12 Rational Numbers

"On the count of 5, I'm going to give you half of my dog biscuits."

Nice!

I knew this was too good to be true.

"1, 2, 3, 4, $4\frac{1}{2}$, $4\frac{3}{4}$, $4\frac{7}{8}$,..."

"I entered a contest for dog biscuits."

"I was notified that the number of biscuits I won was in the three-digit range."

hmm...1.25

Hold off on the tail wagging until the box arrives.

What You Learned Before

"Let's play a game. The goal is to say a positive rational number that is less than the other pet's number... You go first."

Writing Decimals and Fractions (4.NF.6)

Example 1 Write 0.37 as a fraction.

$$0.37 = \frac{37}{100}$$

Example 2 Write $\frac{2}{5}$ as a decimal.

$$\frac{2}{5} = \frac{2 \cdot 2}{5 \cdot 2} = \frac{4}{10} = 0.4$$

Try It Yourself
Write the decimal as a fraction or the fraction as a decimal.

1. 0.51   **2.** 0.731   **3.** $\frac{3}{5}$ **4.** $\frac{7}{8}$

Adding and Subtracting Fractions (5.NF.1)

Example 3 Find $\frac{1}{3} + \frac{1}{5}$.

$$\frac{1}{3} + \frac{1}{5} = \frac{1 \cdot 5}{3 \cdot 5} + \frac{1 \cdot 3}{5 \cdot 3}$$
$$= \frac{5}{15} + \frac{3}{15}$$
$$= \frac{8}{15}$$

Example 4 Find $\frac{1}{4} - \frac{2}{9}$.

$$\frac{1}{4} - \frac{2}{9} = \frac{1 \cdot 9}{4 \cdot 9} - \frac{2 \cdot 4}{9 \cdot 4}$$
$$= \frac{9}{36} - \frac{8}{36}$$
$$= \frac{1}{36}$$

Multiplying and Dividing Fractions (5.NF.4, 6.NS.1)

Example 5 Find $\frac{5}{6} \cdot \frac{3}{4}$.

$$\frac{5}{6} \cdot \frac{3}{4} = \frac{5 \cdot \overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}} \cdot 4}$$
$$= \frac{5}{8}$$

Example 6 Find $\frac{2}{3} \div \frac{9}{10}$.

$$\frac{2}{3} \div \frac{9}{10} = \frac{2}{3} \cdot \frac{10}{9}$$
$$= \frac{2 \cdot 10}{3 \cdot 9}$$
$$= \frac{20}{27}$$

Multiply by the reciprocal of the divisor.

Try It Yourself
Evaluate the expression.

5. $\frac{1}{4} + \frac{13}{20}$ **6.** $\frac{14}{15} - \frac{1}{3}$ **7.** $\frac{3}{7} \cdot \frac{9}{10}$ **8.** $\frac{4}{5} \div \frac{16}{17}$

12.1 Rational Numbers

Essential Question How can you use a number line to order rational numbers?

The Meaning of a Word ● Rational

The word **rational** comes from the word *ratio*. Recall that you can write a ratio using fraction notation.

If you sleep for 8 hours in a day, then the ratio of your sleeping time to the total hours in a day can be written as

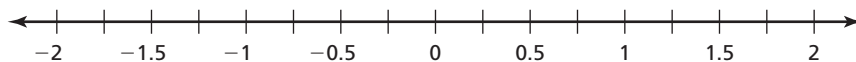$$\frac{8\text{ h}}{24\text{ h}}.$$

A **rational number** is a number that can be written as the ratio of two integers.

$$2 = \frac{2}{1} \qquad -3 = \frac{-3}{1} \qquad -\frac{1}{2} = \frac{-1}{2} \qquad 0.25 = \frac{1}{4}$$

1 ACTIVITY: Ordering Rational Numbers

Work in groups of five. Order the numbers from least to greatest.

● Use masking tape and a marker to make a number line on the floor similar to the one shown.

● Write the numbers on pieces of paper. Then each person should choose one.

● Stand on the location of your number on the number line.

● Use your positions to order the numbers from least to greatest.

a. $-0.5, 1.25, -\dfrac{1}{3}, 0.5, -\dfrac{5}{3}$

b. $-\dfrac{7}{4}, 1.1, \dfrac{1}{2}, -\dfrac{1}{10}, -1.3$

c. $-1.4, -\dfrac{3}{5}, \dfrac{9}{2}, \dfrac{1}{4}, 0.9$

d. $\dfrac{5}{4}, 0.75, -\dfrac{5}{4}, -0.8, -1.1$

ACTIVITY: The Game of Math Card War

Preparation:

- Cut index cards to make 40 playing cards.
- Write each number in the table on a card.

To Play:

- Play with a partner.
- Deal 20 cards to each player facedown.
- Each player turns one card faceup. The player with the greater number wins. The winner collects both cards and places them at the bottom of his or her cards.
- Suppose there is a tie. Each player lays three cards facedown, then a new card faceup. The player with the greater of these new cards wins. The winner collects all ten cards and places them at the bottom of his or her cards.
- Continue playing until one player has all the cards. This player wins the game.
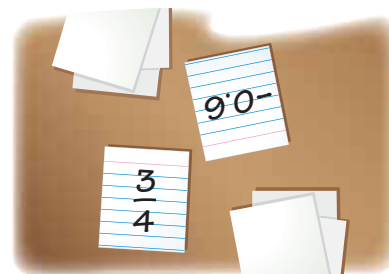
$-\dfrac{3}{2}$	$\dfrac{3}{10}$	$-\dfrac{3}{4}$	-0.6	1.25	-0.15	$\dfrac{5}{4}$	$\dfrac{3}{5}$	-1.6	-0.3
$\dfrac{3}{20}$	$\dfrac{8}{5}$	-1.2	$\dfrac{19}{10}$	0.75	-1.5	$-\dfrac{6}{5}$	$-\dfrac{3}{5}$	1.2	0.3
1.5	1.9	-0.75	-0.4	$\dfrac{3}{4}$	$-\dfrac{5}{4}$	-1.9	$\dfrac{2}{5}$	$-\dfrac{3}{20}$	$-\dfrac{19}{10}$
$\dfrac{6}{5}$	$-\dfrac{3}{10}$	1.6	$-\dfrac{2}{5}$	0.6	0.15	$\dfrac{3}{2}$	-1.25	0.4	$-\dfrac{8}{5}$

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use a number line to order rational numbers? Give an example.

The numbers are in order from least to greatest. Fill in the blank spaces with rational numbers.

4. $-\dfrac{1}{2}$, ▢ , $\dfrac{1}{3}$, ▢ , $\dfrac{7}{5}$, ▢

5. $-\dfrac{5}{2}$, ▢ , -1.9, ▢ , $-\dfrac{2}{3}$, ▢

6. $-\dfrac{1}{3}$, ▢ , -0.1, ▢ , $\dfrac{4}{5}$, ▢

7. -3.4, ▢ , -1.5, ▢ , 2.2, ▢

Practice ➤ Use what you learned about ordering rational numbers to complete Exercises 28–30 on page 522.

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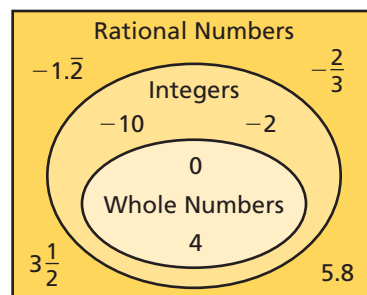
Key Vocabulary 🔊
rational number,
  p. 520
terminating decimal,
  p. 520
repeating decimal,
  p. 520

🔑 Key Idea

Rational Numbers

A **rational number** is a number that can be written as $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

Rational Numbers

$-1.\overline{2}$ $-\frac{2}{3}$

Integers

-10 -2

0

Whole Numbers

4

$3\frac{1}{2}$ 5.8

Because you can divide any integer by any nonzero integer, you can use long division to write fractions and mixed numbers as decimals. These decimals are also rational numbers and will either *terminate* or *repeat*.

A **terminating decimal** is a decimal that ends.

$1.5, \; -0.25, \; 10.625$

A **repeating decimal** is a decimal that has a pattern that repeats.

$-1.333\ldots = -1.\overline{3}$

$0.151515\ldots = 0.\overline{15}$

Use *bar notation* to show which of the digits repeat.

EXAMPLE 1 — Writing Rational Numbers as Decimals

a. Write $-2\frac{1}{4}$ as a decimal.

Notice that $-2\frac{1}{4} = -\frac{9}{4}$.

Divide 9 by 4.

$$\begin{array}{r} 2.25 \\ 4\overline{)9.00} \\ -\,8 \\ \hline 1\,0 \\ -\,8 \\ \hline 20 \\ -\,20 \\ \hline 0 \end{array}$$

The remainder is 0. So, it is a terminating decimal.

∴ So, $-2\frac{1}{4} = -2.25$.

b. Write $\frac{5}{11}$ as a decimal.

Divide 5 by 11.

$$\begin{array}{r} 0.4545 \\ 11\overline{)5.0000} \\ -\,4\,4 \\ \hline 60 \\ -\,55 \\ \hline 50 \\ -\,44 \\ \hline 60 \\ -\,55 \\ \hline 5 \end{array}$$

The remainder repeats. So, it is a repeating decimal.

∴ So, $\frac{5}{11} = 0.\overline{45}$.

🔴 On Your Own

Now You're Ready
Exercises 11–18

Write the rational number as a decimal.

1. $-\frac{6}{5}$

2. $-7\frac{3}{8}$

3. $-\frac{3}{11}$

4. $1\frac{5}{27}$

🔊 **Multi-Language Glossary at BigIdeasMath.com**

EXAMPLE **2** **Writing a Decimal as a Fraction**

Write −0.26 as a fraction in simplest form.

Study Tip

If p and q are integers, then $-\dfrac{p}{q} = \dfrac{-p}{q} = \dfrac{p}{-q}$.

$$-0.26 = -\frac{26}{100}$$

Write the digits after the decimal point in the numerator.

The last digit is in the hundredths place. So, use 100 in the denominator.

$$= -\frac{13}{50}$$ Simplify.

🔴 **On Your Own**

Now You're Ready
Exercises 20–27

Write the decimal as a fraction or a mixed number in simplest form.

5. −0.7 **6.** 0.125 **7.** −3.1 **8.** −10.25

EXAMPLE **3** **Ordering Rational Numbers**

Creature	Elevation (kilometers)
Anglerfish	$-\dfrac{13}{10}$
Squid	$-2\dfrac{1}{5}$
Shark	$-\dfrac{2}{11}$
Whale	−0.8

The table shows the elevations of four sea creatures relative to sea level. Which of the sea creatures are deeper than the whale? Explain.

Write each rational number as a decimal.

$$-\frac{13}{10} = -1.3$$

$$-2\frac{1}{5} = -2.2$$

$$-\frac{2}{11} = -0.\overline{18}$$

Then graph each decimal on a number line.

Squid −2.2 Anglerfish −1.3 Whale −0.8 Shark −0.$\overline{18}$

Elevation (kilometers)

∴ Both −2.2 and −1.3 are less than −0.8. So, the squid and the anglerfish are deeper than the whale.

🔴 **On Your Own**

Now You're Ready
Exercises 28–33

9. **WHAT IF?** The elevation of a dolphin is $-\dfrac{1}{10}$ kilometer. Which of the sea creatures in Example 3 are deeper than the dolphin? Explain.

Vocabulary and Concept Check

1. **VOCABULARY** Is the quotient of two integers always a rational number? Explain.

2. **WRITING** Are all terminating and repeating decimals rational numbers? Explain.

Tell whether the number belongs to each of the following number sets:
rational numbers, integers, whole numbers.

3. -5 4. $-2.1\overline{6}$ 5. 12 6. 0

Tell whether the decimal is *terminating* or *repeating*.

7. $-0.4848\ldots$ 8. -0.151 9. 72.72 10. $-5.2\overline{36}$
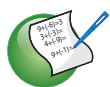
Practice and Problem Solving

Write the rational number as a decimal.

① 11. $\dfrac{7}{8}$ 12. $\dfrac{1}{11}$ 13. $-\dfrac{7}{9}$ 14. $-\dfrac{17}{40}$

15. $1\dfrac{5}{6}$ 16. $-2\dfrac{17}{18}$ 17. $-5\dfrac{7}{12}$ 18. $8\dfrac{15}{22}$

19. **ERROR ANALYSIS** Describe and correct the error in writing the rational number as a decimal.

     ✗ $-\dfrac{7}{11} = -0.6\overline{3}$

Write the decimal as a fraction or a mixed number in simplest form.

② 20. -0.9 21. 0.45 22. -0.258 23. -0.312

24. -2.32 25. -1.64 26. 6.012 27. -12.405

Order the numbers from least to greatest.

③ 28. $-\dfrac{3}{4}, 0.5, \dfrac{2}{3}, -\dfrac{7}{3}, 1.2$ 29. $\dfrac{9}{5}, -2.5, -1.1, -\dfrac{4}{5}, 0.8$ 30. $-1.4, -\dfrac{8}{5}, 0.6, -0.9, \dfrac{1}{4}$

31. $2.1, -\dfrac{6}{10}, -\dfrac{9}{4}, -0.75, \dfrac{5}{3}$ 32. $-\dfrac{7}{2}, -2.8, -\dfrac{5}{4}, \dfrac{4}{3}, 1.3$ 33. $-\dfrac{11}{5}, -2.4, 1.6, \dfrac{15}{10}, -2.25$

34. **COINS** You lose one quarter, two dimes, and two nickels.

     a. Write the amount as a decimal.

     b. Write the amount as a fraction in simplest form.

35. **HIBERNATION** A box turtle hibernates in sand at $-1\dfrac{5}{8}$ feet. A spotted turtle hibernates at $-1\dfrac{16}{25}$ feet. Which turtle is deeper?

Copy and complete the statement using <, >, or =.

36. -2.2 ⬜ -2.42

37. -1.82 ⬜ -1.81

38. $\dfrac{15}{8}$ ⬜ $1\dfrac{7}{8}$

39. $-4\dfrac{6}{10}$ ⬜ -4.65

40. $-5\dfrac{3}{11}$ ⬜ $-5.\overline{2}$

41. $-2\dfrac{13}{16}$ ⬜ $-2\dfrac{11}{14}$

**42. OPEN-ENDED** Find one terminating decimal and one repeating decimal between $-\dfrac{1}{2}$ and $-\dfrac{1}{3}$.

Player	Hits	At Bats
Eva	42	90
Michelle	38	80

43. SOFTBALL In softball, a batting average is the number of hits divided by the number of times at bat. Does Eva or Michelle have the higher batting average?

44. PROBLEM SOLVING You miss 3 out of 10 questions on a science quiz and 4 out of 15 questions on a math quiz. Which quiz has a higher percent of correct answers?

45. SKATING Is the half pipe deeper than the skating pool? Explain.

Skating pool — Lip — −10 ft — Base

Half pipe — Lip — $-9\dfrac{5}{6}$ ft — Base
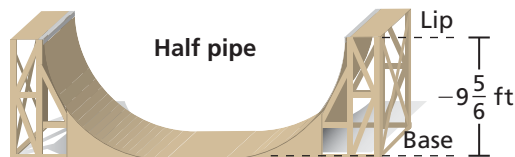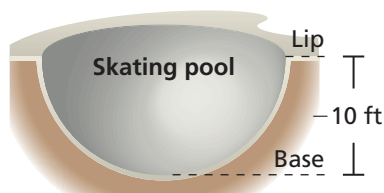
46. ENVIRONMENT The table shows the changes from the average water level of a pond over several weeks. Order the numbers from least to greatest.

Week	1	2	3	4
Change (inches)	$-\dfrac{7}{5}$	$-1\dfrac{5}{11}$	-1.45	$-1\dfrac{91}{200}$

47. **Critical Thinking** Given: a and b are integers.

    a. When is $-\dfrac{1}{a}$ positive?

    b. When is $\dfrac{1}{ab}$ positive?

Fair Game Review What you learned in previous grades & lessons

Add or subtract. *(Section 1.6 and Section 2.4)*

48. $\dfrac{3}{5} + \dfrac{2}{7}$

49. $\dfrac{9}{10} - \dfrac{2}{3}$

50. $8.79 - 4.07$

51. $11.81 + 9.34$

52. MULTIPLE CHOICE In one year, a company has a profit of $-\$2$ million. In the next year, the company has a profit of $\$7$ million. How much more profit did the company make the second year? *(Section 11.3)*

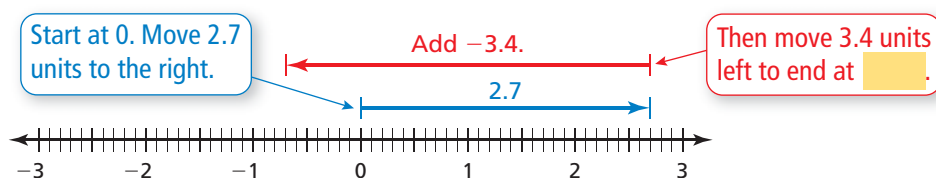 Ⓐ $2 million   Ⓑ $5 million   Ⓒ $7 million   Ⓓ $9 million

12.2 Adding Rational Numbers

Essential Question How can you use what you know about adding integers to add rational numbers?

1 ACTIVITY: Adding Rational Numbers

Work with a partner. Use a number line to find the sum.

a. $2.7 + (-3.4)$

Start at 0. Move 2.7 units to the right.

Add -3.4.

Then move 3.4 units left to end at ▢.

2.7

⋰ So, $2.7 + (-3.4) = $ ▢.
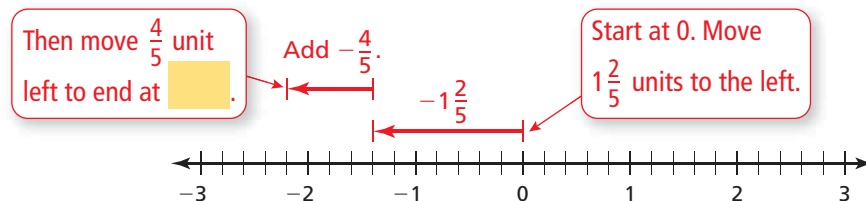
b. $1.3 + (-1.5)$

c. $-2.1 + 0.8$

d. $-1\frac{1}{4} + \frac{3}{4}$

e. $\frac{3}{10} + \left(-\frac{3}{10}\right)$

2 ACTIVITY: Adding Rational Numbers

Work with a partner. Use a number line to find the sum.

a. $-1\frac{2}{5} + \left(-\frac{4}{5}\right)$

Then move $\frac{4}{5}$ unit left to end at ▢.

Add $-\frac{4}{5}$.

Start at 0. Move $1\frac{2}{5}$ units to the left.

$-1\frac{2}{5}$

⋰ So, $-1\frac{2}{5} + \left(-\frac{4}{5}\right) = $ ▢.

b. $-\frac{7}{10} + \left(-1\frac{7}{10}\right)$

c. $-1\frac{2}{3} + \left(-1\frac{1}{3}\right)$

d. $-0.4 + (-1.9)$

e. $-2.3 + (-0.6)$
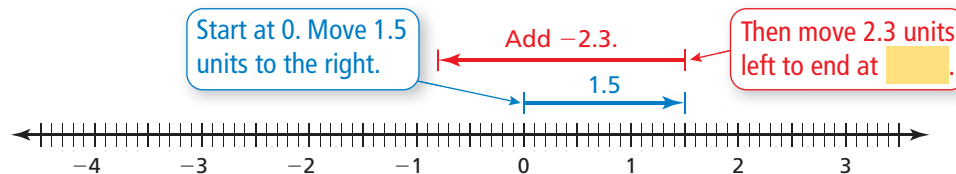
ACTIVITY: Writing Expressions

Work with a partner. Write the addition expression shown. Then find the sum.

Math Practice 2

Use Operations
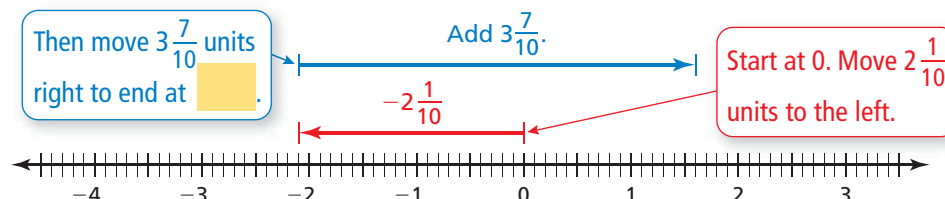What operation is represented in each number line? How does this help you write an expression?
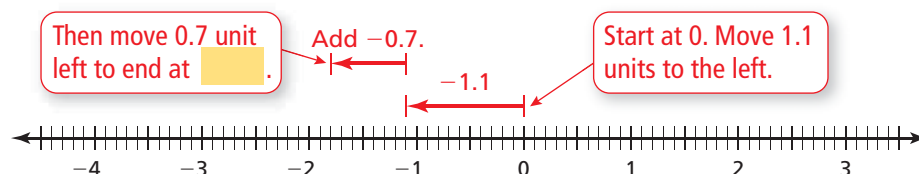
a.

Start at 0. Move 1.5 units to the right.　　Add −2.3.　　Then move 2.3 units left to end at ☐.

1.5

```
←——————————————————————————————→
  −4      −3      −2      −1       0       1       2       3
```

b.

Then move $3\frac{7}{10}$ units right to end at ☐.　　Add $3\frac{7}{10}$.　　Start at 0. Move $2\frac{1}{10}$ units to the left.

$-2\frac{1}{10}$

```
←——————————————————————————————→
  −4      −3      −2      −1       0       1       2       3
```

c.

Then move 0.7 unit left to end at ☐.　　Add −0.7.　　Start at 0. Move 1.1 units to the left.

−1.1

```
←——————————————————————————————→
  −4      −3      −2      −1       0       1       2       3
```

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use what you know about adding integers to add rational numbers?

PUZZLE Find a path through the table so that the numbers add up to the sum. You can move horizontally or vertically.

5. Sum: $\frac{3}{4}$

Start →

$\frac{1}{2}$	$\frac{2}{3}$	$-\frac{5}{7}$
$-\frac{1}{8}$	$-\frac{3}{4}$	$\frac{1}{3}$

←End

6. Sum: −0.07

Start →

2.43	1.75	−0.98
−1.09	3.47	−4.88

←End

Practice → Use what you learned about adding rational numbers to complete Exercises 4–6 on page 528.

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Key Idea

Adding Rational Numbers

Words To add rational numbers, use the same rules for signs as you used for integers.

Numbers $-\dfrac{1}{3} + \dfrac{1}{6} = \dfrac{-2}{6} + \dfrac{1}{6} = \dfrac{-2+1}{6} = \dfrac{-1}{6} = -\dfrac{1}{6}$

EXAMPLE 1 Adding Rational Numbers

Study Tip

In Example 1, notice how $-\dfrac{8}{3}$ is written as $-\dfrac{8}{3} = \dfrac{-8}{3} = \dfrac{-16}{6}$.

Find $-\dfrac{8}{3} + \dfrac{5}{6}$.

Estimate $-3 + 1 = -2$

$$-\dfrac{8}{3} + \dfrac{5}{6} = \dfrac{-16}{6} + \dfrac{5}{6} \qquad \text{Rewrite using the LCD (least common denominator).}$$

$$= \dfrac{-16 + 5}{6} \qquad \text{Write the sum of the numerators over the common denominator.}$$

$$= \dfrac{-11}{6} \qquad \text{Add.}$$

$$= -1\dfrac{5}{6} \qquad \text{Write the improper fraction as a mixed number.}$$

The sum is $-1\dfrac{5}{6}$.   **Reasonable?** $-1\dfrac{5}{6} \approx -2$ ✓

EXAMPLE 2 Adding Rational Numbers

Find $-4.05 + 7.62$.

$$-4.05 + 7.62 = 3.57 \qquad |\,7.62\,| > |-4.05\,|. \text{ So, subtract } |-4.05\,| \text{ from } |\,7.62\,|.$$

Use the sign of 7.62.

The sum is 3.57.

On Your Own

Now You're Ready
Exercises 4–12

Add.

1. $-\dfrac{7}{8} + \dfrac{1}{4}$

2. $-6\dfrac{1}{3} + \dfrac{20}{3}$

3. $2 + \left(-\dfrac{7}{2}\right)$

4. $-12.5 + 15.3$

5. $-8.15 + (-4.3)$

6. $0.65 + (-2.75)$

EXAMPLE 3 Evaluating Expressions

Evaluate $2x + y$ when $x = \frac{1}{4}$ and $y = -\frac{3}{2}$.

$$2x + y = 2\left(\frac{1}{4}\right) + \left(-\frac{3}{2}\right) \qquad \text{Substitute } \frac{1}{4} \text{ for } x \text{ and } -\frac{3}{2} \text{ for } y.$$

$$= \frac{1}{2} + \left(\frac{-3}{2}\right) \qquad \text{Multiply.}$$

$$= \frac{1 + (-3)}{2} \qquad \text{Write the sum of the numerators over the common denominator.}$$

$$= -1 \qquad \text{Simplify.}$$

EXAMPLE 4 Real-Life Application

Year	Profit (billions of dollars)
2008	−1.7
2009	−4.75
2010	1.7
2011	0.85
2012	3.6

The table shows the annual profits (in billions of dollars) of a financial company from 2008 to 2012. Positive numbers represent *gains*, and negative numbers represent *losses*. Which statement describes the profit over the five-year period?

A gain of $0.3 billion   **B** gain of $30 million

C loss of $3 million   **D** loss of $300 million

To determine whether there was a gain or a loss, find the sum of the profits.

$$\text{five-year profit} = -1.7 + (-4.75) + 1.7 + 0.85 + 3.6 \qquad \text{Write the sum.}$$

$$= -1.7 + 1.7 + (-4.75) + 0.85 + 3.6 \qquad \text{Comm. Prop. of Add.}$$

$$= 0 + (-4.75) + 0.85 + 3.6 \qquad \text{Additive Inv. Prop.}$$

$$= -4.75 + 0.85 + 3.6 \qquad \text{Add. Prop. of Zero}$$

$$= -3.9 + 3.6 \qquad \text{Add } -4.75 \text{ and } 0.85.$$

$$= -0.3 \qquad \text{Add } -3.9 \text{ and } 3.6.$$

The five-year profit is −$0.3 billion. So, the company has a five-year loss of $0.3 billion, or $300 million.

∴ The correct answer is **D**.

On Your Own

Now You're Ready
Exercises 15–17

Evaluate the expression when $a = \frac{1}{2}$ and $b = -\frac{5}{2}$.

7. $b + 4a$ **8.** $|a + b|$

9. WHAT IF? In Example 4, the 2013 profit is $1.07 billion. State the company's gain or loss over the six-year period in millions of dollars.
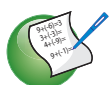
Vocabulary and Concept Check

1. **WRITING** Explain how to find the sum $-8.46 + 5.31$.

2. **OPEN-ENDED** Write an addition expression using fractions that equals $-\frac{1}{2}$.

3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

   Add -4.5 and 3.5.   What is the distance between -4.5 and 3.5?

   What is -4.5 increased by 3.5?   Find the sum of -4.5 and 3.5.

Practice and Problem Solving

Add. Write fractions in simplest form.

1 2   **4.** $\frac{11}{12} + \left(-\frac{7}{12}\right)$ **5.** $-1\frac{1}{5} + \left(-\frac{3}{5}\right)$ **6.** $-4.2 + 3.3$

7. $-\frac{9}{14} + \frac{2}{7}$ **8.** $4 + \left(-1\frac{2}{3}\right)$ **9.** $\frac{15}{4} + \left(-4\frac{1}{3}\right)$

10. $-3.1 + (-0.35)$ **11.** $12.48 + (-10.636)$ **12.** $20.25 + (-15.711)$

ERROR ANALYSIS Describe and correct the error in finding the sum.

13.

❌
$$\begin{array}{r} -3.7 \\ + (-0.25) \\ \hline -0.62 \end{array}$$

14.

❌
$$-\frac{5}{8} + \frac{1}{8} = \frac{-5 + 1}{8} = \frac{-6}{8} = -\frac{3}{4}$$

Evaluate the expression when $x = \frac{1}{3}$ and $y = -\frac{7}{4}$.

3   **15.** $x + y$ **16.** $3x + y$ **17.** $-x + |y|$

18. **BANKING** Your bank account balance is –$20.85. You deposit $15.50. What is your new balance?

19. **HOT DOGS** You eat $\frac{3}{10}$ of a pack of hot dogs. Your friend eats $\frac{1}{5}$ of the pack of hot dogs. What fraction of the pack of hot dogs do you and your friend eat?

Add. Write fractions in simplest form.

20. $6 + \left(-4\frac{3}{4}\right) + \left(-2\frac{1}{8}\right)$

21. $-5\frac{2}{3} + 3\frac{1}{4} + \left(-7\frac{1}{3}\right)$

22. $10.9 + (-15.6) + 2.1$

23. NUMBER SENSE When is the sum of two negative mixed numbers an integer?

24. WRITING You are adding two rational numbers with different signs. How can you tell if the sum will be *positive, negative,* or *zero*?

25. RESERVOIR The table at the left shows the water level (in inches) of a reservoir for three months compared to the yearly average. Is the water level for the three-month period greater than or less than the yearly average? Explain.

June	July	August
$-2\frac{1}{8}$	$1\frac{1}{4}$	$-\frac{9}{16}$

26. BREAK EVEN The table at the right shows the annual profits (in thousands of dollars) of a county fair from 2008 to 2012. What must the 2012 profit be (in hundreds of dollars) to break even over the five-year period?

Year	Profit (thousands of dollars)
2008	2.5
2009	1.75
2010	-3.3
2011	-1.4
2012	?

27. REASONING Is $|a + b| = |a| + |b|$ for all rational numbers a and b? Explain.

28. *Repeated Reasoning* Evaluate the expression.

$$\frac{19}{20} + \left(\frac{-18}{20}\right) + \frac{17}{20} + \left(\frac{-16}{20}\right) + \cdots + \left(\frac{-4}{20}\right) + \frac{3}{20} + \left(\frac{-2}{20}\right) + \frac{1}{20}$$

Fair Game Review *What you learned in previous grades & lessons*

Identify the property. Then simplify. *(Section 3.3)*

29. $8 + (-3) + 2 = 8 + 2 + (-3)$

30. $2 \cdot (4.5 \cdot 9) = (2 \cdot 4.5) \cdot 9$

31. $\frac{1}{4} + \left(\frac{3}{4} + \frac{1}{8}\right) = \left(\frac{1}{4} + \frac{3}{4}\right) + \frac{1}{8}$

32. $\frac{3}{7} \cdot \frac{4}{5} \cdot \frac{14}{27} = \frac{3}{7} \cdot \frac{14}{27} \cdot \frac{4}{5}$

33. MULTIPLE CHOICE The regular price of a photo album is $18. You have a coupon for 15% off. How much is the discount? *(Section 5.6)*
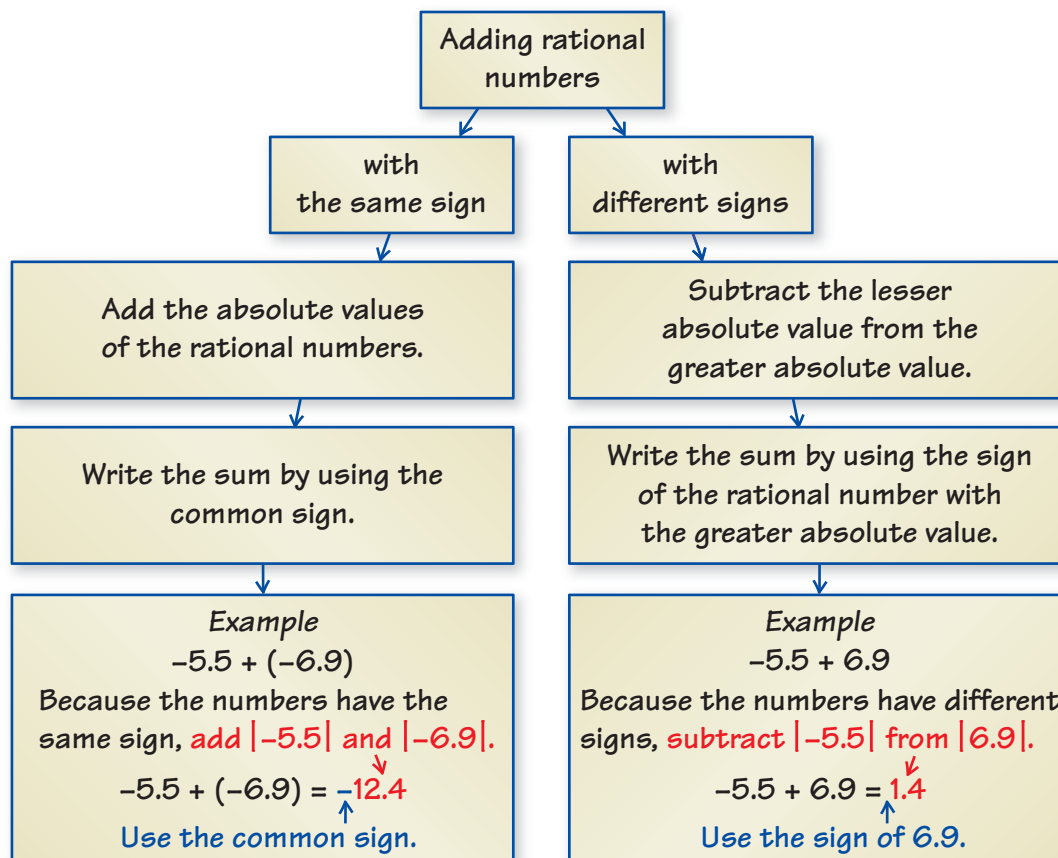
Ⓐ $2.70 　　　Ⓑ $3 　　　Ⓒ $15 　　　Ⓓ $15.30

Check It Out
Graphic Organizer
BigIdeasMath com

You can use a **process diagram** to show the steps involved in a procedure. Here is an example of a process diagram for adding rational numbers.
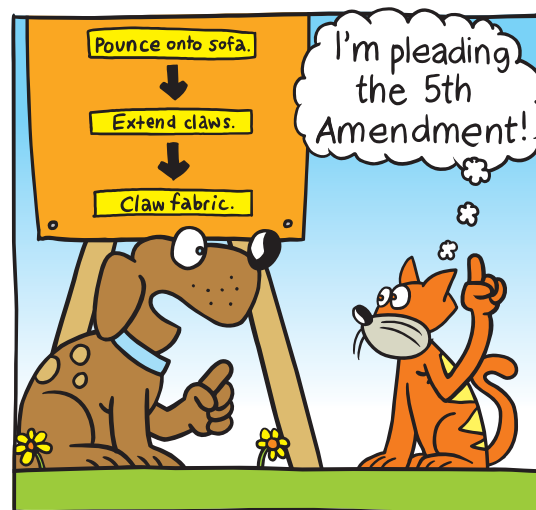
Adding rational numbers

with the same sign

with different signs

Add the absolute values of the rational numbers.

Subtract the lesser absolute value from the greater absolute value.

Write the sum by using the common sign.

Write the sum by using the sign of the rational number with the greater absolute value.

Example
$-5.5 + (-6.9)$
Because the numbers have the same sign, add $|-5.5|$ and $|-6.9|$.
$-5.5 + (-6.9) = -12.4$
Use the common sign.

Example
$-5.5 + 6.9$
Because the numbers have different signs, subtract $|-5.5|$ from $|6.9|$.
$-5.5 + 6.9 = 1.4$
Use the sign of 6.9.

On Your Own

Make a process diagram with examples to help you study the topic.

1. writing rational numbers as decimals

After you complete this chapter, make process diagrams with examples for the following topics.

2. subtracting rational numbers

3. multiplying rational numbers

4. dividing rational numbers

Pounce onto sofa.

Extend claws.

Claw fabric.

I'm pleading the 5th Amendment!

"Does this process diagram accurately show how a cat claws furniture?"

Write the rational number as a decimal.   *(Section 12.1)*

1. $-\dfrac{3}{20}$

2. $-\dfrac{11}{6}$

Write the decimal as a fraction or a mixed number in simplest form.   *(Section 12.1)*

3. -0.325

4. -1.28

Order the numbers from least to greatest.   *(Section 12.1)*

5. $-\dfrac{1}{3}, -0.2, \dfrac{5}{3}, 0.4, 1.3$

6. $-\dfrac{4}{3}, -1.2, 0.3, \dfrac{4}{9}, -0.8$

Add. Write fractions in simplest form.   *(Section 12.2)*

7. $-\dfrac{4}{5} + \left(-\dfrac{3}{8}\right)$

8. $-\dfrac{13}{6} + \dfrac{7}{12}$

9. $-5.8 + 2.6$

10. $-4.28 + (-2.56)$

Evaluate the expression when $x = \dfrac{3}{4}$ and $y = -\dfrac{1}{2}$.   *(Section 12.2)*

11. $x + y$

12. $2x + y$

13. $x + |y|$

14. $|-x + y|$

15. STOCK The value of Stock A changes $-\$3.68$, and the value of Stock B changes $-\$3.72$. Which stock has the greater loss? Explain.   *(Section 12.1)*

16. LEMONADE You drink $\dfrac{2}{7}$ of a pitcher of lemonade. Your friend drinks $\dfrac{3}{14}$ of the pitcher. What fraction of the pitcher do you and your friend drink?   *(Section 12.2)*

17. FOOTBALL The table shows the statistics of a running back in a football game. Did he gain more than 50 yards total? Explain.   *(Section 12.2)*

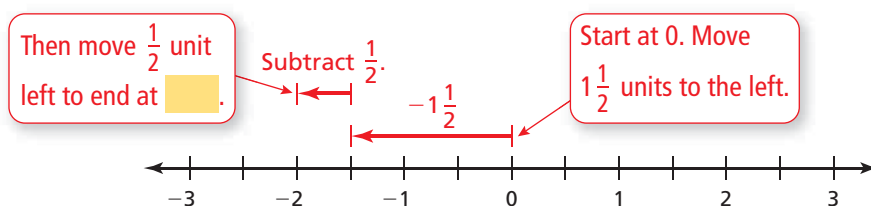| Quarter	1	2	3	4	Total
Yards	$-8\dfrac{1}{2}$	23	$42\dfrac{1}{2}$	$-2\dfrac{1}{4}$	?

12.3 Subtracting Rational Numbers

Essential Question How can you use what you know about subtracting integers to subtract rational numbers?

1 ACTIVITY: Subtracting Rational Numbers

Work with a partner. Use a number line to find the difference.

a. $-1\frac{1}{2} - \frac{1}{2}$

Then move $\frac{1}{2}$ unit left to end at ☐.

Subtract $\frac{1}{2}$.

$-1\frac{1}{2}$

Start at 0. Move $1\frac{1}{2}$ units to the left.

```
←——+——+——+——+——+——+——+——+——+——+——+——+——+——→
   −3       −2       −1        0        1        2        3
```

So, $-1\frac{1}{2} - \frac{1}{2} = $ ☐.

b. $\frac{6}{10} - 1\frac{3}{10}$

c. $-1\frac{1}{4} - 1\frac{3}{4}$

d. $-1.9 - 0.8$

e. $0.2 - 0.7$

2 ACTIVITY: Finding Distances on a Number Line

Work with a partner.

a. Plot -3 and 2 on the number line. Then find $-3 - 2$ and $2 - (-3)$. What do you notice about your results?

```
←——+——+——+——+——+——+——+——+——+——+——+——+——→
  −6  −5  −4  −3  −2  −1   0   1   2   3   4   5   6
```
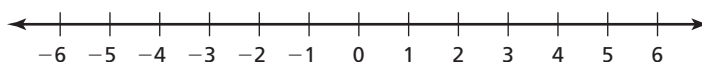
b. Plot $\frac{3}{4}$ and 1 on the number line. Then find $\frac{3}{4} - 1$ and $1 - \frac{3}{4}$. What do you notice about your results?

```
←—+++++++++++++++++++++++++++++++++++++++++—→
  −3        −2        −1         0         1         2         3
```
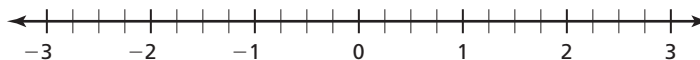
c. Choose any two points a and b on a number line. Find the values of $a - b$ and $b - a$. What do the absolute values of these differences represent? Is this true for any pair of rational numbers? Explain.

ACTIVITY: Financial Literacy

Work with a partner. The table shows the balance in a checkbook.

- Black numbers are amounts added to the account.
- Red numbers are amounts taken from the account.

Date	Check #	Transaction	Amount	Balance
––	––	Previous balance	––	100.00
1/02/2013	124	Groceries	34.57	
1/07/2013		Check deposit	875.50	
1/11/2013		ATM withdrawal	40.00	
1/14/2013	125	Electric company	78.43	
1/17/2013		Music store	10.55	
1/18/2013	126	Shoes	47.21	
1/22/2013		Check deposit	125.00	
1/24/2013		Interest	2.12	
1/25/2013	127	Cell phone	59.99	
1/26/2013	128	Clothes	65.54	
1/30/2013	129	Cable company	75.00	

Math Practice 4

Interpret Results

What does your answer represent? Does your answer make sense?

You can find the balance in the **second row** two different ways.

$$100.00 - 34.57 = 65.43 \qquad \text{Subtract 34.57 from 100.00.}$$
$$100.00 + (-34.57) = 65.43 \qquad \text{Add } -34.57 \text{ to } 100.00.$$

a. Copy the table. Then complete the balance column.

b. How did you find the balance in the **twelfth row**?

c. Use a different way to find the balance in part (b).

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use what you know about subtracting integers to subtract rational numbers?

5. Give two real-life examples of subtracting rational numbers that are not integers.

Practice Use what you learned about subtracting rational numbers to complete Exercises 3–5 on page 536.

Key Idea

Subtracting Rational Numbers

Words To subtract rational numbers, use the same rules for signs as you used for integers.

Numbers $\dfrac{2}{5} - \left(-\dfrac{1}{5}\right) = \dfrac{2}{5} + \dfrac{1}{5} = \dfrac{2+1}{5} = \dfrac{3}{5}$

EXAMPLE 1 Subtracting Rational Numbers

Find $-4\dfrac{1}{7} - \left(-\dfrac{6}{7}\right)$. **Estimate** $-4 - (-1) = -3$

$$-4\dfrac{1}{7} - \left(-\dfrac{6}{7}\right) = -4\dfrac{1}{7} + \dfrac{6}{7}$$ Add the opposite of $-\dfrac{6}{7}$.

$$= -\dfrac{29}{7} + \dfrac{6}{7}$$ Write the mixed number as an improper fraction.

$$= \dfrac{-29 + 6}{7}$$ Write the sum of the numerators over the common denominator.

$$= \dfrac{-23}{7}$$ Add.

$$= -3\dfrac{2}{7}$$ Write the improper fraction as a mixed number.

∴ The difference is $-3\dfrac{2}{7}$. **Reasonable?** $-3\dfrac{2}{7} \approx -3$ ✓

EXAMPLE 2 Subtracting Rational Numbers

Find $12.8 - 21.6$.

$12.8 - 21.6 = 12.8 + (-21.6)$ Add the opposite of 21.6.

$\qquad\qquad = -8.8$ $|-21.6| > |12.8|$. So, subtract $|12.8|$ from $|-21.6|$.

Use the sign of -21.6.

∴ The difference is -8.8.

On Your Own

Now You're Ready
Exercises 3–11

1. $\dfrac{1}{3} - \left(-\dfrac{1}{3}\right)$ **2.** $-3\dfrac{1}{3} - \dfrac{5}{6}$ **3.** $4\dfrac{1}{2} - 5\dfrac{1}{4}$
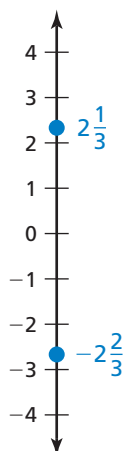
4. $-8.4 - 6.7$ **5.** $-20.5 - (-20.5)$ **6.** $0.41 - (-0.07)$

The distance between any two numbers on a number line is the absolute value of the difference of the numbers.

EXAMPLE 3 Finding Distances Between Numbers on a Number Line

Find the distance between the two numbers on the number line.

To find the distance between the numbers, first find the difference of the numbers.

$$-2\frac{2}{3} - 2\frac{1}{3} = -2\frac{2}{3} + \left(-2\frac{1}{3}\right) \quad \text{Add the opposite of } 2\frac{1}{3}.$$

$$= -\frac{8}{3} + \left(-\frac{7}{3}\right) \quad \text{Write the mixed numbers as improper fractions.}$$

$$= \frac{-15}{3} \quad \text{Add.}$$

$$= -5 \quad \text{Simplify.}$$

∴ Because $|-5| = 5$, the distance between $-2\frac{2}{3}$ and $2\frac{1}{3}$ is 5.

EXAMPLE 4 Real-Life Application

In the water, the bottom of a boat is 2.1 feet below the surface, and the top of the boat is 8.7 feet above it. Towed on a trailer, the bottom of the boat is 1.3 feet above the ground. Can the boat and trailer pass under the bridge?

Step 1: Find the height h of the boat.

$$h = 8.7 - (-2.1) \quad \text{Subtract the lowest point from the highest point.}$$

$$= 8.7 + 2.1 \quad \text{Add the opposite of } -2.1.$$

$$= 10.8 \quad \text{Add.}$$

Clearance: 11 ft 8 in.

Step 2: Find the height t of the boat and trailer.

$$t = 10.8 + 1.3 \quad \text{Add the trailer height to the boat height.}$$

$$= 12.1 \quad \text{Add.}$$

∴ Because 12.1 feet is greater than 11 feet 8 inches, the boat and trailer cannot pass under the bridge.

On Your Own

Now You're Ready
Exercises 13–15

7. Find the distance between -7.5 and -15.3 on a number line.

8. WHAT IF? In Example 4, the clearance is 12 feet 1 inch. Can the boat and trailer pass under the bridge?
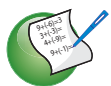
✓ Vocabulary and Concept Check

1. **WRITING** Explain how to find the difference $-\dfrac{4}{5} - \dfrac{3}{5}$.

2. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$$-\dfrac{5}{8} - \dfrac{3}{4}\qquad\qquad -\dfrac{3}{4} + \dfrac{5}{8}\qquad\qquad -\dfrac{5}{8} + \left(-\dfrac{3}{4}\right)\qquad\qquad -\dfrac{3}{4} - \dfrac{5}{8}$$

Practice and Problem Solving

Subtract. Write fractions in simplest form.

① ② 3. $\dfrac{5}{8} - \left(-\dfrac{7}{8}\right)$

4. $-1\dfrac{1}{3} - 1\dfrac{2}{3}$

5. $-1 - 2.5$

6. $-5 - \dfrac{5}{3}$

7. $-8\dfrac{3}{8} - 10\dfrac{1}{6}$

8. $-\dfrac{1}{2} - \left(-\dfrac{5}{9}\right)$

9. $5.5 - 8.1$

10. $-7.34 - (-5.51)$

11. $6.673 - (-8.29)$

12. **ERROR ANALYSIS** Describe and correct the error in finding the difference.

✗ $\dfrac{3}{4} - \dfrac{9}{2} = \dfrac{3-9}{4-2} = \dfrac{-6}{2} = -3$

Find the distance between the two numbers on a number line.

③ 13. $-2\dfrac{1}{2}, -5\dfrac{3}{4}$

14. $-2.2, 8.4$

15. $-7, -3\dfrac{2}{3}$

16. **SPORTS DRINK** Your sports drink bottle is $\dfrac{5}{6}$ full. After practice, the bottle is $\dfrac{3}{8}$ full. Write the difference of the amounts after practice and before practice.

17. **SUBMARINE** The figure shows the depths of a submarine.

    a. Find the vertical distance traveled by the submarine.

    b. Find the mean hourly vertical distance traveled by the submarine.

−314.9 ft (now)

−725.6 ft (3 hours ago)
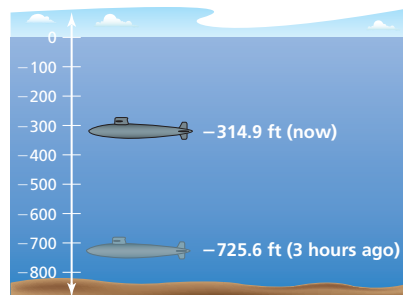
Evaluate.

18. $2\dfrac{1}{6} - \left(-\dfrac{8}{3}\right) + \left(-4\dfrac{7}{9}\right)$

19. $6.59 + (-7.8) - (-2.41)$

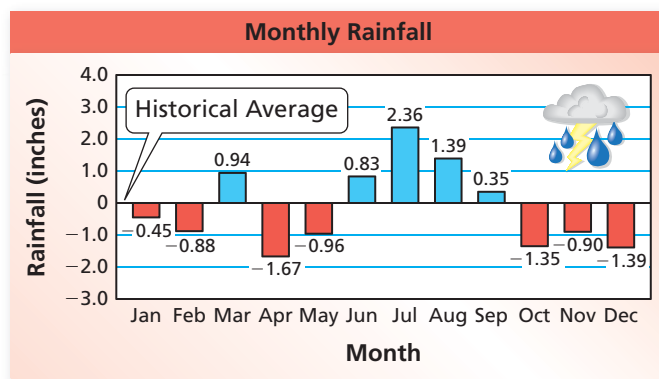20. $-\dfrac{12}{5} + \left|-\dfrac{13}{6}\right| + \left(-3\dfrac{2}{3}\right)$

21. REASONING When is the difference of two decimals an integer? Explain.

22. RECIPE A cook has $2\frac{2}{3}$ cups of flour. A recipe calls for $2\frac{3}{4}$ cups of flour. Does the cook have enough flour? If not, how much more flour is needed?

Springville
new road
$2\frac{3}{8}$ mi
Uniontown
$3\frac{5}{6}$ mi

23. ROADWAY A new road that connects Uniontown to Springville is $4\frac{1}{3}$ miles long. What is the change in distance when using the new road instead of the dirt roads?

RAINFALL In Exercises 24–26, the bar graph shows the differences in a city's rainfall from the historical average.

24. What is the difference in rainfall between the wettest and the driest months?

25. Find the sum of the differences for the year.

26. What does the sum in Exercise 25 tell you about the rainfall for the year?

Monthly Rainfall
Historical Average
2.36
1.39
0.94
0.83
0.35
−0.45
−0.88
−0.96
−1.67
−1.35
−0.90
−1.39
Rainfall (inches)
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec
Month

27. OPEN-ENDED Write two different pairs of negative decimals, x and y, that make the statement $x - y = 0.6$ true.

REASONING Tell whether the difference between the two numbers is *always, sometimes,* or *never* positive. Explain your reasoning.

28. two negative fractions

29. a positive decimal and a negative decimal

30. Structure Fill in the blanks to make the solution correct.

$$5.\boxed{}4 - \left(\boxed{}.8\boxed{}\right) = -3.61$$

Fair Game Review What you learned in previous grades & lessons

Evaluate. *(Section 2.1, Section 2.3, Section 2.5, and Section 2.6)*

31. 5.2×6.9

32. $7.2 \div 2.4$

33. $2\frac{2}{3} \times 3\frac{1}{4}$

34. $9\frac{4}{5} \div 3\frac{1}{2}$

35. MULTIPLE CHOICE A sports store has 116 soccer balls. Over 6 months, it sells 8 soccer balls per month. How many soccer balls are in inventory at the end of the 6 months? *(Section 11.3 and Section 11.4)*

Ⓐ -48 Ⓑ 48 Ⓒ 68 Ⓓ 108

12.4 Multiplying and Dividing Rational Numbers

Essential Question Why is the product of two negative rational numbers positive?

In Section 11.4, you used a table to see that the product of two negative integers is a positive integer. In this activity, you will find that same result another way.

1 ACTIVITY: Showing $(-1)(-1) = 1$

Work with a partner. How can you show that $(-1)(-1) = 1$?

To begin, assume that $(-1)(-1) = 1$ is a true statement. From the Additive Inverse Property, you know that $1 + (-1) = 0$. So, substitute $(-1)(-1)$ for 1 to get $(-1)(-1) + (-1) = 0$. If you can show that $(-1)(-1) + (-1) = 0$ is true, then you have shown that $(-1)(-1) = 1$.

Justify each step.

$$(-1)(-1) + (-1) = (-1)(-1) + 1(-1)$$

$$= (-1)[(-1) + 1]$$

$$= (-1)0$$

$$= 0$$

So, $(-1)(-1) = 1$.

2 ACTIVITY: Multiplying by -1

Work with a partner.

a. Graph each number below on three different number lines. Then multiply each number by -1 and graph the product on the appropriate number line.

$$2 \qquad 8 \qquad -1$$

b. How does multiplying by -1 change the location of the points in part (a)? What is the relationship between the number and the product?

c. Graph each number below on three different number lines. Where do you think the points will be after multiplying by -1? Plot the points. Explain your reasoning.

$$\frac{1}{2} \qquad 2.5 \qquad -\frac{5}{2}$$

d. What is the relationship between a rational number $-a$ and the product $-1(a)$? Explain your reasoning.

ACTIVITY: Understanding the Product of Rational Numbers

Work with a partner. Let a and b be positive rational numbers.

a. Because a and b are positive, what do you know about $-a$ and $-b$?

b. Justify each step.

$$(-a)(-b) = (-1)(a)(-1)(b)$$
$$= (-1)(-1)(a)(b)$$
$$= (1)(a)(b)$$
$$= ab$$

c. Because a and b are positive, what do you know about the product ab?

d. What does this tell you about products of rational numbers? Explain.

4 **ACTIVITY: Writing a Story**

Work with a partner. Write a story that uses addition, subtraction, multiplication, or division of rational numbers.

- At least one of the numbers in the story has to be negative and *not* an integer.
- Draw pictures to help illustrate what is happening in the story.
- Include the solution of the problem in the story.

Math Practice 6

Specify Units
What units are in your story?

If you are having trouble thinking of a story, here are some common uses of negative numbers:

- A profit of $-\$15$ is a loss of $15.
- An elevation of -100 feet is a depth of 100 feet below sea level.
- A gain of -5 yards in football is a loss of 5 yards.
- A score of -4 in golf is 4 strokes under par.

What Is Your Answer?

5. **IN YOUR OWN WORDS** Why is the product of two negative rational numbers positive?

6. **PRECISION** Show that $(-2)(-3) = 6$.

7. How can you show that the product of a negative rational number and a positive rational number is negative?

Practice

Use what you learned about multiplying rational numbers to complete Exercises 7–9 on page 542.

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Idea

Multiplying and Dividing Rational Numbers

Words   To multiply or divide rational numbers, use the same rules for signs as you used for integers.

Remember

The *reciprocal* of $\dfrac{a}{b}$ is $\dfrac{b}{a}$.

Numbers $-\dfrac{2}{7} \cdot \dfrac{1}{3} = \dfrac{-2 \cdot 1}{7 \cdot 3} = \dfrac{-2}{21} = -\dfrac{2}{21}$

$-\dfrac{1}{2} \div \dfrac{4}{9} = \dfrac{-1}{2} \cdot \dfrac{9}{4} = \dfrac{-1 \cdot 9}{2 \cdot 4} = \dfrac{-9}{8} = -\dfrac{9}{8}$

EXAMPLE 1   Dividing Rational Numbers

Find $-5\dfrac{1}{5} \div 2\dfrac{1}{3}$.   **Estimate** $-5 \div 2 = -2\dfrac{1}{2}$

$-5\dfrac{1}{5} \div 2\dfrac{1}{3} = -\dfrac{26}{5} \div \dfrac{7}{3}$ Write mixed numbers as improper fractions.

$= \dfrac{-26}{5} \cdot \dfrac{3}{7}$ Multiply by the reciprocal of $\dfrac{7}{3}$.

$= \dfrac{-26 \cdot 3}{5 \cdot 7}$ Multiply the numerators and the denominators.

$= \dfrac{-78}{35}$, or $-2\dfrac{8}{35}$ Simplify.

The quotient is $-2\dfrac{8}{35}$.   **Reasonable?** $-2\dfrac{8}{35} \approx -2\dfrac{1}{2}$ ✔

EXAMPLE 2   Multiplying Rational Numbers

Find $-2.5 \cdot 3.6$.

$$
\begin{array}{r}
-2.5 \\
\times\ 3.6 \\
\hline
1\,5\,0 \\
7\,5\,0\ \ \\
\hline
-9.0\,0
\end{array}
$$

The decimals have different signs.

The product is negative.

The product is -9.

EXAMPLE 3 Multiplying More Than Two Rational Numbers

Find $-\dfrac{1}{7} \cdot \left[\dfrac{4}{5} \cdot (-7)\right]$.

You can use properties of multiplication to make the product easier to find.

$$-\dfrac{1}{7} \cdot \left[\dfrac{4}{5} \cdot (-7)\right] = -\dfrac{1}{7} \cdot \left(-7 \cdot \dfrac{4}{5}\right) \qquad \text{Commutative Property of Multiplication}$$

$$= -\dfrac{1}{7} \cdot (-7) \cdot \dfrac{4}{5} \qquad \text{Associative Property of Multiplication}$$

$$= 1 \cdot \dfrac{4}{5} \qquad \text{Multiplicative Inverse Property}$$

$$= \dfrac{4}{5} \qquad \text{Multiplication Property of One}$$

⋮ The product is $\dfrac{4}{5}$.

On Your Own

Now You're Ready
Exercises 10–30

Multiply or divide. Write fractions in simplest form.

1. $-\dfrac{6}{5} \div \left(-\dfrac{1}{2}\right)$

2. $\dfrac{1}{3} \div \left(-2\dfrac{2}{3}\right)$

3. $1.8(-5.1)$

4. $-6.3(-0.6)$

5. $-\dfrac{2}{3} \cdot 7\dfrac{7}{8} \cdot \dfrac{3}{2}$

6. $-7.2 \cdot 0.1 \cdot (-100)$

EXAMPLE 4 Real-Life Application

Account Positions	⟳		
Stock	**Original Value**	**Current Value**	**Change**
A	600.54	420.15	−180.39
B	391.10	518.38	127.28
C	380.22	99.70	−280.52

An investor owns Stocks A, B, and C. What is the mean change in the value of the stocks?

$$\text{mean} = \dfrac{-180.39 + 127.28 + (-280.52)}{3} = \dfrac{-333.63}{3} = -111.21$$

⋮ The mean change in the value of the stocks is −$111.21.

On Your Own

7. **WHAT IF?** The change in the value of Stock D is $568.23. What is the mean change in the value of the four stocks?
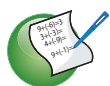
✔ Vocabulary and Concept Check

1. **WRITING** How is multiplying and dividing rational numbers similar to multiplying and dividing integers?

2. **NUMBER SENSE** Find the reciprocal of $-\dfrac{2}{5}$.

Tell whether the expression is *positive* or *negative* without evaluating.

3. $-\dfrac{3}{10} \times \left(-\dfrac{8}{15}\right)$

4. $1\dfrac{1}{2} \div \left(-\dfrac{1}{4}\right)$

5. -6.2×8.18

6. $\dfrac{-8.16}{-2.72}$

✎ Practice and Problem Solving

Multiply.

7. $-1\left(\dfrac{4}{5}\right)$

8. $-1\left(-3\dfrac{1}{2}\right)$

9. $-0.25(-1)$

Divide. Write fractions in simplest form.

① 10. $-\dfrac{7}{10} \div \dfrac{2}{5}$

11. $\dfrac{1}{4} \div \left(-\dfrac{3}{8}\right)$

12. $-\dfrac{8}{9} \div \left(-\dfrac{8}{9}\right)$

13. $-\dfrac{1}{5} \div 20$

14. $-2\dfrac{4}{5} \div (-7)$

15. $-10\dfrac{2}{7} \div \left(-4\dfrac{4}{11}\right)$

16. $-9 \div 7.2$

17. $8 \div 2.2$

18. $-3.45 \div (-15)$

19. $-0.18 \div 0.03$

20. $8.722 \div (-3.56)$

21. $12.42 \div (-4.8)$

Multiply. Write fractions in simplest form.

② ③ 22. $-\dfrac{1}{4} \times \left(-\dfrac{4}{3}\right)$

23. $\dfrac{5}{6}\left(-\dfrac{8}{15}\right)$

24. $-2\left(-1\dfrac{1}{4}\right)$

25. $-3\dfrac{1}{3} \cdot \left(-2\dfrac{7}{10}\right)$

26. $0.4 \times (-0.03)$

27. $-0.05 \times (-0.5)$

28. $-8(0.09)(-0.5)$

29. $\dfrac{5}{6} \cdot \left(-4\dfrac{1}{2}\right) \cdot \left(-2\dfrac{1}{5}\right)$
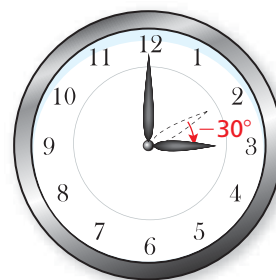
30. $\left(-1\dfrac{2}{3}\right)^3$

ERROR ANALYSIS Describe and correct the error.

31. ✗ $-2.2 \times 3.7 = 8.14$

32. ✗ $-\dfrac{1}{4} \div \dfrac{3}{2} = -\dfrac{4}{1} \times \dfrac{3}{2} = -\dfrac{12}{2} = -6$

33. **HOUR HAND** The hour hand of a clock moves $-30°$ every hour. How many degrees does it move in $2\dfrac{1}{5}$ hours?

34. **SUNFLOWER SEEDS** How many 0.75-pound packages can you make with 6 pounds of sunflower seeds?

−30°

Evaluate.

35. $-4.2 + 8.1 \times (-1.9)$

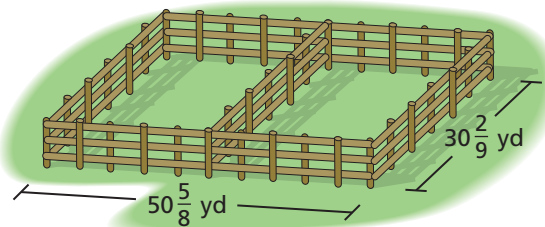**36.** $2.85 - 6.2 \div 2^2$

37. $-3.64 \cdot \left| -5.3 \right| - 1.5^3$

38. $1\frac{5}{9} \div \left(-\frac{2}{3} \right) + \left(-2\frac{3}{5} \right)$

39. $-3\frac{3}{4} \times \frac{5}{6} - 2\frac{1}{3}$

40. $\left(-\frac{2}{3} \right)^2 - \frac{3}{4}\left(2\frac{1}{3} \right)$

41. OPEN-ENDED Write two fractions whose product is $-\dfrac{3}{5}$.

42. FENCING A farmer needs to enclose two adjacent rectangular pastures. How much fencing does the farmer need?

$30\frac{2}{9}$ yd

$50\frac{5}{8}$ yd

43. GASOLINE A 14.5-gallon gasoline tank is $\dfrac{3}{4}$ full. How many gallons will it take to fill the tank?

44. PRECISION A section of a boardwalk is made using 15 boards. Each board is $9\dfrac{1}{4}$ inches wide. The total width of the section is 144 inches. The spacing between each board is equal. What is the width of the spacing between each board?

45. RUNNING The table shows the changes in the times (in seconds) of four teammates. What is the mean change?

46. **Critical Thinking** The daily changes in the barometric pressure for four days are -0.05, 0.09, -0.04, and -0.08 inches.

    a. What is the mean change?

    b. The mean change after five days is -0.01 inch. What is the change on the fifth day? Explain.

Teammate	Change
1	-2.43
2	-1.85
3	0.61
4	-1.45

Fair Game Review What you learned in previous grades & lessons

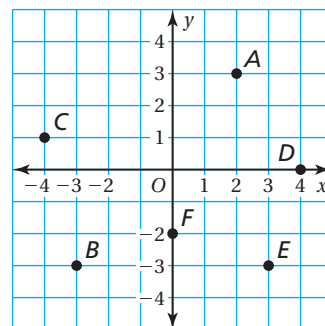**Add or subtract.** *(Section 12.2 and Section 12.3)*

47. $-6.2 + 4.7$

48. $-8.1 - (-2.7)$

49. $\dfrac{9}{5} - \left(-2\dfrac{7}{10} \right)$

50. $-4\dfrac{5}{6} + \left(-3\dfrac{4}{9} \right)$

51. MULTIPLE CHOICE What are the coordinates of the point in Quadrant IV? *(Section 6.5)*

    (A) $(-4, 1)$ **(B)** $(-3, -3)$

    (C) $(0, -2)$ **(D)** $(3, -3)$

Subtract. Write fractions in simplest form. *(Section 12.3)*
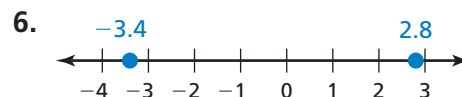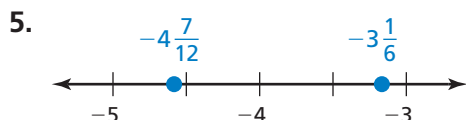
1. $\dfrac{2}{7} - \left(\dfrac{6}{7}\right)$

2. $\dfrac{12}{7} - \left(-\dfrac{2}{9}\right)$

3. $9.1 - 12.9$

4. $5.647 - (-9.24)$

Find the distance between the two numbers on the number line. *(Section 12.3)*

5.
$-4\dfrac{7}{12}$ $-3\dfrac{1}{6}$

$-5 \quad\quad -4 \quad\quad -3$

6.
$-3.4 \quad\quad\quad\quad 2.8$

$-4 \ -3 \ -2 \ -1 \ \ 0 \ \ 1 \ \ 2 \ \ 3$

Divide. Write fractions in simplest form. *(Section 12.4)*

7. $\dfrac{2}{3} \div \left(-\dfrac{5}{6}\right)$

8. $-8\dfrac{5}{9} \div \left(-1\dfrac{4}{7}\right)$

9. $-8.4 \div 2.1$

10. $32.436 \div (-4.24)$

Multiply. Write fractions in simplest form. *(Section 12.4)*

11. $\dfrac{5}{8} \times \left(-\dfrac{4}{15}\right)$

**12.** $-2\dfrac{3}{8} \times \dfrac{8}{5}$

13. $-9.4 \times (-4.7)$

14. $-100(-0.6)(0.01)$

15. PARASAILING A parasail is at 200.6 feet above the water. After 5 minutes, the parasail is at 120.8 feet above the water. What is the change in height of the parasail? *(Section 12.3)*

16. TEMPERATURE Use the thermometer shown. How much did the temperature drop from 5:00 P.M. to 10:00 P.M.? *(Section 12.3)*

17. LATE FEES You were overcharged $4.52 on your cell phone bill 3 months in a row. The cell phone company says that it will add $-\$4.52$ to your next bill for each month you were overcharged. On the next bill, you see an adjustment of -13.28. Is this amount correct? Explain. *(Section 12.4)*

5:00 P.M. ← 7.1°F

10:00 P.M. ← −10.3°F

18. CASHEWS How many $1\dfrac{1}{4}$-pound packages can you make with $7\dfrac{1}{2}$ pounds of cashews? *(Section 12.4)*
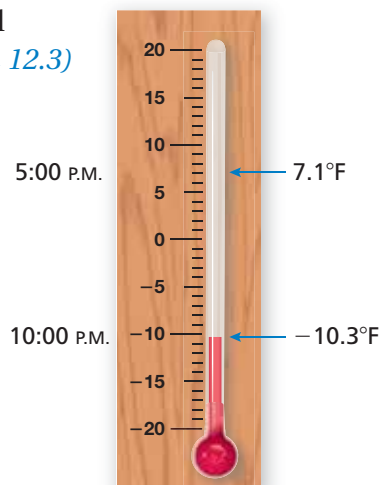
Review Key Vocabulary

rational number, *p. 520*   repeating decimal, *p. 520*
terminating decimal, *p. 520*

Review Examples and Exercises

12.1 **Rational Numbers**   *(pp. 518–523)*

a. Write $4\dfrac{3}{5}$ as a decimal.

Notice that $4\dfrac{3}{5} = \dfrac{23}{5}$.

> Divide 23 by 5. →

$$\begin{array}{r} 4.6 \\ 5\overline{)23.0} \\ -\,20 \\ \hline 3\,0 \\ -\,3\,0 \\ \hline 0 \end{array}$$

> The remainder is 0. So, it is a terminating decimal. → 0

∴ So, $4\dfrac{3}{5} = 4.6$.

b. Write -0.14 as a fraction in simplest form.

> Write the digits after the decimal point in the numerator.

$$-0.14 = -\frac{14}{100}$$

> The last digit is in the hundredths place. So, use 100 in the denominator.

$$= -\frac{7}{50} \qquad \text{Simplify.}$$

Exercises

Write the rational number as a decimal.

1. $-\dfrac{8}{15}$ **2.** $\dfrac{5}{8}$ **3.** $-\dfrac{13}{6}$ **4.** $1\dfrac{7}{16}$

Write the decimal as a fraction or a mixed number in simplest form.

5. -0.6 **6.** -0.35 **7.** -5.8 **8.** 24.23

Find $-\dfrac{7}{2} + \dfrac{5}{4}$.

$$-\frac{7}{2} + \frac{5}{4} = \frac{-14}{4} + \frac{5}{4}$$ Rewrite using the LCD (least common denominator).

$$= \frac{-14 + 5}{4}$$ Write the sum of the numerators over the common denominator.

$$= \frac{-9}{4}$$ Add.

$$= -2\frac{1}{4}$$ Write the improper fraction as a mixed number.

∴ The sum is $-2\dfrac{1}{4}$.

Exercises

Add. Write fractions in simplest form.

9. $\dfrac{9}{10} + \left(-\dfrac{4}{5}\right)$ **10.** $-4\dfrac{5}{9} + \dfrac{8}{9}$ **11.** $-1.6 + (-2.4)$

Find $-4\dfrac{2}{5} - \left(-\dfrac{3}{5}\right)$.

$$-4\frac{2}{5} - \left(-\frac{3}{5}\right) = -4\frac{2}{5} + \frac{3}{5}$$ Add the opposite of $-\dfrac{3}{5}$.

$$= -\frac{22}{5} + \frac{3}{5}$$ Write the mixed number as an improper fraction.

$$= \frac{-22 + 3}{5}$$ Write the sum of the numerators over the common denominator.

$$= \frac{-19}{5}, \text{ or } -3\frac{4}{5}$$ Simplify.

∴ The difference is $-3\dfrac{4}{5}$.

Exercises

Subtract. Write fractions in simplest form.

12. $-\dfrac{5}{12} - \dfrac{3}{10}$ **13.** $3\dfrac{3}{4} - \dfrac{7}{8}$ **14.** $3.8 - (-7.45)$

15. **TURTLE** A turtle is $20\dfrac{5}{6}$ inches below the surface of a pond. It dives to a depth of $32\dfrac{1}{4}$ inches. What is the change in the turtle's position?

Multiplying and Dividing Rational Numbers *(pp. 538–543)*

a. Find $-4\frac{1}{6} \div 1\frac{1}{3}$.

$$-4\frac{1}{6} \div 1\frac{1}{3} = -\frac{25}{6} \div \frac{4}{3}$$ Write mixed numbers as improper fractions.

$$= \frac{-25}{6} \cdot \frac{3}{4}$$ Multiply by the reciprocal of $\frac{4}{3}$.

$$= \frac{-25 \cdot 3}{6 \cdot 4}$$ Multiply the numerators and the denominators.

$$= \frac{-25}{8}, \text{ or } -3\frac{1}{8}$$ Simplify.

∴ The quotient is $-3\frac{1}{8}$.

b. Find $-1.6 \cdot 2.4$.

$$
\begin{array}{r}
-1.6 \\
\times\, 2.4 \\
\hline
64 \\
320 \\
\hline
-3.84
\end{array}
$$

The decimals have different signs.

The product is negative.

∴ The product is -3.84.

Exercises

Divide. Write fractions in simplest form.

16. $\dfrac{9}{10} \div \left(-\dfrac{6}{5}\right)$ **17.** $-\dfrac{4}{11} \div \dfrac{2}{7}$ **18.** $6.4 \div (-3.2)$ **19.** $-15.4 \div (-2.5)$

Multiply. Write fractions in simplest form.

20. $-\dfrac{4}{9}\left(-\dfrac{7}{9}\right)$ **21.** $\dfrac{8}{15}\left(-\dfrac{2}{3}\right)$ **22.** $-5.9(-9.7)$

23. $4.5(-5.26)$ **24.** $-\dfrac{2}{3} \cdot \left(2\dfrac{1}{2}\right) \cdot (-3)$ **25.** $-1.6 \cdot (0.5) \cdot (-20)$

26. SUNKEN SHIP The elevation of a sunken ship is -120 feet. Your elevation is $\dfrac{5}{8}$ of the ship's elevation. What is your elevation?

Write the rational number as a decimal.

1. $\dfrac{7}{40}$

2. $-\dfrac{1}{9}$

3. $-\dfrac{21}{16}$

4. $\dfrac{36}{5}$

Write the decimal as a fraction or a mixed number in simplest form.

5. -0.122

6. 0.33

7. -4.45

8. -7.09

Add or subtract. Write fractions in simplest form.

9. $-\dfrac{4}{9} + \left(-\dfrac{23}{18}\right)$

10. $\dfrac{17}{12} - \left(-\dfrac{1}{8}\right)$

11. $9.2 + (-2.8)$

12. $2.86 - 12.1$

Multiply or divide. Write fractions in simplest form.

13. $3\dfrac{9}{10} \times \left(-\dfrac{8}{3}\right)$

14. $-1\dfrac{5}{6} \div 4\dfrac{1}{6}$

15. $-4.4 \times (-6.02)$

16. $-5 \div 1.5$

17. $-\dfrac{3}{5} \cdot \left(2\dfrac{2}{7}\right) \cdot \left(-3\dfrac{3}{4}\right)$

18. $-6 \cdot (-0.05) \cdot (-0.4)$

19. **ALMONDS** How many 2.25-pound containers can you make with 24.75 pounds of almonds?

20. **FISH** The elevation of a fish is -27 feet.

   a. The fish decreases its elevation by 32 feet, and then increases its elevation by 14 feet. What is its new elevation?

   b. Your elevation is $\dfrac{2}{5}$ of the fish's new elevation. What is your elevation?

21. **RAINFALL** The table shows the rainfall (in inches) for three months compared to the yearly average. Is the total rainfall for the three-month period greater than or less than the yearly average? Explain.

November	December	January
-0.86	2.56	-1.24

22. **BANK ACCOUNTS** Bank Account A has $750.92, and Bank Account B has $675.44. Account A changes by –$216.38, and Account B changes by $-\$168.49$. Which account has the greater balance? Explain.

1. When José and Sean were each 5 years old, José was $1\frac{1}{2}$ inches taller than Sean. José grew at an average rate of $2\frac{3}{4}$ inches per year from the time that he was 5 years old until the time he was 13 years old. José was 63 inches tall when he was 13 years old. How tall was Sean when he was 5 years old? *(7.NS.3)*

   A. $39\frac{1}{2}$ in.　　　　**C.** $44\frac{3}{4}$ in.

   B. $42\frac{1}{2}$ in.　　　　**D.** $47\frac{3}{4}$ in.

One-fourth of the 36 cats in our town are tabbies. How many are not tabbies?

Ⓐ 9　　Ⓑ 18　　Ⓒ 27　　Ⓓ 36

IC.

"Using **estimation** you can see that there are about 10 tabbies. So about 30 are not tabbies."

2. Which expression represents a positive integer? *(7.NS.2a)*

   F. -6^2　　　　**H.** $(-5)^2$

   G. $(-3)^3$　　　　**I.** -2^3

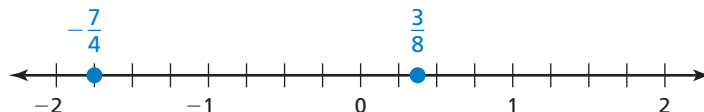3. What is the missing number in the sequence below? *(7.NS.2a)*

   $$\frac{9}{16},\ -\frac{9}{8},\ \frac{9}{4},\ -\frac{9}{2},\ 9,\ \underline{\hspace{1cm}}$$

4. What is the value of the expression below? *(7.NS.1c)*

   $$\left| -2 - (-2.5) \right|$$

   A. -4.5　　　　　　　　　　　　**C.** 0.5

   B. -0.5　　　　　　　　　　　　**D.** 4.5

5. What is the distance between the two numbers on the number line? *(7.NS.1c)*

   $-\dfrac{7}{4}$　　　　　$\dfrac{3}{8}$

   ‹――|――|――|――|――|――|――|――|――|――|――›
   　　-2　　　-1　　　0　　　1　　　2

   F. $-2\frac{1}{8}$　　　　　　　　　　**H.** $1\frac{3}{8}$

   G. $-1\frac{3}{8}$　　　　　　　　　　**I.** $2\frac{1}{8}$

6. Sandra was evaluating an expression in the box below.

$$-4\frac{3}{4} \div 2\frac{1}{5} = -\frac{19}{4} \div \frac{11}{5}$$
$$= \frac{-4}{19} \cdot \frac{5}{11}$$
$$= \frac{-4 \cdot 5}{19 \cdot 11}$$
$$= \frac{-20}{209}$$

What should Sandra do to correct the error that she made?   *(7.NS.3)*

A. Rewrite $-\frac{19}{4}$ as $-\frac{4}{19}$ and multiply by $\frac{11}{5}$.

B. Rewrite $\frac{11}{5}$ as $\frac{5}{11}$ and multiply by $-\frac{19}{4}$.

C. Rewrite $\frac{11}{5}$ as $-\frac{5}{11}$ and multiply by $-\frac{19}{4}$.

D. Rewrite $-4\frac{3}{4}$ as $-\frac{13}{4}$ and multiply by $\frac{5}{11}$.

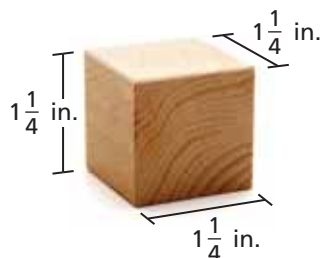**7.** What is the value of the expression below when $q = -2$, $r = -12$, and $s = 8$?   *(7.NS.3)*

$$\frac{-q^2 - r}{s}$$

F. -2 **H.** 1

G. -1 **I.** 2

8. You are stacking wooden blocks with the dimensions shown below. How many blocks do you need to stack to build a block tower that is $7\frac{1}{2}$ inches tall?   *(7.NS.3)*

9. What is the area of a triangle with a base length of $2\frac{1}{2}$ inches and a height of 2 inches? *(7.NS.2c)*

A. $2\frac{1}{4}$ in.^{2}

C. $4\frac{1}{2}$ in.^{2}

B. $2\frac{1}{2}$ in.^{2}

D. 5 in.^{2}

10. What is the value of the expression below? *(7.NS.3)*
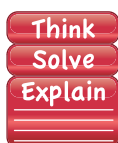$$\frac{-4^2 - (-2)^3}{4}$$

F. -6

H. 2

G. -2

I. 6

11. Four points are graphed on the number line below. *(7.NS.3)*

Part A Choose the two points whose values have the greatest sum. Approximate this sum. Explain your reasoning.

Part B Choose the two points whose values have the greatest difference. Approximate this difference. Explain your reasoning.

Part C Choose the two points whose values have the greatest product. Approximate this product. Explain your reasoning.

Part D Choose the two points whose values have the greatest quotient. Approximate this quotient. Explain your reasoning.

12. What number belongs in the box to make the equation true? *(7.NS.3)*
$$\frac{-0.4}{\boxed{}} + 0.8 = -1.2$$

A. -1

C. 0.2

B. -0.2

D. 1