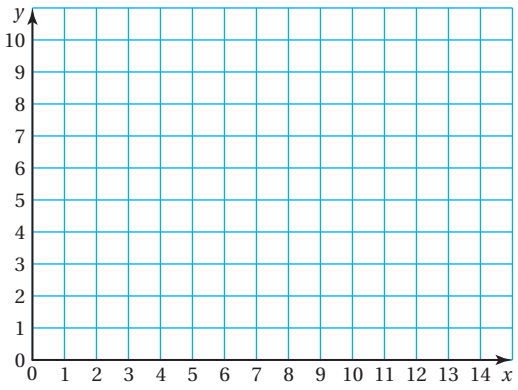
d. Connect the three locations of the public library, City Hall, and the stadium using your answers in part (c). What shapes are formed?

2 ACTIVITY: Graphing Polygons

Work with a partner. Plot and label each set of points in the coordinate plane. Then connect each set of points to form a polygon.

Rectangle: $A(2, 3)$, $B(2, 10)$, $C(6, 10)$, $D(6, 3)$

Triangle: $E(8, 3)$, $F(14, 8)$, $G(14, 3)$



Common Core

Geometry

In this lesson, you will
- draw polygons in the coordinate plane.
- find distances in the coordinate plane.
- solve real-life problems.

Learning Standard 6.G.3

3 ACTIVITY: Finding Distances in a Coordinate Plane

Work with a partner.

a. Find the length of each horizontal line segment in Activity 2.

b. **STRUCTURE** What relationship do you notice between the lengths of the line segments in part (a) and the coordinates of their endpoints? Explain.

c. Find the length of each vertical line segment in Activity 2.

d. **STRUCTURE** What relationship do you notice between the lengths of the line segments in part (c) and the coordinates of their endpoints? Explain.

e. Plot and label the points below in the coordinate plane. Then connect each pair of points with a line segment. Use the relationships you discovered in parts (b) and (d) above to find the length of each line segment. Show your work.

$S(3, 1)$ and $T(14, 1)$ $U(9, 8)$ and $V(9, 0)$

$W(0, 7)$ and $X(0, 10)$ $Y(1, 9)$ and $Z(7, 9)$

f. Check your answers in part (e) by counting grid lines.

Math Practice 8

Repeat Calculations

What calculations are repeated? How can you use this information to write a rule about the length of a line segment?

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you find the lengths of line segments in a coordinate plane? Give examples to support your explanation.

5. Do the methods you used in Activity 3 work for diagonal line segments? Explain why or why not.

6. Use the Internet or some other reference to find an example of how "finding distances in a coordinate plane" is helpful in each of the following careers.

a.

Archaeologist

b.

Surveyor

c.

Pilot

Practice Use what you learned about finding the lengths of line segments to complete Exercises 3–5 on page 178.

4.4 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

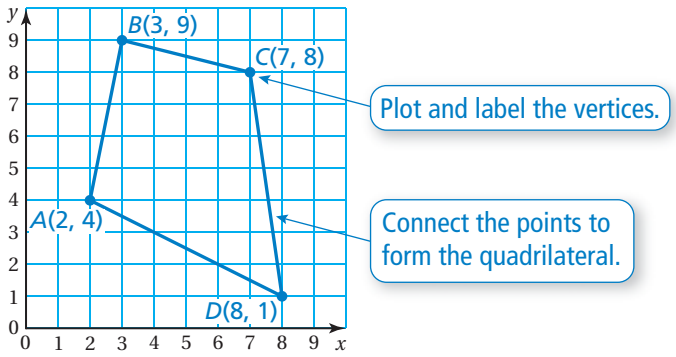
You can use ordered pairs to represent vertices of polygons. To draw a polygon in a coordinate plane, plot and connect the ordered pairs.

EXAMPLE 1 Drawing a Polygon in a Coordinate Plane

The vertices of a quadrilateral are $A(2, 4)$, $B(3, 9)$, $C(7, 8)$, and $D(8, 1)$. Draw the quadrilateral in a coordinate plane.

Study Tip

After you plot the vertices, connect them *in order* to draw the polygon.



On Your Own

Now You're Ready
Exercises 6–11

Draw the polygon with the given vertices in a coordinate plane.

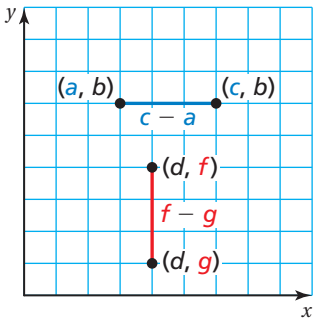
1. $A(0, 0)$, $B(5, 7)$, $C(7, 4)$
2. $W(4, 4)$, $X(7, 4)$, $Y(7, 1)$, $Z(4, 1)$
3. $F(1, 3)$, $G(3, 6)$, $H(5, 6)$, $J(3, 3)$
4. $P(1, 4)$, $Q(3, 5)$, $R(7, 3)$, $S\left(6, \frac{1}{2}\right)$, $T\left(2, \frac{1}{2}\right)$

Key Idea

Finding Distances in the First Quadrant

You can find the length of a horizontal or vertical line segment in a coordinate plane by using the coordinates of the endpoints.

- When the x-coordinates are the same, the vertical distance between the points is the difference of the y-coordinates.
- When the y-coordinates are the same, the horizontal distance between the points is the difference of the x-coordinates.



Be sure to subtract the lesser coordinate from the greater coordinate.

EXAMPLE 2 Finding a Perimeter

The vertices of a rectangle are $F(1, 6)$, $G(7, 6)$, $H(7, 2)$, and $J(1, 2)$. Draw the rectangle in a coordinate plane and find its perimeter.

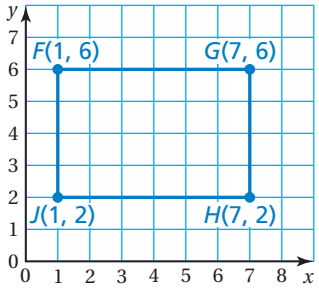
Draw the rectangle and use the vertices to find its dimensions.

The length is the horizontal distance between $F(1, 6)$ and $G(7, 6)$, which is the difference of the x-coordinates.

$$\text{length} = 7 - 1 = 6 \text{ units}$$

The width is the vertical distance between $G(7, 6)$ and $H(7, 2)$, which is the difference of the y-coordinates.

$$\text{width} = 6 - 2 = 4 \text{ units}$$



So, the perimeter of the rectangle is $2(6) + 2(4) = 20$ units.

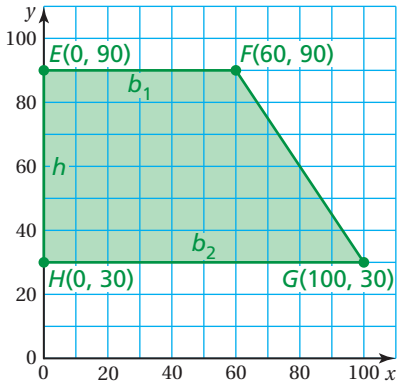
Study Tip

You can also find the length using vertices H and J. You can find the width using vertices F and J.

EXAMPLE 3 Real-Life Application

In a grid of the exhibits at a zoo, the vertices of the giraffe exhibit are $E(0, 90)$, $F(60, 90)$, $G(100, 30)$, and $H(0, 30)$. The coordinates are measured in feet. What is the area of the giraffe exhibit?

Plot and connect the vertices using a coordinate grid to form a trapezoid. Use the coordinates to find the lengths of the bases and the height.



$$b_1 = 60 - 0 = 60$$

$$b_2 = 100 - 0 = 100$$

$$h = 90 - 30 = 60$$

Use the formula for the area of a trapezoid.

$$A = \frac{1}{2}(60)(60 + 100)$$

$$= \frac{1}{2}(60)(160) = 4800$$

The area of the giraffe exhibit is 4800 square feet.

Common Error

You can count grid lines to find the dimensions, but make sure you consider the scale of the axes.

On Your Own

Now You're Ready
Exercises 12–15

5. The vertices of a rectangle are $J(2, 7)$, $K(4, 7)$, $L(4, 1.5)$, and $M(2, 1.5)$. Find the perimeter and the area of the rectangle.
6. **WHAT IF?** In Example 3, the giraffe exhibit is enlarged by moving vertex F to $(80, 90)$. How does this affect the area? Explain.

4.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** How can you use a coordinate plane to draw a polygon?
2. **WRITING** How can you find the perimeter of a rectangle in a coordinate plane?

Practice and Problem Solving

Plot and label each pair of points in a coordinate plane. Find the length of the line segment connecting the points.

3. $C(0, 1), D(8, 1)$
4. $K(5, 2), L(5, 6)$
5. $Q(3, 4), R(3, 9)$

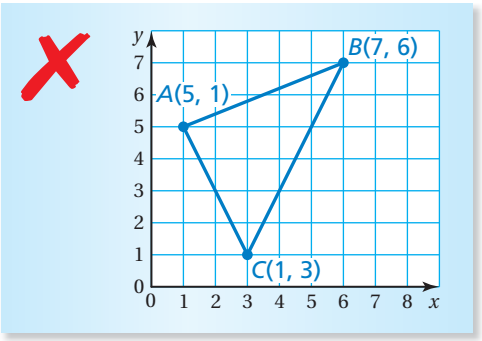
Draw the polygon with the given vertices in a coordinate plane.

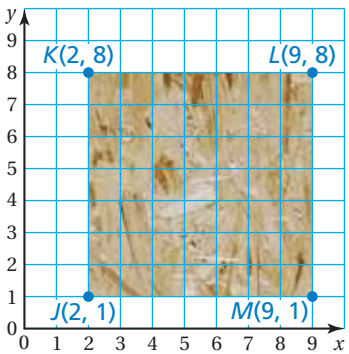
1 6. $A(4, 7), B(6, 2), C(0, 0)$
7. $D\left(\frac{1}{2}, 2\right), E(5, 5), F(4, 1)$
8. $G\left(1\frac{1}{2}, 4\right), H\left(1\frac{1}{2}, 8\right), J(5, 8), K(5, 4)$
9. $L(3, 2), M(3, 5), N(9, 5), P(9, 2)$
10. $Q(0, 4), R(10, 8), S(7, 4), T(10, 2), U(5, 0)$
11. $V(2, 2), W\left(3, 7\frac{1}{2}\right), X\left(8, 7\frac{1}{2}\right), Y(10, 4), Z(7, 0)$

Find the perimeter and the area of the polygon with the given vertices.

2 12. $C(1, 1), D(1, 4), E(4, 4), F(4, 1)$
13. $J(1, 2), K(7, 2), L(7, 8), M(1, 8)$
14. $N(0, 2), P(5, 2), Q(5, 5), R(0, 5)$
15. $S(3, 0), T(3, 9), U(8, 9), V(8, 0)$

16. **ERROR ANALYSIS** Describe and correct the error in drawing a triangle with vertices $A(5, 1)$, $B(7, 6)$, and $C(1, 3)$.





17. **TREE HOUSE** You design a tree house using a coordinate plane. You plot the vertices of the floor at $J(2, 1)$, $K(2, 8)$, $L(9, 8)$, and $M(9, 1)$. The coordinates are measured in feet.
   a. What is the shape of the floor?
   b. What are the perimeter and the area of the floor?

OPEN-ENDED **Draw a polygon with the given conditions in a coordinate plane.**

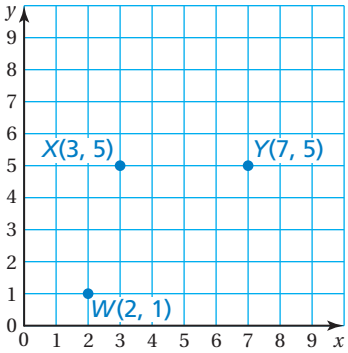
18. a square with a perimeter of 20 units

19. a rectangle with a perimeter of 18 units

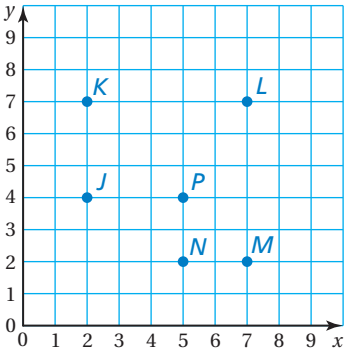
20. a rectangle with an area of 24 units^2

21. a triangle with an area of 15 units^2

22. **STRUCTURE** The coordinate plane shows three vertices of a parallelogram. Find two possible points that could represent the fourth vertex.



23. **BUS ROUTE** Polygon *JKLMNP* represents a bus route. Each grid square represents 9 square miles. What is the shortest distance, in miles, from station *P* to station *L* using the bus route? Explain your reasoning.



24. **CITY LIMITS** In a topographical map of a city, the vertices of the city limits are $A(10, 9)$, $B(18, 9)$, $C(18, 2)$, $D(14, 4.5)$, and $E(10, 4.5)$. The coordinates are measured in miles. What is the area of the city?

25. **BACKYARD** The vertices of a backyard are $W(10, 30)$, $X(10, 100)$, $Y(110, 100)$, and $Z(50, 30)$. The coordinates are measured in feet. The line segment XZ separates the backyard into a lawn and a garden. The area of the lawn is greater than the area of the garden. How many times larger is the lawn than the garden?

26. **Precision** The vertices of a rectangle are $(1, 0)$, $(1, a)$, $(5, a)$, and $(5, 0)$. The vertices of a parallelogram are $(1, 0)$, $(2, b)$, $(6, b)$, and $(5, 0)$. The value of a is greater than the value of b. Which polygon has a greater area? Explain your reasoning.

Fair Game Review What you learned in previous grades & lessons

Divide. Write the answer in simplest form. *(Section 2.3)*

27. $1\frac{1}{3} \div \frac{2}{3}$

28. $6\frac{3}{5} \div \frac{3}{4}$

29. $2\frac{1}{2} \div 8$

30. $4\frac{1}{6} \div 1\frac{1}{8}$

31. **MULTIPLE CHOICE** You are filling bottles from 5 gallons of lemonade. How many bottles can you fill when each bottle is $\frac{3}{8}$ of a gallon? *(Section 2.2)*

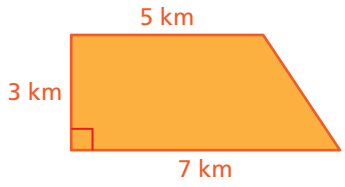
A $1\frac{7}{8}$ **B** 3 **C** 8 **D** $13\frac{1}{3}$

4.3–4.4 Quiz

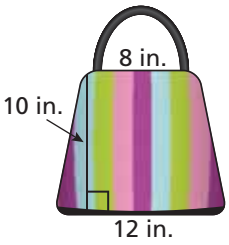
Check It Out
Progress Check
BigIdeasMath.com

Find the area of the trapezoid. *(Section 4.3)*

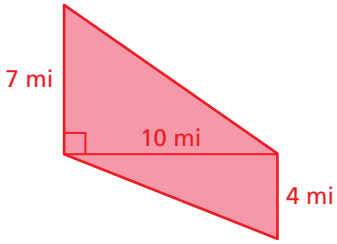
1.



2.

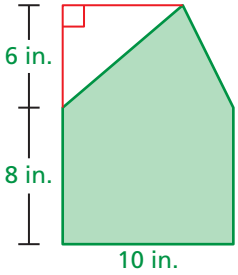


3.

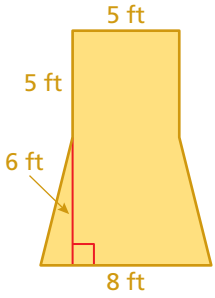


Find the area of the figure. *(Section 4.3)*

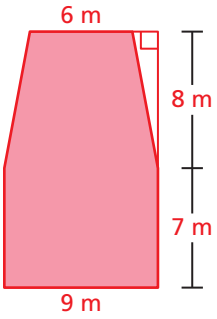
4.



5.



6.



Draw the polygon with the given vertices in a coordinate plane. *(Section 4.4)*

7. $A(1, 2)$, $B(3, 5)$, $C(6, 1)$

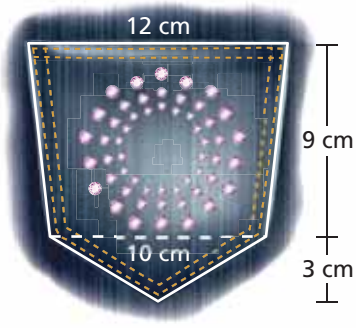
8. $E(1, 2)$, $F(3, 6)$, $G(8, 6)$, $H(6, 2)$

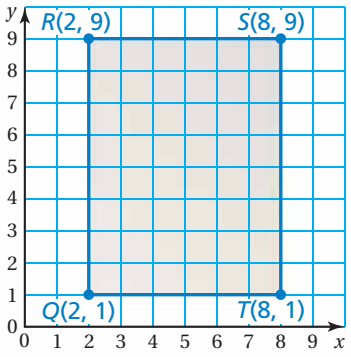
Find the perimeter and the area of the polygon with the given vertices. *(Section 4.4)*

9. $J(1, 3)$, $K(1, 8)$, $L(5, 8)$, $M(5, 3)$

10. $P(1, 2)$, $Q(1, 7)$, $R(7, 7)$, $S(7, 2)$

11. **BACK POCKET** How much material do you need to make two back pockets? *(Section 4.3)*





12. **PATIO** Plans for a patio are shown in the coordinate plane at the left. The coordinates are measured in feet. Find the perimeter and the area of the patio. *(Section 4.4)*

4 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com

Review Key Vocabulary

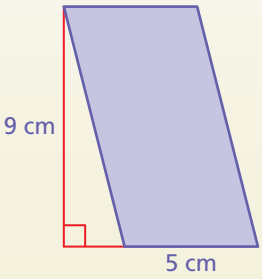
polygon, *p. 152*

composite figure, *p. 172*

Review Examples and Exercises

4.1 Areas of Parallelograms *(pp. 152–157)*

Find the area of the parallelogram.



$A = bh$ Write formula.

$= 5(9)$ Substitute 5 for b and 9 for h.

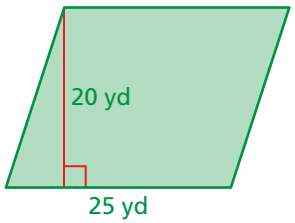
$= 45$ Multiply.

∴ The area of the parallelogram is 45 square centimeters.

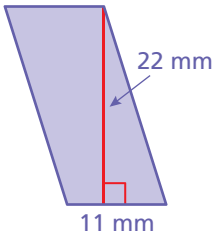
Exercises

Find the area of the parallelogram.

1.



2.



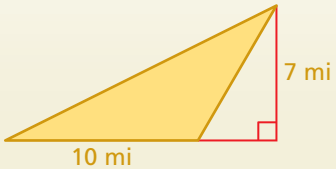
4.2 Areas of Triangles *(pp. 158–163)*

Find the area of the triangle.

$A = \frac{1}{2}bh$ Write formula.

$= \frac{1}{2}(10)(7)$ Substitute.

$= 35$ Multiply.

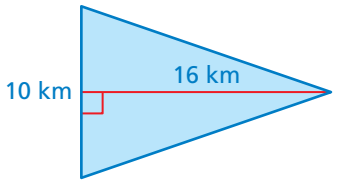


∴ The area of the triangle is 35 square miles.

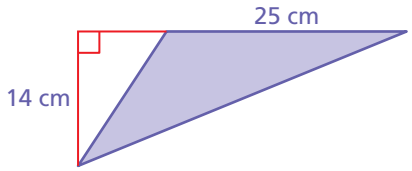
Exercises

Find the area of the triangle.

3.



4.



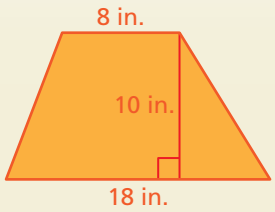
4.3 Areas of Trapezoids *(pp. 166–173)*

Find the area of the trapezoid.

$$A = \frac{1}{2}h(b_1 + b_2) \quad \text{Write formula.}$$

$$= \frac{1}{2}(10)(8 + 18) \quad \text{Substitute.}$$

$$= \frac{1}{2}(10)(26) = 130 \quad \text{Multiply.}$$

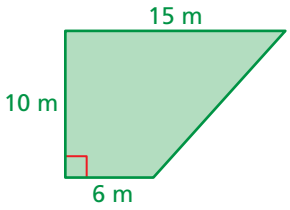


∴ The area of the trapezoid is 130 square inches.

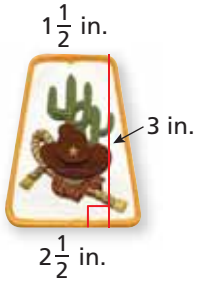
Exercises

Find the area of the trapezoid.

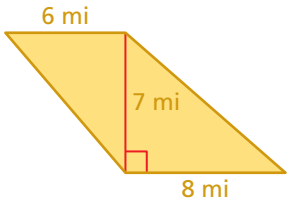
5.



6.

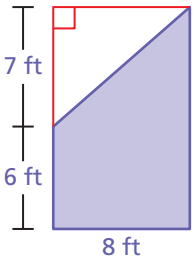


7.

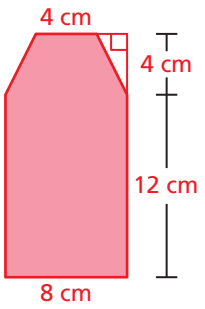


Find the area of the figure.

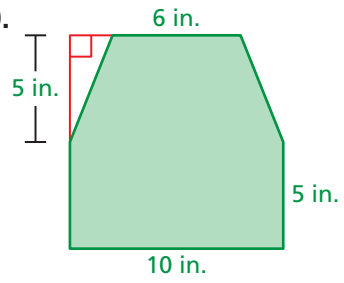
8.



9.

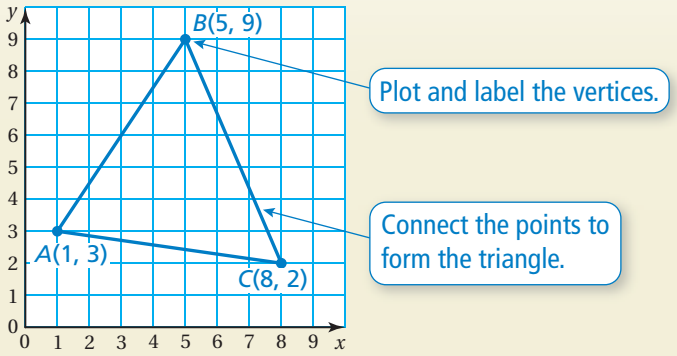


10.



4.4 Polygons in the Coordinate Plane *(pp. 174–179)*

a. The vertices of a triangle are $A(1, 3)$, $B(5, 9)$, and $C(8, 2)$. Draw the triangle in a coordinate plane.



b. The vertices of a rectangle are $F(2, 6)$, $G(8, 6)$, $H(8, 1)$, and $J(2, 1)$. Draw the rectangle in a coordinate plane and find its perimeter.

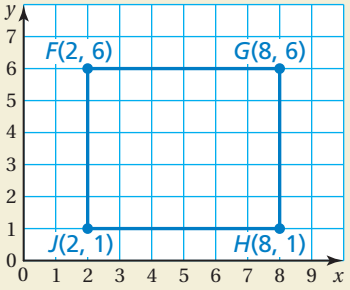
Draw the rectangle and use the vertices to find its dimensions.

The length is the horizontal distance between $F(2, 6)$ and $G(8, 6)$, which is the difference of the x-coordinates.

$$\text{length} = 8 - 2 = 6 \text{ units}$$

The width is the vertical distance between $G(8, 6)$ and $H(8, 1)$, which is the difference of the y-coordinates.

$$\text{width} = 6 - 1 = 5 \text{ units}$$



∴ So, the perimeter of the rectangle is $2(6) + 2(5) = 22$ units.

Exercises

Draw the polygon with the given vertices in a coordinate plane.

11. $A(3, 2)$, $B(4, 7)$, $C(6, 0)$

12. $D(1, 1)$, $E(1, 5)$, $F(4, 5)$, $G(4, 1)$

13. $J(1, 2)$, $K(1, 7)$, $L(5, 7)$, $M(8, 2)$

14. $K\left(3, 3\frac{1}{2}\right)$, $L(5, 7)$, $M(8, 7)$, $N\left(6, 3\frac{1}{2}\right)$

Find the perimeter and the area of the polygon with the given vertices.

15. $P(4, 3)$, $Q(4, 7)$, $R(9, 7)$, $S(9, 3)$

16. $T(2, 7)$, $U(2, 9)$, $V(5, 9)$, $W(5, 7)$

17. $W(11, 2)$, $X(11, 8)$, $Y(14, 8)$, $Z(14, 2)$

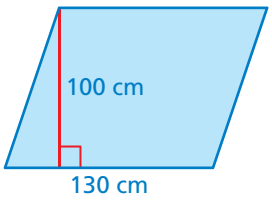
18. $A(12, 2)$, $B(12, 13)$, $C(15, 13)$, $D(15, 2)$

4 Chapter Test

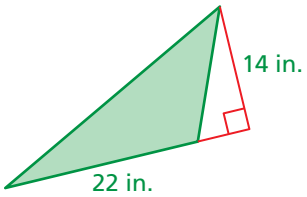
Check It Out
Test Practice
BigIdeasMath.com

Find the area of the parallelogram, triangle, or trapezoid.

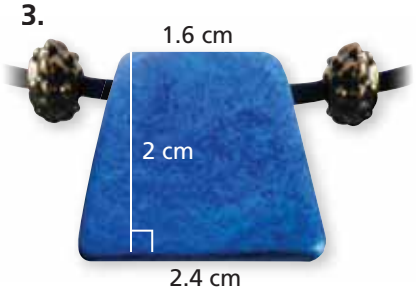
1.



2.

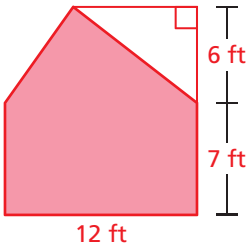


3.

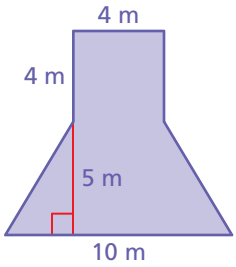


Find the area of the figure.

4.



5.



6.

Draw the polygon with the given vertices in a coordinate plane.

7. $A(4, 2)$, $B(5, 6)$, $C(7, 4)$

8. $D(3, 4)$, $E(5, 8)$, $F(8, 8)$, $G(6, 4)$

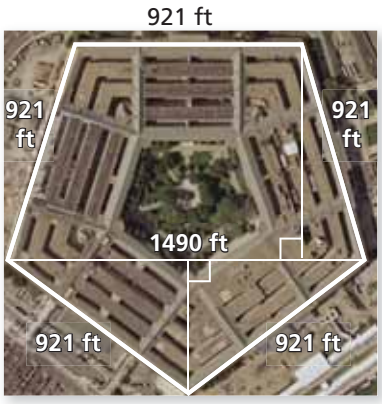
Find the perimeter and the area of the polygon with the given vertices.

9. $Q(5, 6)$, $R(5, 10)$, $S(9, 10)$, $T(9, 6)$

10. $W(2, 8)$, $X(2, 16)$, $Y(8, 16)$, $Z(8, 8)$

11. **TABLETOP** The base lengths of a trapezoidal tabletop are 6 feet and 8 feet. The height is 5 feet. What is the area of the tabletop?

12. **PENTAGON** The Pentagon in Arlington, Virginia, is the headquarters of the U.S. Department of Defense.

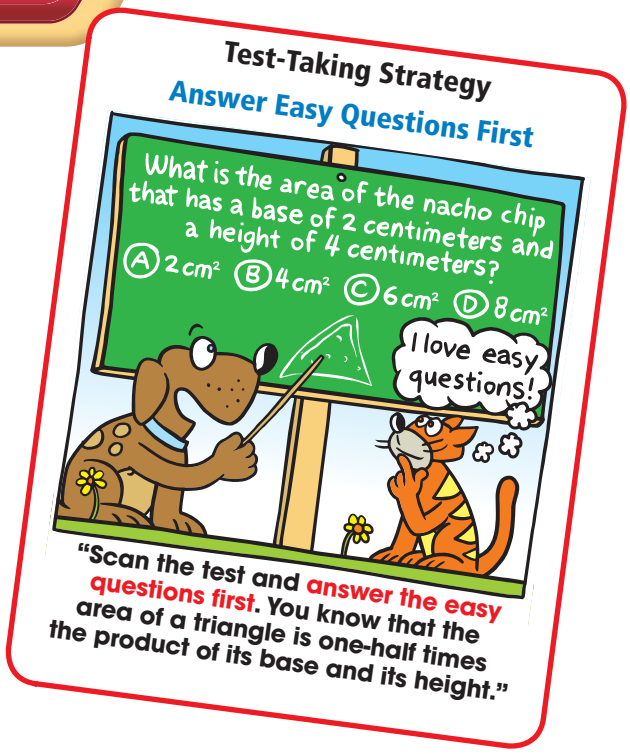


a. Find the perimeter of the Pentagon.

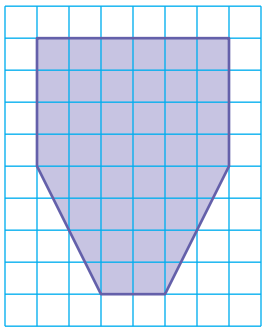
b. A pentagon is made of a triangle and a trapezoid. The height of the triangle shown is about 541 feet, and the height of the trapezoid shown is about 876 feet. Estimate the land area of the Pentagon.

13. **CAMPING** The vertices of a campsite are (25, 15), (25, 30), (55, 30), and (55, 15). The vertices of your tent are (30, 20), (30, 25), (40, 25), and (40, 20). The coordinates are measured in feet. What is the area of the campsite not covered by your tent?

4 Standards Assessment



1. What is the area of the shaded figure shown below? *(6.G.1)*



A. 32 units^2

B. 40 units^2

C. 44 units^2

D. 56 units^2

2. What is the value of the expression below? *(6.EE.1)*

18^3

3. You have 36 red apples and 42 green apples. What is the greatest number of identical fruit baskets you can make with no apples left over? *(6.NS.4)*

F. 6

G. 9

H. 12

I. 18

4. What is the perimeter of the rectangle with the vertices shown below? *(6.G.3)*

$A(4, 7), B(4, 15), C(9, 15), D(9,7)$

A. 8 units

B. 13 units

C. 26 units

D. 70 units

5. What property was used to simplify the expression? *(6.EE.3)*

$$\begin{aligned} 5 \times 78 &= 5(70 + 8) \\ &= 5(70) + 5(8) \\ &= 350 + 40 \\ &= 390 \end{aligned}$$

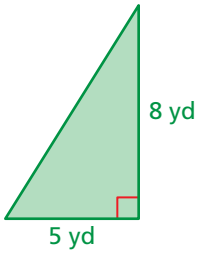
F. Associative Property of Multiplication

G. Commutative Property of Addition

H. Distributive Property

I. Multiplication Property of One

6. What is the area, in square yards, of the triangle below? *(6.G.1)*



7. Which of the following is equivalent to $\frac{12}{35}$? *(6.NS.1)*

A. $\frac{5}{6} \div \frac{2}{7}$

B. $\frac{2}{7} \div \frac{6}{5}$

C. $\frac{2}{7} \div \frac{5}{6}$

D. $\frac{5}{6} \div \frac{7}{2}$

8. The description below represents the area of which polygon? *(6.G.1)*

"one-half the product of its height and the sum of the lengths of its bases"

F. rectangle

G. square

H. trapezoid

I. triangle

9. Edward was evaluating the expression in the box.

$$\begin{aligned} 180 \div 9 + 3^4 - 1 &= 180 \div 9 + 81 - 1 \\ &= 180 \div 90 - 1 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

What should Edward do to correct the error that he made? *(6.EE.1)*

A. Add 9 and 81 then subtract 1 before dividing.

B. Divide 180 by 9 before adding or subtracting.

C. Divide 180 by 9 then subtract 1 before adding 3^4.

D. Subtract 1 from 90 before dividing.

10. You have 3 times as many guitar picks as your cousin. Let v be the number of guitar picks that your cousin has. Which expression represents the number of guitar picks you have? *(6.EE.2a)*

F. $3v$

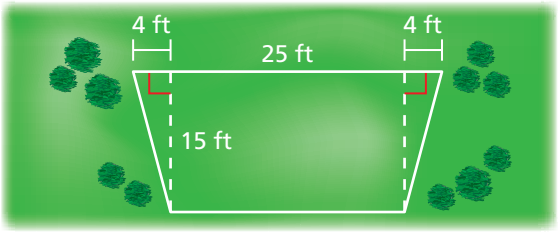
G. $v + 3$

H. $3 - v$

I. $\frac{v}{3}$

11. Your family hires a company to install invisible fencing around your yard. *(6.G.1)*

Think
Solve
Explain



Part A Find the area of the yard using only the area formulas for rectangles and triangles. Show your work.

Part B Find the area of the yard using the area formula for trapezoids.

Part C Explain why the two methods of finding the area of the yard give the same result. Describe the advantages of each method.