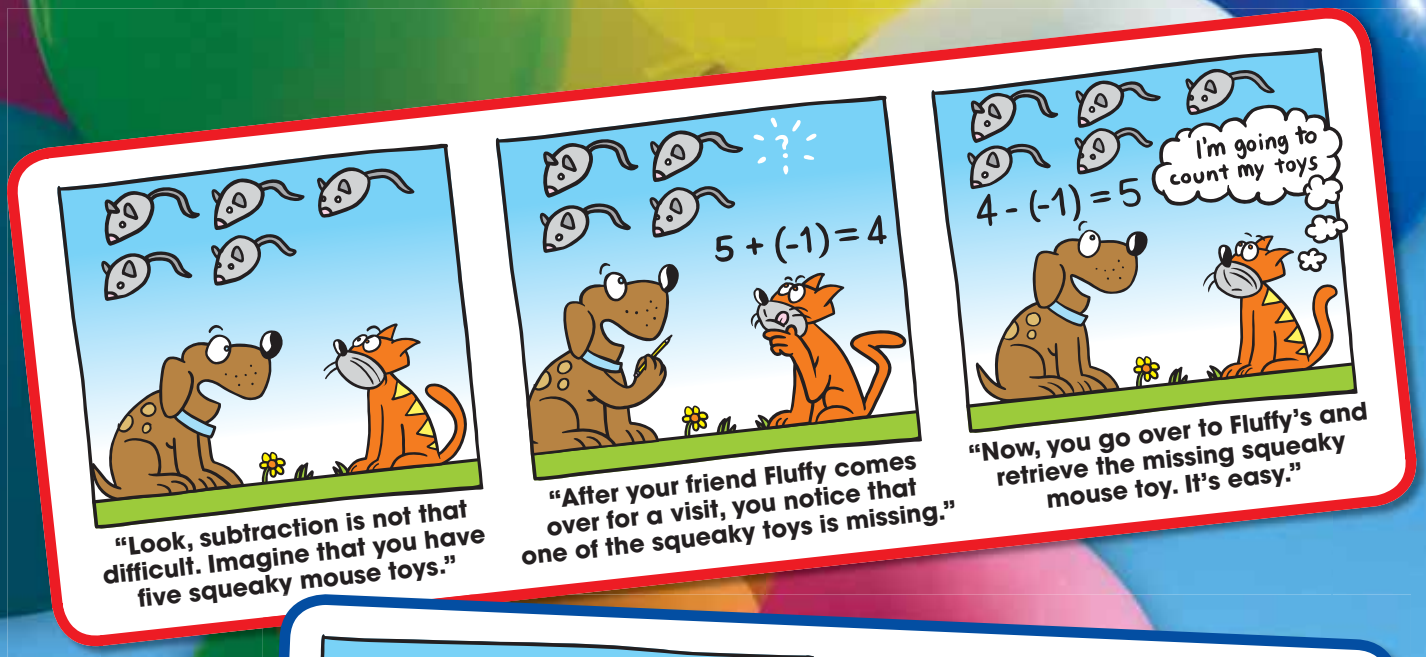
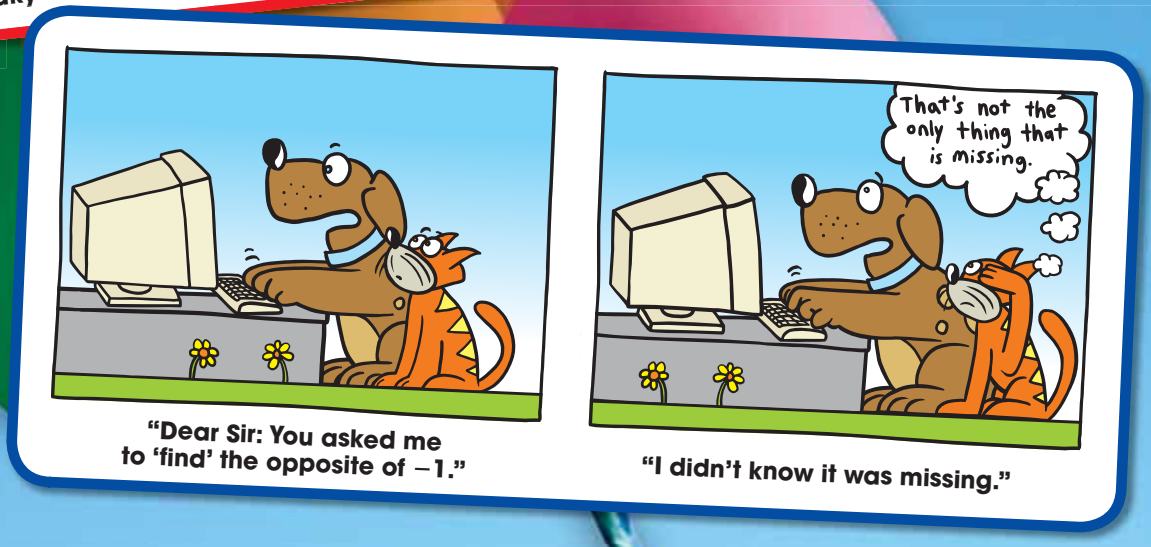
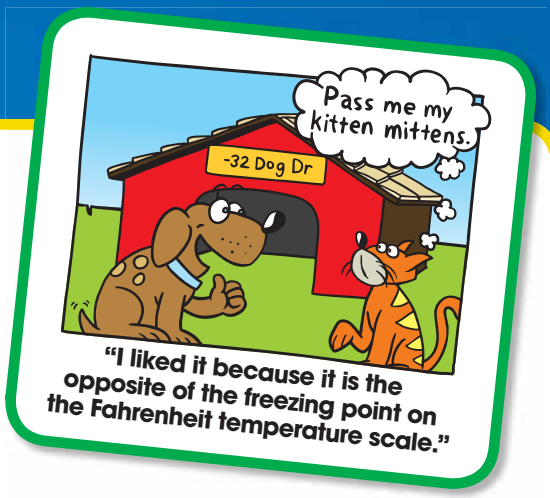
11 Integers





What You Learned Before



Commutative and Associative Properties (6.EE.3)

Example 1 **a. Simplify the expression $6 + (14 + x)$.**

$$6 + (14 + x) = (6 + 14) + x \qquad \text{Associative Property of Addition}$$
$$= 20 + x \qquad \text{Add 6 and 14.}$$

b. Simplify the expression $(3.1 + x) + 7.4$.

$$(3.1 + x) + 7.4 = (x + 3.1) + 7.4 \qquad \text{Commutative Property of Addition}$$
$$= x + (3.1 + 7.4) \qquad \text{Associative Property of Addition}$$
$$= x + 10.5 \qquad \text{Add 3.1 and 7.4.}$$

c. Simplify the expression $5(12y)$.

$$5(12y) = (5 \cdot 12)y \qquad \text{Associative Property of Multiplication}$$
$$= 60y \qquad \text{Multiply 5 and 12.}$$

Try It Yourself

Simplify the expression. Explain each step.

1. $3 + (b + 8)$ **2.** $(d + 4) + 6$ **3.** $6(5p)$

Properties of Zero and One (6.EE.3)

Example 2 **a. Simplify the expression $6 \cdot 0 \cdot q$.**

$$6 \cdot 0 \cdot q = (6 \cdot 0) \cdot q \qquad \text{Associative Property of Multiplication}$$
$$= 0 \cdot q = 0 \qquad \text{Multiplication Property of Zero}$$

b. Simplify the expression $3.6 \cdot s \cdot 1$.

$$3.6 \cdot s \cdot 1 = 3.6 \cdot (s \cdot 1) \qquad \text{Associative Property of Multiplication}$$
$$= 3.6 \cdot s \qquad \text{Multiplication Property of One}$$
$$= 3.6s$$

Try It Yourself

Simplify the expression. Explain each step.

4. $13 \cdot m \cdot 0$ **5.** $1 \cdot x \cdot 29$ **6.** $(n + 14) + 0$

11.1 Integers and Absolute Value

Essential Question How can you use integers to represent the velocity and the speed of an object?

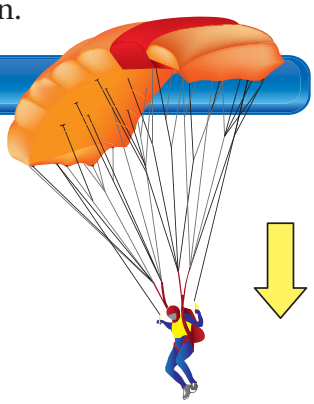
On these two pages, you will investigate vertical motion (up or down).

- Speed tells how fast an object is moving, but it does not tell the direction.
- Velocity tells how fast an object is moving, and it also tells the direction.

  When velocity is positive, the object is moving up.

  When velocity is negative, the object is moving down.

1 ACTIVITY: Falling Parachute



Work with a partner. You are gliding to the ground wearing a parachute. The table shows your height above the ground at different times.

Time (seconds)	0	1	2	3
Height (feet)	90	75	60	45

a. Describe the pattern in the table. How many feet do you move each second? After how many seconds will you land on the ground?

b. What integer represents your speed? Give the units.

c. Do you think your velocity should be represented by a positive or negative integer? Explain your reasoning.

d. What integer represents your velocity? Give the units.

2 ACTIVITY: Rising Balloons

Work with a partner. You release a group of balloons. The table shows the height of the balloons above the ground at different times.

Time (seconds)	0	1	2	3
Height (feet)	8	12	16	20

a. Describe the pattern in the table. How many feet do the balloons move each second? After how many seconds will the balloons be at a height of 40 feet?

b. What integer represents the speed of the balloons? Give the units.

c. Do you think the velocity of the balloons should be represented by a positive or negative integer? Explain your reasoning.

d. What integer represents the velocity of the balloons? Give the units.

Common Core

Integers

In this lesson, you will

- define the absolute value of a number.
- find absolute values of numbers.
- solve real-life problems.

Preparing for Standards
7.NS.1
7.NS.2
7.NS.3

3 ACTIVITY: Firework Parachute

Work with a partner. The table shows the height of a firework's parachute above the ground at different times.

Time (seconds)	Height (feet)
0	480
1	360
2	240
3	120
4	0

Math Practice 6

Use Clear Definitions

What information can you use to support your answer?

a. Describe the pattern in the table. How many feet does the parachute move each second?

b. What integer represents the speed of the parachute? What integer represents the velocity? How are these integers similar in their relation to 0 on a number line?

Inductive Reasoning

4. Copy and complete the table.

Velocity (feet per second)	−14	20	−2	0	25	−15
Speed (feet per second)						

5. Find two different velocities for which the speed is 16 feet per second.

6. Which number is greater: −4 or 3? Use a number line to explain your reasoning.

7. One object has a velocity of −4 feet per second. Another object has a velocity of 3 feet per second. Which object has the greater speed? Explain your answer.

What Is Your Answer?

8. IN YOUR OWN WORDS How can you use integers to represent the velocity and the speed of an object?

9. LOGIC In this lesson, you will study *absolute value*. Here are some examples:

$|-16| = 16$ $|16| = 16$ $|0| = 0$ $|-2| = 2$

Which of the following is a true statement? Explain your reasoning.

$|\text{velocity}| = \text{speed}$ $|\text{speed}| = \text{velocity}$

Practice Use what you learned about absolute value to complete Exercises 4–11 on page 480.

11.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
integer, *p. 478*
absolute value, *p. 478*

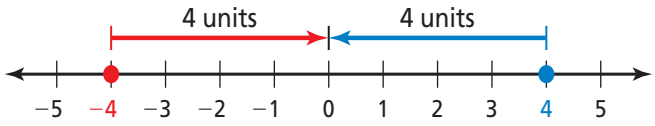
The following numbers are **integers:**

. . . , −3, −2, −1, 0, 1, 2, 3, . . .

Key Idea

Absolute Value

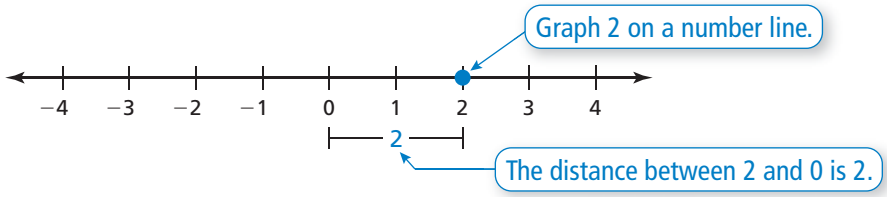
Words The **absolute value** of an integer is the distance between the number and 0 on a number line. The absolute value of a number a is written as $|a|$.



Numbers $|-4| = 4$ $|4| = 4$

EXAMPLE 1 Finding Absolute Value

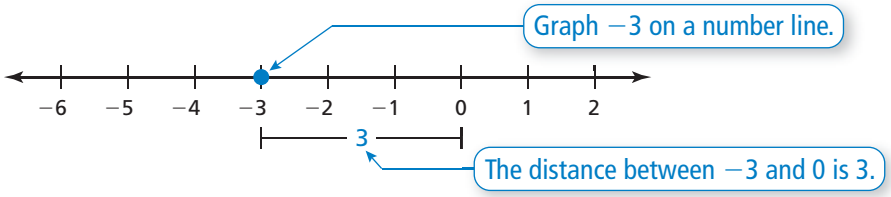
Find the absolute value of 2.



∴ So, $|2| = 2$.

EXAMPLE 2 Finding Absolute Value

Find the absolute value of −3.



∴ So, $|-3| = 3$.

On Your Own

Now You're Ready
Exercises 4–19

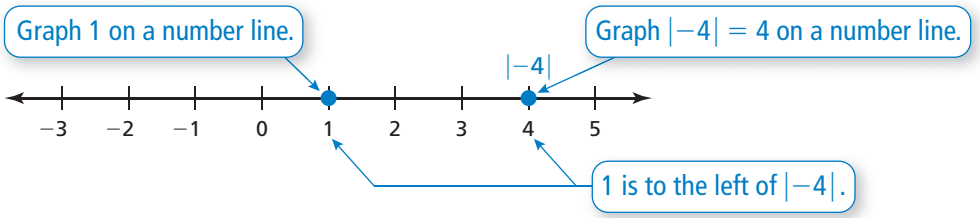
Find the absolute value.

1. $|7|$ **2.** $|-1|$ **3.** $|-5|$ **4.** $|14|$

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 3 Comparing Values

Compare 1 and $|-4|$.



∴ So, $1 < |-4|$.

Remember

A number line can be used to compare and order integers. Numbers to the left are less than numbers to the right. Numbers to the right are greater than numbers to the left.

On Your Own

Now You're Ready
Exercises 20–25

Copy and complete the statement using <, >, or =.

5. $|-2|$ ☐ -1

6. -7 ☐ $|6|$

7. $|10|$ ☐ 11

8. 9 ☐ $|-9|$

EXAMPLE 4 Real-Life Application

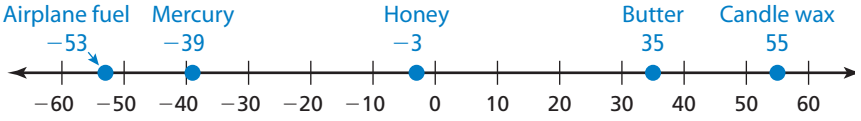
Substance	Freezing Point (°C)
Butter	35
Airplane fuel	−53
Honey	−3
Mercury	−39
Candle wax	55

The *freezing point* is the temperature at which a liquid becomes a solid.

a. Which substance in the table has the lowest freezing point?

b. Is the freezing point of mercury or butter closer to the freezing point of water, 0°C?

a. Graph each freezing point.



∴ Airplane fuel has the lowest freezing point, −53°C.

b. The freezing point of water is 0°C, so you can use absolute values.

Mercury: $|-39| = 39$ **Butter:** $|35| = 35$

∴ Because 35 is less than 39, the freezing point of butter is closer to the freezing point of water.

On Your Own

9. Is the freezing point of airplane fuel or candle wax closer to the freezing point of water? Explain your reasoning.

11.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **VOCABULARY** Which of the following numbers are integers?

$$9,\ 3.2,\ -1,\ \frac{1}{2},\ -0.25,\ 15$$

2. **VOCABULARY** What is the absolute value of an integer?

3. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$|6|$ $\quad$ 6 $\quad$ -6 $\quad$ $|-6|$

Practice and Problem Solving

Find the absolute value.

1 2 **4.** $|9|$ **5.** $|-6|$ **6.** $|-10|$ **7.** $|10|$

8. $|-15|$ **9.** $|13|$ **10.** $|-7|$ **11.** $|-12|$

12. $|5|$ **13.** $|-8|$ **14.** $|0|$ **15.** $|18|$

16. $|-24|$ **17.** $|-45|$ **18.** $|60|$ **19.** $|-125|$

Copy and complete the statement using <, >, or =.

3 **20.** $2 \square |-5|$ **21.** $|-4| \square 7$ **22.** $-5 \square |-9|$

23. $|-4| \square -6$ **24.** $|-1| \square |-8|$ **25.** $|5| \square |-5|$

ERROR ANALYSIS **Describe and correct the error.**

26. ✗ $|10| = -10$

27. ✗ $|-5| < 4$

28. SAVINGS You deposit \$50 in your savings account. One week later, you withdraw \$20. Write each amount as an integer.

29. ELEVATOR You go down 8 floors in an elevator. Your friend goes up 5 floors in an elevator. Write each amount as an integer.

Order the values from least to greatest.

30. $8, |3|, -5, |-2|, -2$

31. $|-6|, -7, 8, |5|, -6$

32. $-12, |-26|, -15, |-12|, |10|$

33. $|-34|, 21, -17, |20|, |-11|$

Simplify the expression.

34. $|-30|$ **35.** $-|4|$ **36.** $-|-15|$

37. PUZZLE Use a number line.

a. Graph and label the following points on a number line: $A = -3$, $E = 2$, $M = -6$, $T = 0$. What word do the letters spell?

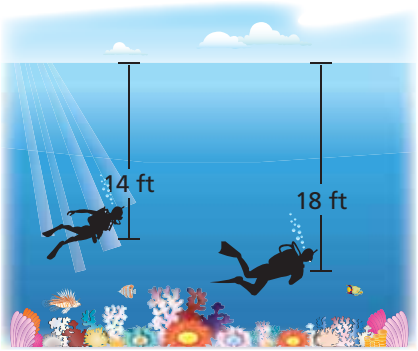
b. Graph and label the absolute value of each point in part (a). What word do the letters spell now?

38. OPEN-ENDED Write a negative integer whose absolute value is greater than 3.

REASONING Determine whether $n \geq 0$ or $n \leq 0$.

39. $n + |-n| = 2n$

40. $n + |-n| = 0$



41. CORAL REEF The depths of two scuba divers exploring a living coral reef are shown.

a. Write an integer for the position of each diver relative to sea level.

b. Which integer in part (a) is greater?

c. Which integer in part (a) has the greater absolute value? Compare this absolute value with the depth of that diver.

42. VOLCANOES The *summit elevation* of a volcano is the elevation of the top of the volcano relative to sea level. The summit elevation of the volcano Kilauea in Hawaii is 1277 meters. The summit elevation of the underwater volcano Loihi in the Pacific Ocean is −969 meters. Which summit is closer to sea level?

43. MINIATURE GOLF The table shows golf scores, relative to *par*.

Player	Score
1	+5
2	0
3	−4
4	−1
5	+2

a. The player with the lowest score wins. Which player wins?

b. Which player is at par?

c. Which player is farthest from par?

True or false? Determine whether the statement is *true* or *false*. Explain your reasoning.

44. If $x < 0$, then $|x| = -x$.

45. The absolute value of every integer is positive.

Fair Game Review What you learned in previous grades & lessons

Add. *(Section 1.1)*

46. 19 + 32 **47.** 50 + 94 **48.** 181 + 217 **49.** 1149 + 2021

50. MULTIPLE CHOICE Which value is *not* a whole number? *(Skills Review Handbook)*

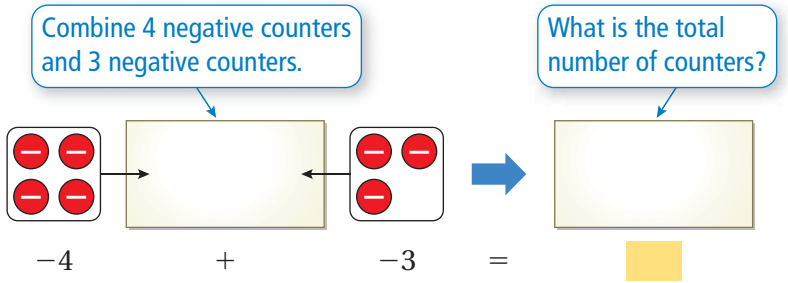
Ⓐ −5 Ⓑ 0 Ⓒ 4 Ⓓ 113

11.2 Adding Integers

Essential Question Is the sum of two integers *positive*, *negative*, or *zero*? How can you tell?

1 ACTIVITY: Adding Integers with the Same Sign

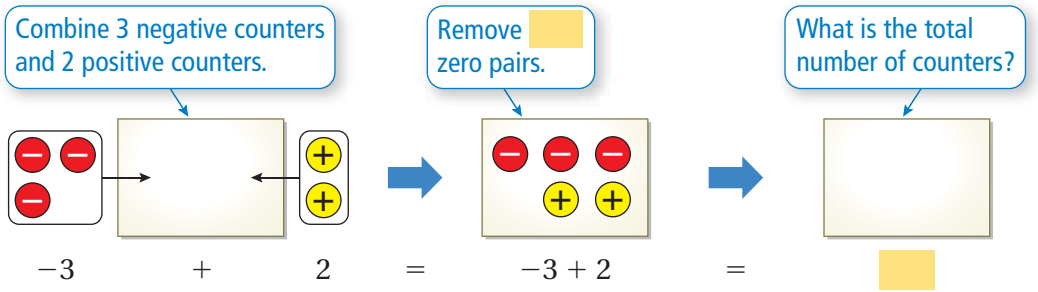
Work with a partner. Use integer counters to find $-4 + (-3)$.



So, $-4 + (-3) =$ ____.

2 ACTIVITY: Adding Integers with Different Signs

Work with a partner. Use integer counters to find $-3 + 2$.



So, $-3 + 2 =$ ____.

Common Core

Integers

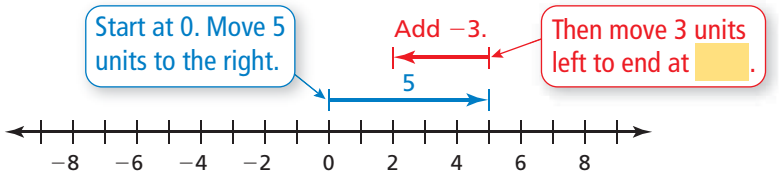
In this lesson, you will

- add integers.
- show that the sum of a number and its opposite is 0.
- solve real-life problems.

Learning Standards
7.NS.1a
7.NS.1b
7.NS.1d
7.NS.3

3 ACTIVITY: Adding Integers with Different Signs

Work with a partner. Use a number line to find $5 + (-3)$.



So, $5 + (-3) =$ ____.

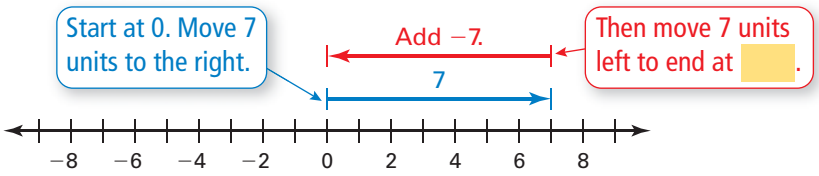
4 ACTIVITY: Adding Integers with Different Signs

Math Practice 3

Make Conjectures

How can the relationship between the integers help you write a rule?

Work with a partner. Write the addition expression shown. Then find the sum. How are the integers in the expression related to 0 on a number line?



Inductive Reasoning

Work with a partner. Use integer counters or a number line to complete the table.

Exercise	Type of Sum	Sum	Sum: Positive, Negative, or Zero
5. $-4 + (-3)$	Integers with the same sign		
6. $-3 + 2$			
7. $5 + (-3)$			
8. $7 + (-7)$			
9. $2 + 4$			
10. $-6 + (-2)$			
11. $-5 + 9$			
12. $15 + (-9)$			
13. $-10 + 10$			
14. $-6 + (-6)$			
15. $13 + (-13)$			

What Is Your Answer?

16. IN YOUR OWN WORDS Is the sum of two integers *positive*, *negative*, or *zero*? How can you tell?

17. STRUCTURE Write general rules for adding (a) two integers with the same sign, (b) two integers with different signs, and (c) two integers that vary only in sign.

Practice Use what you learned about adding integers to complete Exercises 8–15 on page 486.

11.2 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
opposites, *p. 484*
additive inverse, *p. 484*

Key Idea

Adding Integers with the Same Sign

Words Add the absolute values of the integers. Then use the common sign.

Numbers $2 + 5 = 7$ $-2 + (-5) = -7$

EXAMPLE 1 Adding Integers with the Same Sign

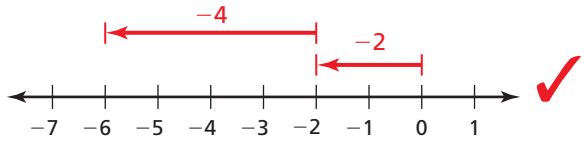
Find $-2 + (-4)$. Use a number line to check your answer.

$-2 + (-4) = -6$ Add $|-2|$ and $|-4|$.

Use the common sign.

∴ The sum is -6.

Check



✓

On Your Own

Add.

1. $7 + 13$ **2.** $-8 + (-5)$ **3.** $-20 + (-15)$

The Meaning of a Word

Opposite

When you walk across a street, you are moving to the **opposite** side of the street.

Two numbers that are the same distance from 0, but on opposite sides of 0, are called **opposites.** For example, -3 and 3 are opposites.

Key Ideas

Adding Integers with Different Signs

Words Subtract the lesser absolute value from the greater absolute value. Then use the sign of the integer with the greater absolute value.

Numbers $8 + (-10) = -2$ $-13 + 17 = 4$

Additive Inverse Property

Words The sum of an integer and its **additive inverse,** or opposite, is 0.

Numbers $6 + (-6) = 0$ $-25 + 25 = 0$ **Algebra** $a + (-a) = 0$

Multi-Language Glossary at BigIdeasMath.com

EXAMPLE 2 Adding Integers with Different Signs

a. Find $5 + (-10)$.

$5 + (-10) = -5$ — $|-10| > |5|$. So, subtract $|5|$ from $|-10|$.

Use the sign of -10.

∴ The sum is -5.

b. Find $-3 + 7$.

$-3 + 7 = 4$ — $|7| > |-3|$. So, subtract $|-3|$ from $|7|$.

Use the sign of 7.

∴ The sum is 4.

c. Find $-12 + 12$.

$-12 + 12 = 0$ — The sum is 0 by the Additive Inverse Property.

-12 and 12 are opposites.

∴ The sum is 0.

EXAMPLE 3 Adding More Than Two Integers

The list shows four bank account transactions in July. Find the change C in the account balance.

JULY TRANSACTIONS	
Withdrawal	-$40
Deposit	$50
Deposit	$75
Withdrawal	-$50

Study Tip

A deposit of \$50 and a withdrawal of \$50 represent opposite quantities, $+50$ and -50, which have a sum of 0.

Find the sum of the four transactions.

$C = -40 + 50 + 75 + (-50)$ — Write the sum.

$= -40 + 75 + 50 + (-50)$ — Commutative Property of Addition

$= -40 + 75 + [50 + (-50)]$ — Associative Property of Addition

$= -40 + 75 + 0$ — Additive Inverse Property

$= 35 + 0$ — Add -40 and 75.

$= 35$ — Addition Property of Zero

∴ Because $C = 35$, the account balance increased \$35 in July.

On Your Own

Add.

4. $-2 + 11$ **5.** $9 + (-10)$ **6.** $-31 + 31$

7. WHAT IF? In Example 3, the deposit amounts are \$30 and \$40. Find the change C in the account balance.

Now You're Ready
Exercises 8–23 and 28–39

11.2 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** How do you find the additive inverse of an integer?
2. **NUMBER SENSE** Is $3 + (-4)$ the same as $-4 + 3$? Explain.

Tell whether the sum is *positive*, *negative*, or *zero* without adding. Explain your reasoning.

3. $-8 + 20$
4. $30 + (-30)$
5. $-10 + (-18)$

Tell whether the statement is *true* or *false*. Explain your reasoning.

6. The sum of two negative integers is always negative.
7. An integer and its absolute value are always opposites.

Practice and Problem Solving

Add.

1 2 8. $6 + 4$
9. $-4 + (-6)$
10. $-2 + (-3)$
11. $-5 + 12$
12. $5 + (-7)$
13. $8 + (-8)$
14. $9 + (-11)$
15. $-3 + 13$
16. $-4 + (-16)$
17. $-3 + (-1)$
18. $14 + (-5)$
19. $0 + (-11)$
20. $-10 + (-15)$
21. $-13 + 9$
22. $18 + (-18)$
23. $-25 + (-9)$

ERROR ANALYSIS **Describe and correct the error in finding the sum.**

24. ✗ $9 + (-6) = -3$

25. ✗ $-10 + (-10) = 0$

26. **TEMPERATURE** The temperature is $-3°$F at 7:00 A.M. During the next 4 hours, the temperature increases $21°$F. What is the temperature at 11:00 A.M.?
27. **BANKING** Your bank account has a balance of $-\$12$. You deposit \$60. What is your new balance?

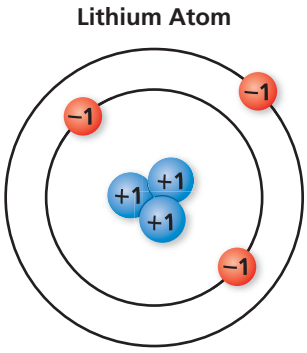
Tell how the Commutative and Associative Properties of Addition can help you find the sum mentally. Then find the sum.

3 28. $9 + 6 + (-6)$
29. $-8 + 13 + (-13)$
30. $9 + (-17) + (-9)$
31. $7 + (-12) + (-7)$
32. $-12 + 25 + (-15)$
33. $6 + (-9) + 14$

Add.

34. $13 + (-21) + 16$
35. $22 + (-14) + (-35)$
36. $-13 + 27 + (-18)$
37. $-19 + 26 + 14$
38. $-32 + (-17) + 42$
39. $-41 + (-15) + (-29)$

40. **SCIENCE** A lithium atom has positively charged protons and negatively charged electrons. The sum of the charges represents the charge of the lithium atom. Find the charge of the atom.



41. **OPEN-ENDED** Write two integers with different signs that have a sum of −25. Write two integers with the same sign that have a sum of −25.

ALGEBRA **Evaluate the expression when $a = 4$, $b = -5$, and $c = -8$.**

42. $a + b$ **43.** $-b + c$ **44.** $|a + b + c|$

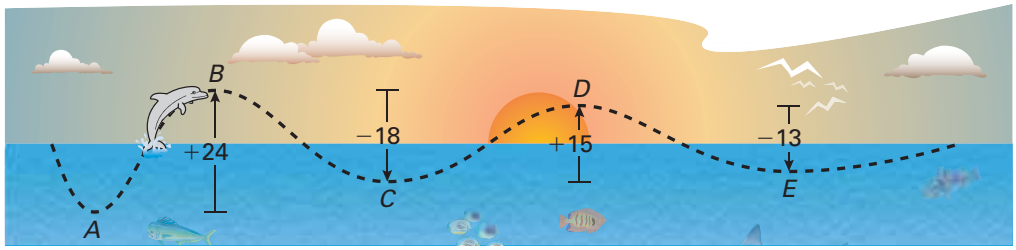
MENTAL MATH **Use mental math to solve the equation.**

45. $d + 12 = 2$ **46.** $b + (-2) = 0$ **47.** $-8 + m = -15$

48. **PROBLEM SOLVING** Starting at point A, the path of a dolphin jumping out of the water is shown.

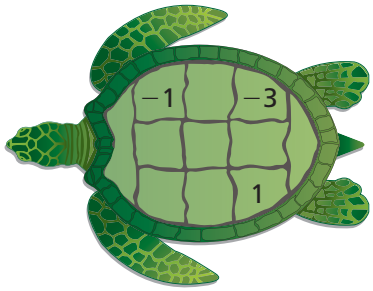
a. Is the dolphin deeper at point C or point E? Explain your reasoning.

b. Is the dolphin higher at point B or point D? Explain your reasoning.



49. **Puzzle** According to a legend, the Chinese Emperor Yu-Huang saw a magic square on the back of a turtle. In a *magic square*, the numbers in each row and in each column have the same sum. This sum is called the *magic sum*.

Copy and complete the magic square so that each row and each column has a magic sum of 0. Use each integer from −4 to 4 exactly once.



Fair Game Review What you learned in previous grades & lessons

Subtract. *(Section 1.1)*

50. 69 − 38 **51.** 82 − 74 **52.** 177 − 63 **53.** 451 − 268

54. **MULTIPLE CHOICE** What is the range of the numbers below? *(Section 9.4)*

12, 8, 17, 12, 15, 18, 30

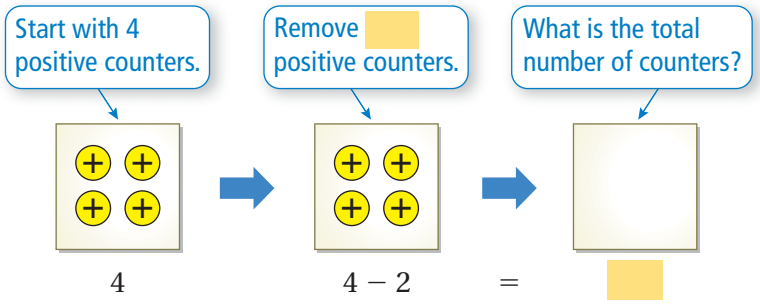
(A) 12 (B) 15 (C) 18 (D) 22

11.3 Subtracting Integers

Essential Question How are adding integers and subtracting integers related?

1 ACTIVITY: Subtracting Integers

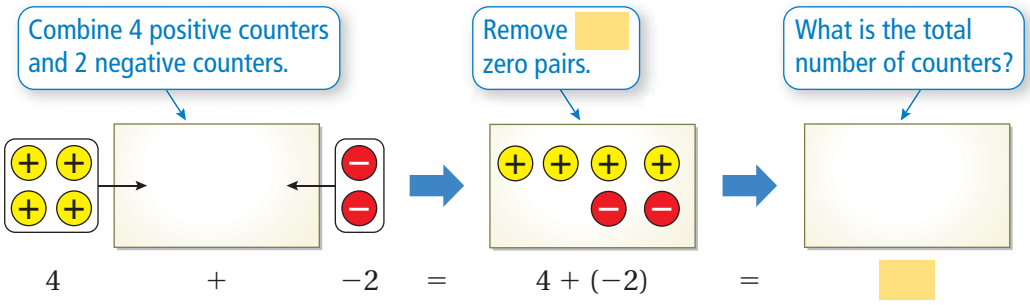
Work with a partner. Use integer counters to find 4 − 2.



So, 4 − 2 = ____.

2 ACTIVITY: Adding Integers

Work with a partner. Use integer counters to find 4 + (−2).



So, 4 + (−2) = ____.

Common Core

Integers

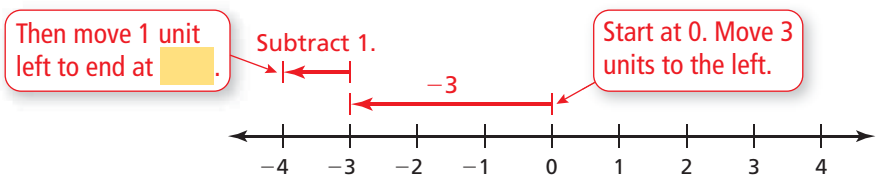
In this lesson, you will

- subtract integers.
- solve real-life problems.

Learning Standards
7.NS.1c
7.NS.1d
7.NS.3

3 ACTIVITY: Subtracting Integers

Work with a partner. Use a number line to find −3 − 1.



So, −3 − 1 = ____.

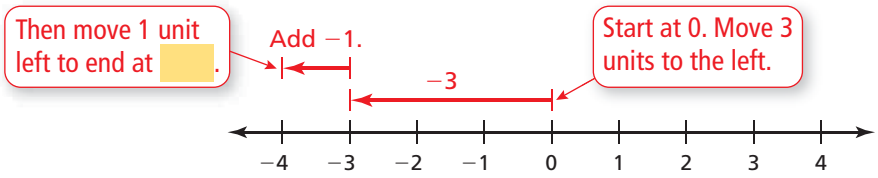
4 ACTIVITY: Adding Integers

Math Practice 2

Make Sense of Quantities

What integers will you use in your addition expression?

Work with a partner. Write the addition expression shown. Then find the sum.



Inductive Reasoning

Work with a partner. Use integer counters or a number line to complete the table.

	Exercise	Operation: Add or Subtract	Answer
1	**5.** $4 - 2$	Subtract 2	
2	**6.** $4 + (-2)$		
3	**7.** $-3 - 1$		
4	**8.** $-3 + (-1)$		
	9. $3 - 8$		
	10. $3 + (-8)$		
	11. $9 - 13$		
	12. $9 + (-13)$		
	13. $-6 - (-3)$		
	14. $-6 + 3$		
	15. $-5 - (-12)$		
	16. $-5 + 12$		

What Is Your Answer?

17. **IN YOUR OWN WORDS** How are adding integers and subtracting integers related?

18. **STRUCTURE** Write a general rule for subtracting integers.

19. Use a number line to find the value of the expression $-4 + 4 - 9$. What property can you use to make your calculation easier? Explain.

Practice Use what you learned about subtracting integers to complete Exercises 8–15 on page 492.

11.3 Lesson

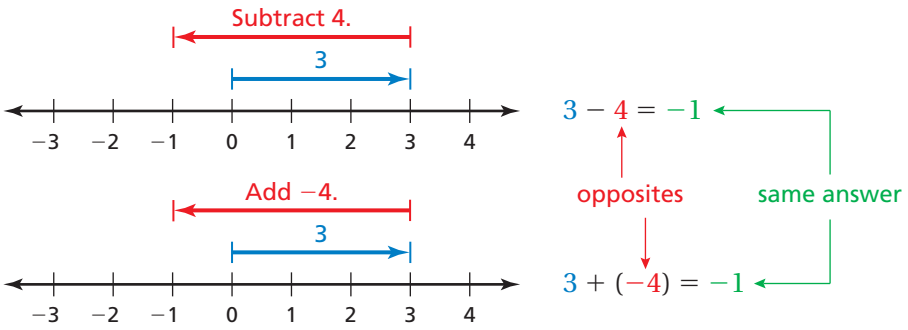
Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Idea

Subtracting Integers

Words To subtract an integer, add its opposite.

Numbers $3 - 4 = 3 + (-4) = -1$



EXAMPLE 1 Subtracting Integers

a. Find 3 − 12.

$3 - 12 = 3 + (-12)$ Add the opposite of 12.

$= -9$ Add.

∴ The difference is −9.

b. Find −8 − (−13).

$-8 - (-13) = -8 + 13$ Add the opposite of −13.

$= 5$ Add.

∴ The difference is 5.

c. Find 5 − (−4).

$5 - (-4) = 5 + 4$ Add the opposite of −4.

$= 9$ Add.

∴ The difference is 9.

On Your Own

Now You're Ready
Exercises 8–23

Subtract.

1. $8 - 3$
2. $9 - 17$
3. $-3 - 3$
4. $-14 - 9$
5. $9 - (-8)$
6. $-12 - (-12)$

EXAMPLE 2 Subtracting Integers

Evaluate $-7 - (-12) - 14$.

$$\begin{aligned} -7 - (-12) - 14 &= -7 + 12 - 14 && \text{Add the opposite of } -12. \\ &= 5 - 14 && \text{Add } -7 \text{ and } 12. \\ &= 5 + (-14) && \text{Add the opposite of } 14. \\ &= -9 && \text{Add.} \end{aligned}$$

∴ So, $-7 - (-12) - 14 = -9$.

On Your Own

Now You're Ready
Exercises 27–32

Evaluate the expression.

7. $-9 - 16 - 8$

8. $-4 - 20 - 9$

9. $0 - 9 - (-5)$

10. $-8 - (-6) - 0$

11. $15 - (-20) - 20$

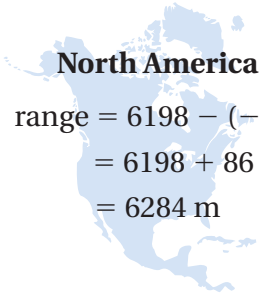
12. $-14 - 9 - 36$

EXAMPLE 3 Real-Life Application

Which continent has the greater range of elevations?

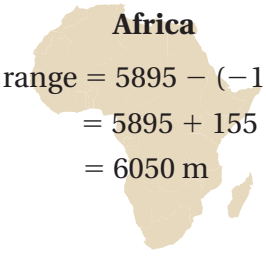
	North America	Africa
Highest Elevation	6198 m	5895 m
Lowest Elevation	−86 m	−155 m

To find the range of elevations for each continent, subtract the lowest elevation from the highest elevation.



North America

$$\begin{aligned} \text{range} &= 6198 - (-86) \\ &= 6198 + 86 \\ &= 6284 \text{ m} \end{aligned}$$



Africa

$$\begin{aligned} \text{range} &= 5895 - (-155) \\ &= 5895 + 155 \\ &= 6050 \text{ m} \end{aligned}$$

∴ Because 6284 is greater than 6050, North America has the greater range of elevations.

On Your Own

13. The highest elevation in Mexico is 5700 meters, on Pico de Orizaba. The lowest elevation in Mexico is −10 meters, in Laguna Salada. Find the range of elevations in Mexico.

11.3 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** How do you subtract one integer from another?
2. **OPEN-ENDED** Write two integers that are opposites.
3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Find the difference of 3 and −2.

What is 3 less than −2?

How much less is −2 than 3?

Subtract −2 from 3.

MATCHING **Match the subtraction expression with the corresponding addition expression.**

4. $9 - (-5)$
5. $-9 - 5$
6. $-9 - (-5)$
7. $9 - 5$

A. $-9 + 5$
B. $9 + (-5)$
C. $-9 + (-5)$
D. $9 + 5$

Practice and Problem Solving

Subtract.

1 **8.** $4 - 7$
9. $8 - (-5)$
10. $-6 - (-7)$
11. $-2 - 3$
12. $5 - 8$
13. $-4 - 6$
14. $-8 - (-3)$
15. $10 - 7$
16. $-8 - 13$
17. $15 - (-2)$
18. $-9 - (-13)$
19. $-7 - (-8)$
20. $-6 - (-6)$
21. $-10 - 12$
22. $32 - (-6)$
23. $0 - 20$

24. **ERROR ANALYSIS** Describe and correct the error in finding the difference $7 - (-12)$.

✗ $7 - (-12) = 7 + (-12) = -5$

25. **SWIMMING POOL** The floor of the shallow end of a swimming pool is at −3 feet. The floor of the deep end is 9 feet deeper. Which expression can be used to find the depth of the deep end?

$-3 + 9$
$-3 - 9$
$9 - 3$

26. **SHARKS** A shark is at −80 feet. It swims up and jumps out of the water to a height of 15 feet. Write a subtraction expression for the vertical distance the shark travels.

Evaluate the expression.

2 **27.** $-2 - 7 + 15$
28. $-9 + 6 - (-2)$
29. $12 - (-5) - 8$
30. $-87 - 5 - 13$
31. $-6 - (-8) + 6$
32. $-15 - 7 - (-11)$

MENTAL MATH **Use mental math to solve the equation.**

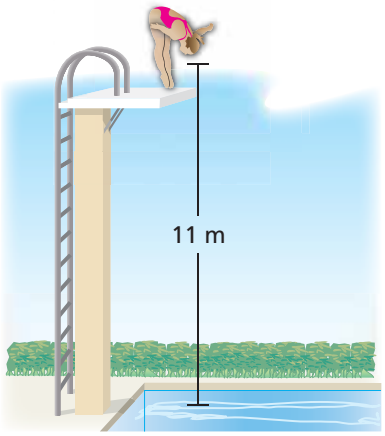
33. $m - 5 = 9$ **34.** $w - (-3) = 7$ **35.** $6 - c = -9$

ALGEBRA **Evaluate the expression when $k = -3$, $m = -6$, and $n = 9$.**

36. $4 - n$ **37.** $m - (-8)$

38. $-5 + k - n$ **39.** $|m - k|$

40. **PLATFORM DIVING** The figure shows a diver diving from a platform. The diver reaches a depth of 4 meters. What is the change in elevation of the diver?



41. **OPEN-ENDED** Write two different pairs of negative integers, x and y, that make the statement $x - y = -1$ true.

42. **TEMPERATURE** The table shows the record monthly high and low temperatures for a city in Alaska.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
High (°F)	56	57	56	72	82	92	84	85	73	64	62	53
Low (°F)	−35	−38	−24	−15	1	29	34	31	19	−6	−21	−36

a. Find the range of temperatures for each month.

b. What are the all-time high and all-time low temperatures?

c. What is the range of the temperatures in part (b)?

REASONING **Tell whether the difference between the two integers is *always*, *sometimes*, or *never* positive. Explain your reasoning.**

43. two positive integers **44.** two negative integers

45. a positive integer and a negative integer **46.** a negative integer and a positive integer

Number Sense **For what values of a and b is the statement true?**

47. $|a - b| = |b - a|$ **48.** $|a + b| = |a| + |b|$ **49.** $|a - b| = |a| - |b|$

Fair Game Review What you learned in previous grades & lessons

Add. *(Section 11.2)*

50. $-5 + (-5) + (-5) + (-5)$ **51.** $-9 + (-9) + (-9) + (-9) + (-9)$

Multiply. *(Section 1.1)*

52. 8×5 **53.** 6×78 **54.** 36×41 **55.** 82×29

56. **MULTIPLE CHOICE** Which value of n makes the value of the expression $4n + 3$ a composite number? *(Skills Review Handbook)*

(A) 1 (B) 2 (C) 3 (D) 4

11 Study Help

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use an **idea and examples chart** to organize information about a concept. Here is an example of an idea and examples chart for absolute value.

Absolute Value: the distance between a number and 0 on the number line

Example

$|3| = 3$

Example

$|-5| = 5$

Example

$|0| = 0$

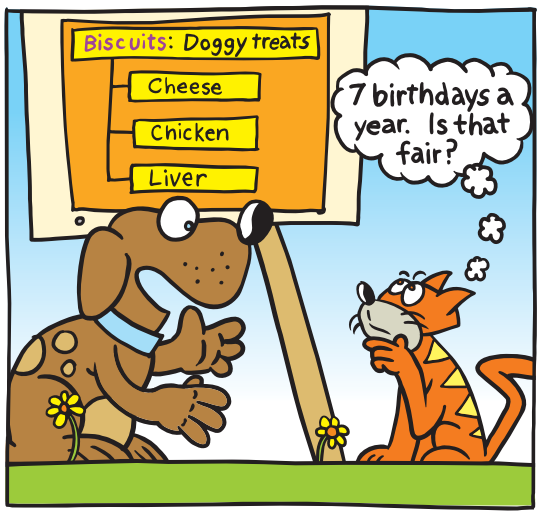
On Your Own

Make idea and examples charts to help you study these topics.

1. integers
2. adding integers
   - **a.** with the same sign
   - **b.** with different signs
3. Additive Inverse Property
4. subtracting integers

After you complete this chapter, make idea and examples charts for the following topics.

5. multiplying integers
   - **a.** with the same sign
   - **b.** with different signs
6. dividing integers
   - **a.** with the same sign
   - **b.** with different signs



"I made an idea and examples chart to give my owner ideas for my birthday next week."

11.1–11.3 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Copy and complete the statement using <, >, or =. *(Section 11.1)*

1. $|-8|$ ☐ 3

2. 7 ☐ $|-7|$

Order the values from least to greatest. *(Section 11.1)*

3. $-4, |-5|, |-4|, 3, -6$

4. $12, -8, |-15|, -10, |-9|$

Evaluate the expression. *(Section 11.2 and Section 11.3)*

5. $-3 + (-8)$

6. $-4 + 16$

7. $3 - 9$

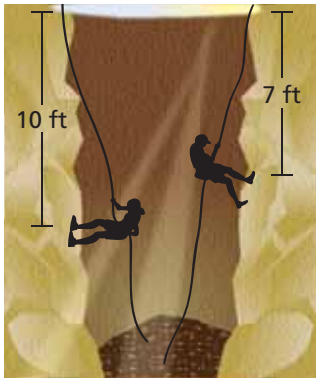
8. $-5 - (-5)$

Evaluate the expression when $a = -2$, $b = -8$, and $c = 5$. *(Section 11.2 and Section 11.3)*

9. $4 - a - c$

10. $|b - c|$

11. **EXPLORING** Two climbers explore a cave. *(Section 11.1)*



a. Write an integer for the position of each climber relative to the surface.

b. Which integer in part (a) is greater?

c. Which integer in part (a) has the greater absolute value?

12. **SCHOOL CARNIVAL** The table shows the income and expenses for a school carnival. The school's goal was to raise $1100. Did the school reach its goal? Explain. *(Section 11.2)*

Games	Concessions	Donations	Flyers	Decorations
$650	$530	$52	−$28	−$75

13. **TEMPERATURE** Temperatures in the Gobi Desert reach −40°F in the winter and 90°F in the summer. Find the range of the temperatures. *(Section 11.3)*

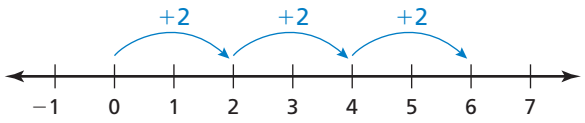
11.4 Multiplying Integers

Essential Question Is the product of two integers *positive*, *negative*, or *zero*? How can you tell?

1 ACTIVITY: Multiplying Integers with the Same Sign

Work with a partner. Use repeated addition to find 3 • 2.

Recall that multiplication is repeated addition. 3 • 2 means to add 3 groups of 2.



Now you can write

$3 \cdot 2 = \square + \square + \square$

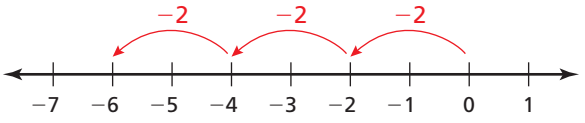
$= \square$.

So, $3 \cdot 2 = \square$.

2 ACTIVITY: Multiplying Integers with Different Signs

Work with a partner. Use repeated addition to find 3 • (−2).

3 • (−2) means to add 3 groups of −2.



Now you can write

$3 \cdot (-2) = \square + \square + \square$

$= \square$.

So, $3 \cdot (-2) = \square$.

3 ACTIVITY: Multiplying Integers with Different Signs

Work with a partner. Use a table to find −3 • 2.

Describe the pattern of the products in the table. Then complete the table.

2	•	2	=	4
1	•	2	=	2
0	•	2	=	0
−1	•	2	=	
−2	•	2	=	
−3	•	2	=	

So, $-3 \cdot 2 = \square$.

Common Core

Integers

In this lesson, you will

- multiply integers.
- solve real-life problems.

Learning Standards
7.NS.2a
7.NS.2c
7.NS.3

4 ACTIVITY: Multiplying Integers with the Same Sign

Work with a partner. Use a table to find $-3 \cdot (-2)$.

Math Practice 7

Look for Patterns

How can you use the pattern to complete the table?

Describe the pattern of the products in the table. Then complete the table.

-3	$\cdot$	3	$=$	-9
-3	$\cdot$	2	$=$	-6
-3	$\cdot$	1	$=$	-3
-3	$\cdot$	0	$=$	
-3	$\cdot$	-1	$=$	
-3	$\cdot$	-2	$=$	

So, $-3 \cdot (-2) =$ ____.

Inductive Reasoning

Work with a partner. Complete the table.

Exercise	Type of Product	Product	Product: Positive or Negative
5. $3 \cdot 2$	Integers with the same sign		
6. $3 \cdot (-2)$			
7. $-3 \cdot 2$			
8. $-3 \cdot (-2)$			
9. $6 \cdot 3$			
10. $2 \cdot (-5)$			
11. $-6 \cdot 5$			
12. $-5 \cdot (-3)$			

What Is Your Answer?

13. Write two integers whose product is 0.

14. **IN YOUR OWN WORDS** Is the product of two integers *positive*, *negative*, or *zero*? How can you tell?

15. **STRUCTURE** Write general rules for multiplying (a) two integers with the same sign and (b) two integers with different signs.

Practice Use what you learned about multiplying integers to complete Exercises 8–15 on page 500.

11.4 Lesson

Check It Out
Lesson Tutorials
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Key Ideas

Multiplying Integers with the Same Sign

Words The product of two integers with the same sign is positive.

Numbers $2 \cdot 3 = 6$ $-2 \cdot (-3) = 6$

Multiplying Integers with Different Signs

Words The product of two integers with different signs is negative.

Numbers $2 \cdot (-3) = -6$ $-2 \cdot 3 = -6$

EXAMPLE 1 Multiplying Integers with the Same Sign

Find $-5 \cdot (-6)$.

The integers have the same sign.

$-5 \cdot (-6) = 30$

The product is positive.

∴ The product is 30.

EXAMPLE 2 Multiplying Integers with Different Signs

Multiply.

a. $3(-4)$

b. $-7 \cdot 4$

The integers have different signs.

$3(-4) = -12$ $-7 \cdot 4 = -28$

The product is negative.

∴ The product is -12.

∴ The product is -28.

On Your Own

Now You're Ready
Exercises 8–23

Multiply.

1. $5 \cdot 5$
2. $4(11)$
3. $-1(-9)$
4. $-7 \cdot (-8)$
5. $12 \cdot (-2)$
6. $4(-6)$
7. $-10(-6)(0)$
8. $-7 \cdot (-5) \cdot (-4)$

EXAMPLE 3 Using Exponents

Study Tip

Place parentheses around a negative number to raise it to a power.

a. Evaluate $(-2)^2$.

$(-2)^2 = (-2) \cdot (-2)$ Write $(-2)^2$ as repeated multiplication.

$= 4$ Multiply.

b. Evaluate -5^2.

$-5^2 = -(5 \cdot 5)$ Write 5^2 as repeated multiplication.

$= -25$ Multiply.

c. Evaluate $(-4)^3$.

$(-4)^3 = (-4) \cdot (-4) \cdot (-4)$ Write $(-4)^3$ as repeated multiplication.

$= 16 \cdot (-4)$ Multiply.

$= -64$ Multiply.

On Your Own

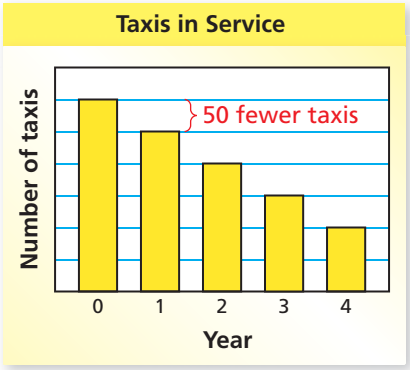
Now You're Ready
Exercises 32–37

Evaluate the expression.

9. $(-3)^2$ **10.** $(-2)^3$ **11.** -7^2 **12.** -6^3

EXAMPLE 4 Real-Life Application

The bar graph shows the number of taxis a company has in service. The number of taxis decreases by the same amount each year for 4 years. Find the total change in the number of taxis.



The bar graph shows that the number of taxis in service decreases by 50 each year. Use a model to solve the problem.

total change = change per year • number of years

$= -50 \cdot 4$

$= -200$

Use -50 for the change per year because the number *decreases* each year.

∴ The total change in the number of taxis is -200.

On Your Own

13. A manatee population decreases by 15 manatees each year for 3 years. Find the total change in the manatee population.

11.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** What can you conclude about the signs of two integers whose product is (a) positive and (b) negative?
2. **OPEN-ENDED** Write two integers whose product is negative.

Tell whether the product is *positive* or *negative* without multiplying. Explain your reasoning.

3. $4(-8)$
4. $-5(-7)$
5. $-3 \cdot 12$

Tell whether the statement is *true* or *false*. Explain your reasoning.

6. The product of three positive integers is positive.
7. The product of three negative integers is positive.

Practice and Problem Solving

Multiply.

8. $6 \cdot 4$
9. $7(-3)$
10. $-2(8)$
11. $-3(-4)$
12. $-6 \cdot 7$
13. $3 \cdot 9$
14. $8 \cdot (-5)$
15. $-1 \cdot (-12)$
16. $-5(10)$
17. $-13(0)$
18. $-9 \cdot 9$
19. $15(-2)$
20. $-10 \cdot 11$
21. $-6 \cdot (-13)$
22. $7(-14)$
23. $-11 \cdot (-11)$

24. **JOGGING** You burn 10 calories each minute you jog. What integer represents the change in your calories after you jog for 20 minutes?
25. **WETLANDS** About 60,000 acres of wetlands are lost each year in the United States. What integer represents the change in wetlands after 4 years?

Multiply.

26. $3 \cdot (-8) \cdot (-2)$
27. $6(-9)(-1)$
28. $-3(-5)(-4)$
29. $(-5)(-7)(-20)$
30. $-6 \cdot 3 \cdot (-2)$
31. $3 \cdot (-12) \cdot 0$

Evaluate the expression.

32. $(-4)^2$
33. $(-1)^3$
34. -8^2
35. -6^2
36. $-5^2 \cdot 4$
37. $-2 \cdot (-3)^3$

ERROR ANALYSIS **Describe and correct the error in evaluating the expression.**

38. ✗ $-2(-7) = -14$

39. ✗ $-10^2 = 100$

ALGEBRA **Evaluate the expression when $a = -2$, $b = 3$, and $c = -8$.**

40. ab **41.** $|a^2c|$ **42.** $-ab^3 - ac$

NUMBER SENSE **Find the next two numbers in the pattern.**

43. $-12, 60, -300, 1500, \ldots$ **44.** $7, -28, 112, -448, \ldots$

45. **GYM CLASS** You lose four points each time you attend gym class without sneakers. You forget your sneakers three times. What integer represents the change in your points?

46. **MODELING** The height of an airplane during a landing is given by $22{,}000 + (-480t)$, where t is the time in minutes.

a. Copy and complete the table.

b. Estimate how many minutes it takes the plane to land. Explain your reasoning.

Time (minutes)	5	10	15	20
Height (feet)				

47. **INLINE SKATES** In June, the price of a pair of inline skates is \$165. The price changes each of the next 3 months.

a. Copy and complete the table.

Month	Price of Skates
June	165 $= \$165$
July	$165 + (-12) = \$\_\_\_\_$
August	$165 + 2(-12) = \$\_\_\_\_$
September	$165 + 3(-12) = \$\_\_\_\_$

b. Describe the change in the price of the inline skates for each month.

c. The table at the right shows the amount of money you save each month to buy the inline skates. Do you have enough money saved to buy the inline skates in August? September? Explain your reasoning.

Amount Saved	
June	\$35
July	\$55
August	\$45
September	\$18

48. Reasoning Two integers, a and b, have a product of 24. What is the least possible sum of a and b?

Fair Game Review What you learned in previous grades & lessons

Divide. *(Section 1.1)*

49. $27 \div 9$ **50.** $48 \div 6$ **51.** $56 \div 4$ **52.** $153 \div 9$

53. **MULTIPLE CHOICE** What is the prime factorization of 84? *(Section 1.4)*

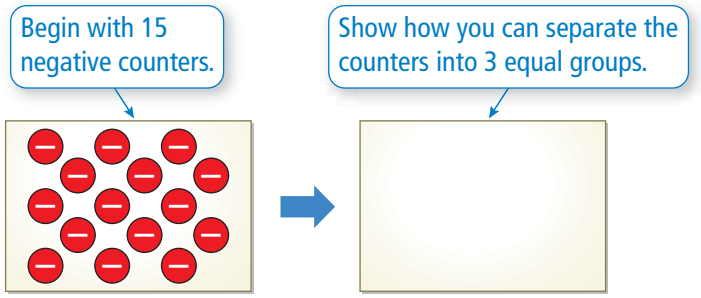
(A) $2^2 \times 3^2$ (B) $2^3 \times 7$ (C) $3^3 \times 7$ (D) $2^2 \times 3 \times 7$

11.5 Dividing Integers

Essential Question Is the quotient of two integers *positive*, *negative*, or *zero*? How can you tell?

1 ACTIVITY: Dividing Integers with Different Signs

Work with a partner. Use integer counters to find $-15 \div 3$.



∴ Because there are ____ negative counters in each group, $-15 \div 3 =$ ____.

2 ACTIVITY: Rewriting a Product as a Quotient

Work with a partner. Rewrite the product $3 \cdot 4 = 12$ as a quotient in two different ways.

First Way

12 is equal to 3 groups of ____.

∴ So, $12 \div 3 =$ ____.

Second Way

12 is equal to 4 groups of ____.

∴ So, $12 \div 4 =$ ____.

Common Core

Integers

In this lesson, you will

- divide integers.
- solve real-life problems.

Learning Standards
7.NS.2b
7.NS.3

3 ACTIVITY: Dividing Integers with Different Signs

Work with a partner. Rewrite the product $-3 \cdot (-4) = 12$ as a quotient in two different ways. What can you conclude?

First Way

$12 \div (\quad) =$ ____

Second Way

$12 \div (\quad) =$ ____

∴ In each case, when you divide a ____ integer by a ____ integer, you get a ____ integer.

4 ACTIVITY: Dividing Negative Integers

Math Practice 8

Maintain Oversight

How do you know what the sign will be when you divide two integers?

Work with a partner. Rewrite the product $3 \cdot (-4) = -12$ as a quotient in two different ways. What can you conclude?

First Way

$-12 \div (\quad) = \quad$

Second Way

$-12 \div (\quad) = \quad$

When you divide a _____ integer by a _____ integer, you get a _____ integer. When you divide a _____ integer by a _____ integer, you get a _____ integer.

Inductive Reasoning

Work with a partner. Complete the table.

	Exercise	Type of Quotient	Quotient	Quotient: Positive, Negative, or Zero
1	**5.** $-15 \div 3$	Integers with different signs		
2	**6.** $12 \div 4$			
3	**7.** $12 \div (-3)$			
4	**8.** $-12 \div (-4)$			
	9. $-6 \div 2$			
	10. $-21 \div (-7)$			
	11. $10 \div (-2)$			
	12. $12 \div (-6)$			
	13. $0 \div (-15)$			
	14. $0 \div 4$			

What Is Your Answer?

15. IN YOUR OWN WORDS Is the quotient of two integers *positive*, *negative*, or *zero*? How can you tell?

16. STRUCTURE Write general rules for dividing (a) two integers with the same sign and (b) two integers with different signs.

Practice Use what you learned about dividing integers to complete Exercises 8–15 on page 506.

11.5 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Ideas

Remember
Division by 0 is undefined.

Dividing Integers with the Same Sign

Words The quotient of two integers with the same sign is positive.

Numbers $8 \div 2 = 4$ $\quad -8 \div (-2) = 4$

Dividing Integers with Different Signs

Words The quotient of two integers with different signs is negative.

Numbers $8 \div (-2) = -4$ $\quad -8 \div 2 = -4$

EXAMPLE 1 Dividing Integers with the Same Sign

Find $-18 \div (-6)$.

The integers have the same sign.

$-18 \div (-6) = 3$

The quotient is positive.

∴ The quotient is 3.

EXAMPLE 2 Dividing Integers with Different Signs

Divide.

a. $75 \div (-25)$ **b.** $\frac{-54}{6}$

The integers have different signs.

$75 \div (-25) = -3$ $\quad \frac{-54}{6} = -9$

The quotient is negative.

∴ The quotient is -3. ∴ The quotient is -9.

On Your Own

Now You're Ready
Exercises 8–23

Divide.

1. $14 \div 2$
2. $-32 \div (-4)$
3. $-40 \div (-8)$
4. $0 \div (-6)$
5. $\frac{-49}{7}$
6. $\frac{21}{-3}$

EXAMPLE 3 Evaluating an Expression

Evaluate $10 - x^2 \div y$ when $x = 8$ and $y = -4$.

$$\begin{aligned} 10 - x^2 \div y &= 10 - 8^2 \div (-4) && \text{Substitute 8 for } x \text{ and } -4 \text{ for } y. \\ &= 10 - 8 \cdot 8 \div (-4) && \text{Write } 8^2 \text{ as repeated multiplication.} \\ &= 10 - 64 \div (-4) && \text{Multiply 8 and 8.} \\ &= 10 - (-16) && \text{Divide 64 by } -4. \\ &= 26 && \text{Subtract.} \end{aligned}$$

Remember

Use order of operations when evaluating an expression.

On Your Own

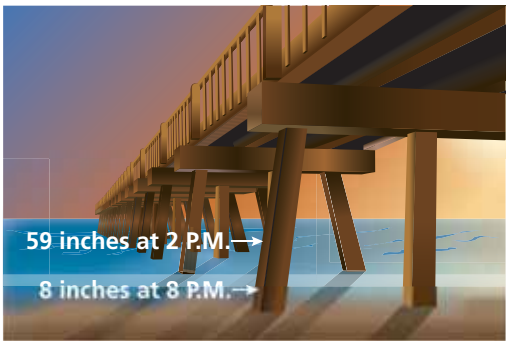
Now You're Ready
Exercises 28–31

Evaluate the expression when $a = -18$ and $b = -6$.

7. $a \div b$ **8.** $\dfrac{a + 6}{3}$ **9.** $\dfrac{b^2}{a} + 4$

EXAMPLE 4 Real-Life Application

You measure the height of the tide using the support beams of a pier. Your measurements are shown in the picture. What is the mean hourly change in the height?



Use a model to solve the problem.

$$\text{mean hourly change} = \frac{\text{final height} - \text{initial height}}{\text{elapsed time}}$$

$$\begin{aligned} &= \frac{8 - 59}{6} && \text{Substitute. The elapsed time from 2 P.M. to 8 P.M. is 6 hours.} \\ &= \frac{-51}{6} && \text{Subtract.} \\ &= -8.5 && \text{Divide.} \end{aligned}$$

∴ The mean change in the height of the tide is -8.5 inches per hour.

On Your Own

10. The height of the tide at the Bay of Fundy in New Brunswick decreases 36 feet in 6 hours. What is the mean hourly change in the height?

11.5 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** What can you tell about two integers when their quotient is positive? negative? zero?
2. **VOCABULARY** A quotient is undefined. What does this mean?
3. **OPEN-ENDED** Write two integers whose quotient is negative.
4. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$\frac{10}{-5}$ $\quad \frac{-10}{5}$ $\quad \frac{-10}{-5}$ $\quad -\left(\frac{10}{5}\right)$

Tell whether the quotient is *positive* or *negative* without dividing.

5. $-12 \div 4$
6. $\frac{-6}{-2}$
7. $15 \div (-3)$

Practice and Problem Solving

Divide, if possible.

8. $4 \div (-2)$
9. $21 \div (-7)$
10. $-20 \div 4$
11. $-18 \div (-3)$
12. $\frac{-14}{7}$
13. $\frac{0}{6}$
14. $\frac{-15}{-5}$
15. $\frac{54}{-9}$
16. $-33 \div 11$
17. $-49 \div (-7)$
18. $0 \div (-2)$
19. $60 \div (-6)$
20. $\frac{-56}{14}$
21. $\frac{18}{0}$
22. $\frac{65}{-5}$
23. $\frac{-84}{-7}$

ERROR ANALYSIS **Describe and correct the error in finding the quotient.**

24. ✗ $\frac{-63}{-9} = -7$

25. ✗ $0 \div (-5) = -5$

26. **ALLIGATORS** An alligator population in a nature preserve in the Everglades decreases by 60 alligators over 5 years. What is the mean yearly change in the alligator population?
27. **READING** You read 105 pages of a novel over 7 days. What is the mean number of pages you read each day?

ALGEBRA **Evaluate the expression when $x = 10$, $y = -2$, and $z = -5$.**

28. $x \div y$
29. $\frac{10y^2}{z}$
30. $\left|\frac{xz}{-y}\right|$
31. $\frac{-x^2 + 6z}{y}$

Find the mean of the integers.

32. $3, -10, -2, 13, 11$

33. $-26, 39, -10, -16, 12, 31$

Evaluate the expression.

34. $-8 - 14 \div 2 + 5$

35. $24 \div (-4) + (-2) \bullet (-5)$

36. PATTERN Find the next two numbers in the pattern $-128, 64, -32, 16, \ldots$. Explain your reasoning.

37. SNOWBOARDING A snowboarder descends a 1200-foot hill in 3 minutes. What is the mean change in elevation per minute?

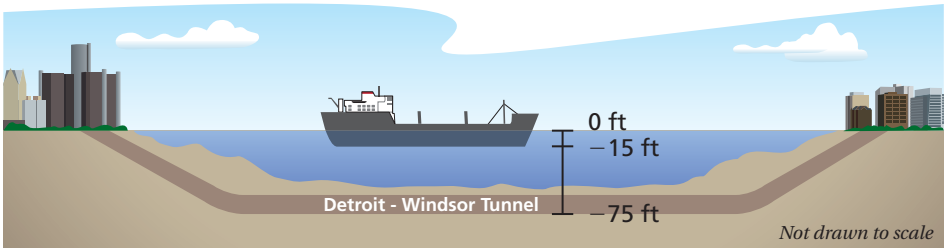
38. GOLF The table shows a golfer's score for each round of a tournament.

Scorecard	
Round 1	-2
Round 2	-6
Round 3	-7
Round 4	-3

a. What was the golfer's total score?

b. What was the golfer's mean score per round?

39. TUNNEL The Detroit-Windsor Tunnel is an underwater highway that connects the cities of Detroit, Michigan, and Windsor, Ontario. How many times deeper is the roadway than the bottom of the ship?



40. AMUSEMENT PARK The regular admission price for an amusement park is \$72. For a group of 15 or more, the admission price is reduced by \$25. How many people need to be in a group to save \$500?

41. Number Sense Write five different integers that have a mean of -10. Explain how you found your answer.

Fair Game Review What you learned in previous grades & lessons

Graph the values on a number line. Then order the values from least to greatest. *(Section 11.1)*

42. $-6, 4, |2|, -1, |-10|$

43. $3, |0|, |-4|, -3, -8$

44. $|5|, -2, -5, |-2|, -7$

45. MULTIPLE CHOICE What is the value of $4 \bullet 3 + (12 \div 2)^2$? *(Section 1.3)*

(A) 15 (B) 48 (C) 156 (D) 324

11.4–11.5 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Evaluate the expression. *(Section 11.4 and Section 11.5)*

1. $-7(6)$

2. $-1(-10)$

3. $\frac{-72}{-9}$

4. $-24 \div 3$

5. $-3 \cdot 4 \cdot (-6)$

6. $(-3)^3$

Evaluate the expression when $a = 4$, $b = -6$, and $c = -12$. *(Section 11.4 and Section 11.5)*

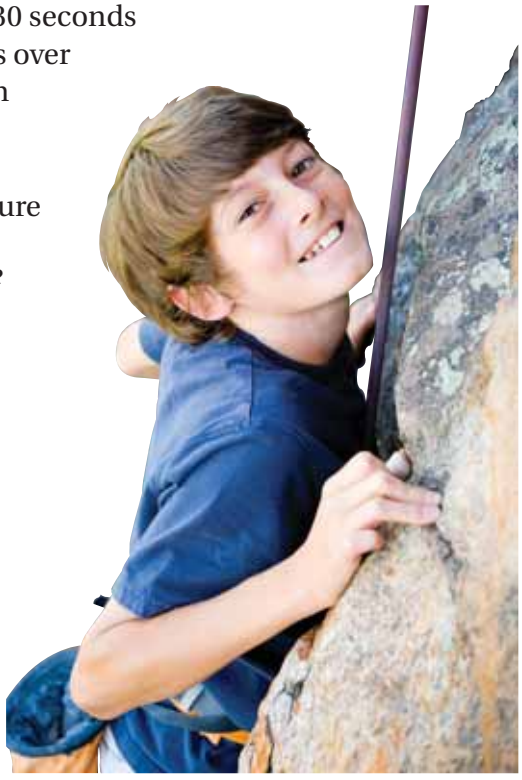
7. c^2

8. bc

9. $\frac{ab}{c}$

10. $\frac{|c - b|}{a}$

11. **SPEECH** In speech class, you lose 3 points for every 30 seconds you go over the time limit. Your speech is 90 seconds over the time limit. What integer represents the change in your points? *(Section 11.4)*



12. **MOUNTAIN CLIMBING** On a mountain, the temperature decreases by 18°F every 5000 feet. What integer represents the change in temperature at 20,000 feet? *(Section 11.4)*

13. **GAMING** You play a video game for 15 minutes. You lose 165 points. What is the mean change in points per minute? *(Section 11.5)*

14. **DIVING** You dive 21 feet from the surface of a lake in 7 seconds. *(Section 11.4 and Section 11.5)*

a. What is the mean change in your position in feet per second?

b. You continue diving. What is your position relative to the surface after 5 more seconds?

15. **HIBERNATION** A female grizzly bear weighs 500 pounds. After hibernating for 6 months, she weighs only 200 pounds. What is the mean change in weight per month? *(Section 11.5)*

11 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com

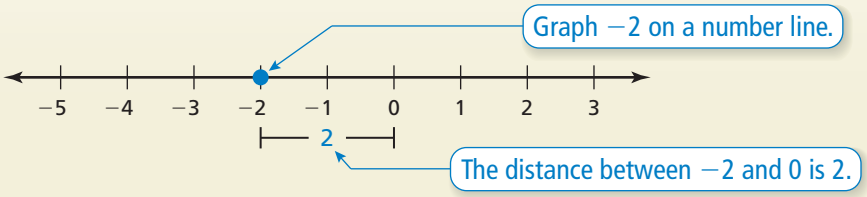
Review Key Vocabulary

integer, *p. 478*
absolute value, *p. 478*
opposites, *p. 484*
additive inverse, *p. 484*

Review Examples and Exercises

11.1 Integers and Absolute Value *(pp. 476–481)*

Find the absolute value of −2.



∴ So, $|-2| = 2$.

Exercises

Find the absolute value.

1. $|3|$ **2.** $|-9|$ **3.** $|-17|$ **4.** $|8|$

5. ELEVATION The elevation of Death Valley, California, is −282 feet. The Mississippi River in Illinois has an elevation of 279 feet. Which is closer to sea level?

11.2 Adding Integers *(pp. 482–487)*

Find 6 + (−14).

$6 + (-14) = -8$ $|-14| > |6|$. So, subtract $|6|$ from $|-14|$.

Use the sign of −14.

∴ The sum is −8.

Exercises

Add.

6. $-16 + (-11)$ **7.** $-15 + 5$ **8.** $100 + (-75)$ **9.** $-32 + (-2)$

11.3 Subtracting Integers *(pp. 488–493)*

Subtract.

a. $7 - 19 = 7 + (-19)$ Add the opposite of 19.

$= -12$ Add.

∴ The difference is -12.

b. $-6 - (-10) = -6 + 10$ Add the opposite of -10.

$= 4$ Add.

∴ The difference is 4.

Exercises

Subtract.

10. $8 - 18$ **11.** $-16 - (-5)$ **12.** $-18 - 7$ **13.** $-12 - (-27)$

14. GAME SHOW Your score on a game show is -300. You answer the final question incorrectly, so you lose 400 points. What is your final score?

11.4 Multiplying Integers *(pp. 496–501)*

a. Find $-7 \cdot (-9)$.

The integers have the same sign.

$-7 \cdot (-9) = 63$

The product is positive.

∴ The product is 63.

b. Find $-6(14)$.

The integers have different signs.

$-6(14) = -84$

The product is negative.

∴ The product is -84.

Exercises

Multiply.

15. $-8 \cdot 6$ **16.** $10(-7)$ **17.** $-3 \cdot (-6)$ **18.** $-12(5)$

11.5 Dividing Integers *(pp. 502–507)*

a. Find $30 \div (-10)$.

The integers have different signs.

$$30 \div (-10) = -3$$

The quotient is negative.

∴ The quotient is -3.

b. Find $\frac{-72}{-9}$.

The integers have the same sign.

$$\frac{-72}{-9} = 8$$

The quotient is positive.

∴ The quotient is 8.

Exercises

Divide.

19. $-18 \div 9$ **20.** $\frac{-42}{-6}$ **21.** $\frac{-30}{6}$ **22.** $84 \div (-7)$

Evaluate the expression when $x = 3$, $y = -4$, and $z = -6$.

23. $z \div x$ **24.** $\frac{xy}{z}$ **25.** $\frac{z - 2x}{y}$

Find the mean of the integers.

26. $-3, -8, 12, -15, 9$ **27.** $-54, -32, -70, -25, -65, -42$

28. PROFITS The table shows the weekly profits of a fruit vendor. What is the mean profit for these weeks?

Week	1	2	3	4
Profit	−$125	−$86	$54	−$35

29. RETURNS You return several shirts to a store. The receipt shows that the amount placed back on your credit card is −$30.60. Each shirt is −$6.12. How many shirts did you return?

11 Chapter Test

Check It Out
Test Practice
BigIdeasMath.com

Find the absolute value.

1. $|-9|$
2. $|64|$
3. $|-22|$

Copy and complete the statement using <, >, or =.

4. $4 \square |-8|$
5. $|-7| \square -12$
6. $-7 \square |3|$

Evaluate the expression.

7. $-6 + (-11)$
8. $2 - (-9)$
9. $-9 \cdot 2$
10. $-72 \div (-3)$

Evaluate the expression when $x = 5$, $y = -3$, and $z = -2$.

11. $\frac{y + z}{x}$
12. $\frac{x - 5z}{y}$

Find the mean of the integers.

13. 11, −7, −14, 10, −5
14. −32, −41, −39, −27, −33, −44

15. **NASCAR** A driver receives −25 points for each rule violation. What integer represents the change in points after 4 rule violations?

16. **GOLF** The table shows your scores, relative to *par*, for nine holes of golf. What is your total score for the nine holes?

Hole	1	2	3	4	5	6	7	8	9	Total
Score	+1	−2	−1	0	−1	+3	−1	−3	+1	?

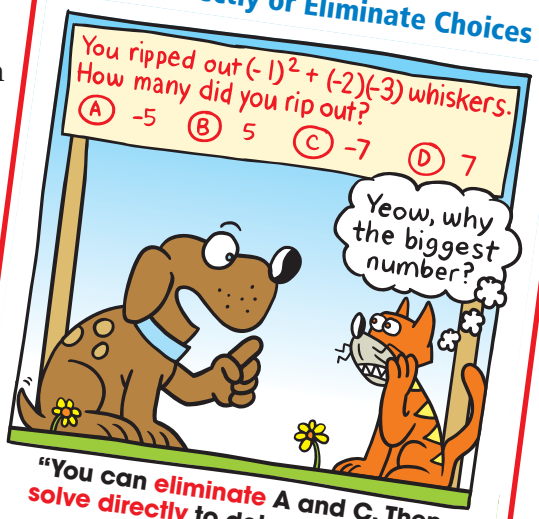
17. **VISITORS** In a recent 10-year period, the change in the number of visitors to U.S. national parks was about −11,150,000 visitors.

   a. What was the mean yearly change in the number of visitors?

   b. During the seventh year, the change in the number of visitors was about 10,800,000. Explain how the change for the 10-year period can be negative.

11 Standards Assessment

Test-Taking Strategy
Solve Directly or Eliminate Choices



"You can eliminate A and C. Then, solve directly to determine that the correct answer is D."

1. A football team gains 2 yards on the first play, loses 5 yards on the second play, loses 3 yards on the third play, and gains 4 yards on the fourth play. What is the team's overall gain or loss for all four plays? *(7.NS.1b)*

A. a gain of 14 yards
B. a gain of 2 yards
C. a loss of 2 yards
D. a loss of 14 yards

2. Which expression is *not* equal to the number 0? *(7.NS.1a)*

F. $5 - 5$
G. $-7 + 7$
H. $6 - (-6)$
I. $-8 - (-8)$

3. What is the value of the expression below when $a = -2$, $b = 3$, and $c = -5$? *(7.NS.3)*

$$|a^2 - 2ac + 5b|$$

A. -9
B. -1
C. 1
D. 9

4. What is the value of the expression below? *(7.NS.1c)*

$$17 - (-8)$$

5. Sam was evaluating an expression in the box below.

$$\begin{aligned}(-2)^3 \cdot 3 - (-5) &= 8 \cdot 3 - (-5)\\ &= 24 + 5\\ &= 29\end{aligned}$$

What should Sam do to correct the error that he made? *(7.NS.3)*

F. Subtract 5 from 24 instead of adding.
G. Rewrite $(-2)^3$ as -8.
H. Subtract -5 from 3 before multiplying by $(-2)^3$.
I. Multiply -2 by 3 before raising the quantity to the third power.

6. What is the value of the expression below when $x = 6$, $y = -4$, and $z = -2$? *(7.NS.3)*

$$\frac{x - 2y}{-z}$$

A. -7

B. -1

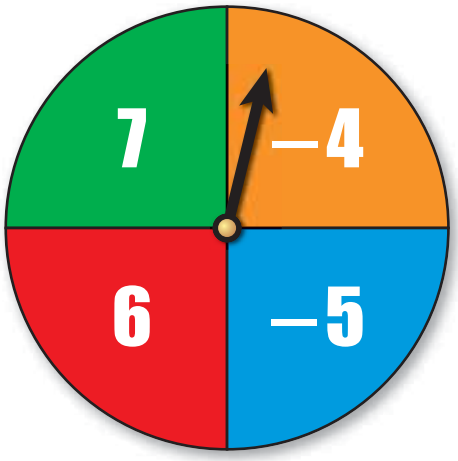
C. 1

D. 7

7. What is the missing number in the sequence below? *(7.NS.1c)*

39, 24, 9, ___, -21

8. You are playing a game using the spinner shown. You start with a score of 0 and spin the spinner four times. When you spin blue or green, you add the number to your score. When you spin red or orange, you subtract the number from your score. Which sequence of colors represents the greatest score? *(7.NS.3)*



F. red, green, green, red

G. orange, orange, green, blue

H. red, blue, orange, green

I. blue, red, blue, red

9. Which expression represents a negative integer? *(7.NS.3)*

A. $5 - (-6)$

B. $(-3)^3$

C. $-12 \div (-6)$

D. $(-2)(-4)$

10. Which expression has the greatest value when $x = -2$ and $y = -3$? *(7.NS.3)*

F. $-xy$

G. xy

H. $x - y$

I. $-x - y$

11. What is the value of the expression below? *(7.NS.3)*

$$-5 \cdot (-4)^2 - (-3)$$

A. -83

B. -77

C. 77

D. 83

12. Which property does the equation below represent? *(7.NS.1d)*

$$-80 + 30 + (-30) = -80 + [30 + (-30)]$$

F. Commutative Property of Addition

G. Associative Property of Addition

H. Additive Inverse Property

I. Addition Property of Zero

13. What is the mean of the data set in the box below? *(7.NS.3)*

$-8, -6, -2, 0, -6, -8, 4, -7, -8, 1$

A. -8

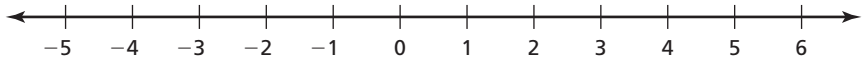
B. -7

C. -6

D. -4

14. Consider the number line shown below. *(7.NS.1b, 7.NS.1c)*

Think
Solve
Explain



Part A Use the number line to explain how to add -2 and -3.

Part B Use the number line to explain how to subtract 5 from 2.

15. What is the value of the expression below? *(7.NS.3)*

$$\frac{-3 - 2^2}{-1}$$

F. -25

G. -1

H. 7

I. 25