1 Numerical Expressions and Factors

- **1.1 Whole Number Operations**
- **1.2** Powers and Exponents
- **1.3 Order of Operations**
- **1.4 Prime Factorization**
- **1.5 Greatest Common Factor**
- 1.6 Least Common Multiple



"Dear Sir: You say that x³ is called x-cubed."



"And you say that x² is called x-squared."



"So, why isn't x¹ called *x*-lined?"



"My sign on adding fractions with unlike denominators is keeping the hyenas away."



"See, it's working."

What You Learned Before



Identifying Prime and Composite Numbers (4.0A.4)

- **Example 1** Determine whether 26 is prime or composite. Because the factors of 26 are 1, 2, 13, and 26, it is composite.
- **Example 2** Determine whether 37 is prime or composite. Because the only factors of 37 are 1 and 37, it is prime.

Try It Yourself

Determine whether the number is prime or composite.

1. 5	2. 14	3. 17
4. 23	5. 28	6. 33
7. 43	8. 57	9. 64

Adding and Subtracting Mixed Numbers with Like Denominators (4.NF.3c)

Example 3 Find $2\frac{3}{5} + 4\frac{1}{5}$.
$2\frac{3}{5} + 4\frac{1}{5} = \frac{2 \cdot 5 + 3}{5} + \frac{4 \cdot 5 + 1}{5}$
$=\frac{13}{5}+\frac{21}{5}$
$=\frac{13+21}{5}$
$=\frac{34}{5}$, or $6\frac{4}{5}$

Rewrite the mixed numbers as improper fractions.

Simplify.

Add the numerators.

Simplify.

Try It Yourself Add or subtract.

10. $4\frac{1}{9} + 2\frac{7}{9}$	11. $6\frac{1}{11} + 3\frac{6}{11}$	12. $3\frac{7}{8} + 4\frac{3}{8}$
13. $5\frac{8}{13} - 1\frac{2}{13}$	14. $7\frac{1}{4} - 3\frac{3}{4}$	15. $4\frac{1}{6} - 2\frac{5}{6}$

Essential Question How do you know which operation to choose

when solving a real-life problem?

ACTIVITY: Choosing an Operation

Work with a partner. The double bar graph shows the history of a citywide cleanup day.



- Copy each question below.
- Underline a key word or phrase that helps you know which operation to use to answer the question. State the operation. Why do you think the key word or phrase indicates the operation you chose?
- Write an expression you can use to answer the question.
- Find the value of your expression.
- **a.** What is the total amount of trash collected from 2010 to 2013?
- **b.** How many more pounds of recyclables were collected in 2013 than in 2010?
- c. How many times more recyclables were collected in 2012 than in 2010?
- **d.** The amount of trash collected in 2014 is estimated to be twice the amount collected in 2011. What is that amount?



COMMON

determine which operation

• divide multi-digit numbers.

CORE

Whole Numbers

to perform.

Learning Standard

6.NS.2

In this lesson, you will

2 ACTIVITY: Checking Answers



Communicate Precisely

What key words should you use so that your partner understands your explanation?

Work with a partner.

- **a.** Explain how you can use estimation to check the reasonableness of the value of your expression in Activity 1(a).
- **b.** Explain how you can use addition to check the value of your expression in Activity 1(b).
- **c.** Explain how you can use estimation to check the reasonableness of the value of your expression in Activity 1(c).
- **d.** Use mental math to check the value of your expression in Activity 1(d). Describe your strategy.

3 ACTIVITY: Using Estimation

Work with a partner. Use the map. Explain how you found each answer.

- **a.** Which two lakes have a combined area of about 33,000 square miles?
- **b.** Which lake covers an area about three times greater than the area of Lake Erie?
- **c.** Which lake covers an area that is about 16,000 square miles greater than the area of Lake Ontario?
- **d.** Estimate the total area covered by the Great Lakes.



-What Is Your Answer?

- **4. IN YOUR OWN WORDS** How do you know which operation to choose when solving a real-life problem?
- 5. In a *magic square*, the sum of the numbers in each row, column, and diagonal is the same and each number from 1 to 9 is used only once. Complete the magic square. Explain how you found the missing numbers.

;	9	2
?	5	?
8	?	?



Use what you learned about choosing operations to complete Exercises 8–11 on page 7.



Recall the four basic operations: addition, subtraction, multiplication, and division.

Operation	Words	Algebra
Addition	the sum of	a + b
Subtraction	the <i>difference</i> of	a-b
Multiplication	the <i>product</i> of	$a \times b$ $a \cdot b$
Division	the <i>quotient</i> of	$a \div b \frac{a}{b} b)\overline{a}$

EXAMPLE 1 Adding and Subtracting Whole Numbers

The bar graph shows the attendance at a three-day art festival.



a. What is the total attendance for the art festival?

You want to find the total attendance for the three days. In this case, the phrase <i>total attendance</i> indicates you need to find the sum of the daily attendances.	
Line up the numbers by their place values, then add.	
The total attendance is 0501 people	

The total attendance is 9591 people.

b. What is the increase in attendance from Day 1 to Day 2?

You want to find how many more people attended on Day 2 than on Day 1. In this case, the phrase *how many more* indicates you need to find the difference of the attendances on Day 2 and Day 1.

Lir	ne up the numbers by their place values, then subtract.	2Ø14 3X45
֥	The increase in attendance from Day 1 to Day 2	-2570
	is 575 people.	575

EXAMPLE



reasonable.

A school lunch contains 12 chicken nuggets. Ninety-five students buy the lunch. What is the total number of chicken nuggets served?

You want to find the total number of chicken nuggets in 95 groups of 12 chicken nuggets. The phrase 95 groups of 12 indicates you need to find the product of 95 and 12.

 $\begin{array}{r}
12 \\
\times 95 \\
60 \\
\underline{108} \\
1140
\end{array}$

Multiplying Whole Numbers

Multiply 12 by the ones digit, 5. Multiply 12 by the tens digit, 9. Add.

There were 1140 chicken nuggets served.

2





Find the value of the expression. Use estimation to check your answer.

2. 912 - 799 **1.** 1745 + 682

3. 42×118

Dividing Whole Numbers: No Remainder 3 **EXAMPLE**

You make 24 equal payments for a go-kart. You pay a total of \$840. How much is each payment?

You want to find the number of groups of 24 in \$840. The phrase groups of 24 in \$840 indicates you need to find the quotient of 840 and 24.



Use long division to find the quotient. Decide where to write the first digit of the quotient.



Do not use the hundreds place because 24 is greater than 8.

24)840

Use the tens place because 24 is less than 84.

So, divide the tens and write the first digit of the quotient in the tens place.

3	
24)840	Divide 84 by 24: There are three groups of 24 in 84.
-72	Multiply 3 and 24.
12	Subtract 72 from 84.

Next, bring down the 0 and divide the ones.

35 24)840 Divide 120 by 24: There are five groups of 24 in 120. Remember -72120 **Check** Find the dividend = quotient Multiply 5 and 24. - 120 divisor product of the quotient Subtract 120 from 120. 0 and the divisor. So, quotient \times divisor = dividend. The quotient of 840 and 24 is 35. • So, each payment is \$35.

35 quotient $\times 24$ divisor 140 70 840 dividend





Find the value of the expression. Use estimation to check your answer.

- **4.** $234 \div 9$ **5.** $\frac{986}{58}$ **6.** $840 \div 105$
- 7. Find the quotient of 9920 and 320.

When you use long division to divide whole numbers and you obtain a remainder, you can write the quotient as a mixed number using the rule

dividend \div divisor = quotient + $\frac{\text{remainder}}{\text{divisor}}$.

EXAMPLE 4 Real-Life Application



A 301-foot-high swing at an amusement park can take 64 people on each ride. A total of 8983 people ride the swing today. All the rides are full except for the last ride. How many rides are given? How many people are on the last ride?

To find the number of rides given, you need to find the number of groups of 64 people in 8983 people. The phrase *groups of 64 people in 8983 people* indicates you need to find the quotient of 8983 and 64.

Divide the place-value positions from left to right.



must write a 0 in the ones 2place of the quotient. -

Do not stop here. You

The quotient is $140\frac{23}{64}$. This indicates 140 groups of 64, with 23 remaining.

So, 141 rides are given, with 23 people on the last ride.

On Your Own



6

Find the value of the expression. Use estimation to check your answer.

 $\frac{6096}{30}$

8.

- **9.** 45,691 ÷ 28 **10.** 3215 ÷ 430
- **11. WHAT IF?** In Example 4, 9038 people ride the swing. What is the least number of rides possible?



Vocabulary and Concept Check

VOCABULARY Determine which operation the word or phrase represents.

1. sum	2. times	3. the quotient of
4. decreased by	5. total of	6. minus
	e the division problem s ber is the divisor, divide b. 26	26



Practice and Problem Solving

The bar graph shows the attendance at a food festival. Write an expression you can use to answer the question. Then find the value of your expression.

- **8.** What is the total attendance at the food festival from 2010 to 2013?
- **9.** How many more people attended the food festival in 2012 than in 2011?
- **10.** How many times more people attended the food festival in 2013 than in 2010?
- **11.** The festival projects that the total attendance for 2014 will be twice the attendance in 2012. What is the projected attendance for 2014?



Find the value of the expression. Use estimation to check your answer.

112. 2219 + 872	13. 5351 + 1730	14. 3968 + 1879
15. 7694 – 5232	16. 9165 – 4729	17. 2416 <u>- 1983</u>
2 18. 84 <u>× 37</u>	19. 124 × 56	20. 419 × 236
3 21. 837 ÷ 27	22. $\frac{588}{84}$	23. 7440 ÷ 124
4 24. 6409 ÷ 61	25. 8241 ÷ 173	26. $\frac{33,505}{160}$

ERROR ANALYSIS Describe and correct the error in finding the value of the expression.



Determine the operation you would use to solve the problem. Do not answer the question.

- **29.** Gymnastic lessons cost \$30 per week. How much will 18 weeks of gymnastic lessons cost?
- **30.** The scores on your first two tests were 82 and 93. By how many points did your score improve?
- **31.** You are setting up tables for a banquet for 150 guests. Each table seats 12 people. What is the minimum number of tables you will need?
- **32.** A store has 15 boxes of peaches. Each box contains 45 peaches. How many peaches does the store have?
- **33.** Two shirts cost \$18 and \$25. What is the total cost of the shirts?
- **34.** A gardener works for 14 hours during a week and charges \$168. How much does the gardener charge for each hour?





- **38. BOX OFFICE** The number of tickets sold for the opening weekend of a movie is 879,575. The movie was shown in 755 theaters across the nation. What was the average number of tickets sold at each theater?
- **39.** LOGIC You find that the product of 93 and 6 is 558. How can you use addition to check your answer? How can you use division to check your answer?
- **40. NUMBER SENSE** Without calculating, decide which is greater: 3999 ÷ 129 or 3834 ÷ 142. Explain.

- **41. REASONING** In a division problem, can the remainder be greater than the divisor? Explain.
- **42. WATER COOLER** You change the water jug on the water cooler. How many cups can be completely filled before you need to change the water jug again?
- **43. ARCADE** You have \$9, one of your friends has \$10, and two of your other friends each have \$13. You combine your money to buy arcade tokens. You use a coupon to buy 8 tokens for \$1. The cost of the remaining tokens is four for \$1. You and your friends share the tokens evenly. How many tokens does each person get?





- **44. BOOK SALE** You borrow bookcases like the one shown to display 943 books at a book sale. You plan to put 22 books on each shelf. No books will be on top of the bookcases.
 - **a.** How many bookcases must you borrow to display all the books?
 - **b.** You fill the shelves of each bookcase in order, starting with the top shelf. How many books are on the third shelf of the last bookcase?
- **45. MODELING** The siding of a house is 2250 square feet. The siding needs two coats of paint. The table shows information about the paint.

Can Size	Cost	Coverage
1 quart	\$18	80 square feet
1 gallon	\$29	320 square feet

- **a.** What is the minimum cost of the paint needed to complete the job?
- **b.** How much paint is left over?
- **46.** Use the digits 3, 4, 6, and 9 to complete the division problem. Use each digit once.





1.2 Powers and Exponents

Essential Question How can you use repeated factors in

real-life situations?

As I was going to St. Ives I met a man with seven wives Each wife had seven sacks Each sack had seven cats Each cat had seven kits Kits, cats, sacks, wives How many were going to St. Ives? Nursery Rhyme, 1730



ACTIVITY: Analyzing a Math Poem

Work with a partner. Here is a "St. Ives" poem written by two students. Answer the question in the poem.

0	As I was walking into town
	I met a ringmaster with five clowns
	Each clown had five magicians
	Each magician had five bunnies
	Each bunny had five fleas
	Fleas, bunnies, magicians, clowns
	How many were going into town?



Numerical Expressions
In this lesson, you will
write expressions as powers.
find values of powers.
Preparing for Standard
6.EE.1



So, the number of fleas, bunnies, magicians, and clowns is Explain how you found your answer.

2 ACTIVITY: Writing Repeated Factors



Repeat Calculations What patterns do you notice with each problem? How does this help you write exponents?

Work with a partner. Copy and complete the table
--

	Repeated Factors	Using an Exponent	Value
a.	4×4		
b.	6×6		
c.	10 imes 10 imes 10		
d.	$100 \times 100 \times 100$		
e.	$3 \times 3 \times 3 \times 3$		
f.	$4 \times 4 \times 4 \times 4 \times 4$		
g.	$2\times 2\times 2\times 2\times 2\times 2$		

h. In your own words, describe what the two numbers in the expression 3⁵ mean.

3 ACTIVITY: Writing and Analyzing a Math Poem

Work with a partner.

- **a.** Write your own "St. Ives" poem.
- **b.** Draw pictures for your poem.
- **c.** Answer the question in your poem.
- **d.** Show how you can use exponents to write your answer.

-What Is Your Answer?

- **4. IN YOUR OWN WORDS** How can you use repeated factors in real-life situations? Give an example.
- 5. **STRUCTURE** Use exponents to complete the table. Describe the pattern.

10	100	1000	10,000	100,000	1,000,000
10^1	10 ²				



Use what you learned about exponents to complete Exercises 4–6 on page 14.



Key Vocabulary 📢

power, p. 12 base, p. 12 exponent, p. 12 perfect square, p. 13 A **power** is a product of repeated factors. The **base** of a power is the repeated factor. The **exponent** of a power indicates the number of times the base is used as a factor.

base exponent

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$$

power 3 is used as a factor 4 times.

Power	Words
3 ²	Three <i>squared</i> , or three to the second
3 ³	Three <i>cubed</i> , or three to the third
3^{4}	Three to the fourth

EXAMPLE (1) Writing Expressions as Powers

Write each product as a power.

a. 4 • 4 • 4 • 4 • 4

Because 4 is used as a factor 5 times, its exponent is 5.

So, $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5$.

b. $12 \times 12 \times 12$

Because 12 is used as a factor 3 times, its exponent is 3.

So, $12 \times 12 \times 12 = 12^3$.

👂 On Your Own



Write the product as a power.

1. 6 • 6 • 6 • 6 • 6 • 6

2. $15 \times 15 \times 15 \times 15$



Math Practice 5 Choose Tools Why are calculators more efficient when finding the values

involving exponents?

of expressions

The square of a whole number is a **perfect square**.



On Your Own

11. What is the area of the square traffic sign in square inches? in square feet?

STOP

AHEAD

24 in.

24 in.







Vocabulary and Concept Check

- 1. VOCABULARY How are exponents and powers different?
- 2. VOCABULARY Is 10 a perfect square? Is 100 a perfect square? Explain.
- **3. WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.

 $2^{4} = 2 \times 2 \times 2 \times 2$ 3 + 3 + 3 = 3(4) 3² = 3 × 3 5 • 5 • 5 = 5³



Write the product as a power.

1 4. 9×9	5. 13 × 13	6. $15 \times 15 \times 15$
7. 2 • 2 • 2 • 2 • 2	8. $14 \times 14 \times 14$	9. 8 • 8 • 8 • 8
10. $11 \times 11 \times 11 \times 11 \times 11$	11. 7 • 7 • 7 • 7 • 7 • 7	12. 16 • 16 • 16 • 16



Find the value of the power. 2 14. 5² 15. 4³ 16. 2⁵

Use a calculate	or to find the value of the	e power.	
18. 7 ⁶	19. 4 ⁸	20. 12 ⁴	21.

22. ERROR ANALYSIS Describe and correct the error in finding the value of the power.



17. 14²

 17^{5}

- **23. POPULATION** The population of Virginia is about 8×10^6 . About how many people live in Virginia?
- 24. FIGURINES The smallest figurine in a gift shop is 2 inches tall. The height of each figurine is twice the height of the previous figurine. Write a power to represent the height of the tallest figurine. Then find the height.



Determine whether the number is a perfect square.

- 3
 25. 8
 26. 4
 27. 81
 28. 44

 29. 49
 30. 125
 31. 150
 32. 144
 - **33. PAINTING** A square painting measures 2 meters on each side. What is the area of the painting in square centimeters?



- **34. NUMBER SENSE** Write three powers that have values greater than 120 and less than 130.
- **35. CHECKERS** A checkers board has 64 squares. How many squares are in each row?
- **36. PATIO** A landscaper has 125 tiles to build a square patio. The patio must have an area of at least 80 square feet.
 - a. What are the possible arrangements for the patio?
 - b. How many tiles are not used in each arrangement?
- **37. PATTERNS** Copy and complete the table. Describe what happens to the value of the power as the exponent decreases. Use this pattern to find the value of 4^0 .

Power	4^{6}	4^{5}	4^4	4^{3}	4^{2}	4^1
Value	4096	1024				

- **38. REASONING** Consider the equation $56 = 10^{2}$. The missing number is between what two whole numbers?
- **39.** Repeated How many blocks do you need to add to Square 6 to get Square 7? to Square 9 to get Square 10? to Square 19 to get Square 20? Explain.



Fair Game Review What you learned in previous grades & lessons

Find the value of the expression. (Skills Review Handbook)

40. 6×14 **41.** 11×15 **42.** $56 \div 7$ **43.** $112 \div 16$

44. MULTIPLE CHOICE You buy a box of sugar-free gum that has 12 packs. Each pack has 5 pieces. Which expression represents the total number of pieces of gum? (*Skills Review Handbook*)

(A) 12+5 (B) 12-5 (C) 12×5 (D) $12 \div 5$



12 in.

12 in.

1.3 Order of Operations

1

Essential Question What is the effect of inserting parentheses

into a numerical expression?

ACTIVITY: Comparing Different Orders

Work with a partner. Find the value of the expression by using different orders of operations. Are your answers the same? (Circle *yes* or *no.*)

a.	Add, then multiply.	Multiply, then add.	Same?
	$3 + 4 \times 2 =$	$3 + 4 \times 2 =$	Yes No
b.	Add, then subtract.	Subtract, then add.	Same?
	5 + 3 - 1 =	5 + 3 - 1 =	Yes No
0	Divide then multiply	Multiply then divide	Samaj
c.	Divide, then multiply.	Multiply, then divide.	Same?
	$12 \div 3 \bullet 2 =$	$12 \div 3 \bullet 2 =$	Yes No
d.	Divide, then add.	Add, then divide.	Same?
	$16 \div 4 + 4 =$	$16 \div 4 + 4 =$	
			Yes No
e.	Multiply, then subtract.	Subtract, then multiply.	Same?
e.	Multiply, then subtract. $8 \times 4 - 2 =$	Subtract, then multiply. $8 \times 4 - 2 =$	Same? Yes No
	$8 \times 4 - 2 =$	$8 \times 4 - 2 =$	Yes No
e. f.			
	$8 \times 4 - 2 =$	$8 \times 4 - 2 =$	Yes No
f.	$8 \times 4 - 2 =$ Multiply, then divide. $8 \cdot 4 \div 2 =$	$8 \times 4 - 2 =$ Divide, then multiply. $8 \cdot 4 \div 2 =$	Yes No Same? Yes No
	$8 \times 4 - 2 =$ Multiply, then divide. $8 \cdot 4 \div 2 =$ Subtract, then add.	$8 \times 4 - 2 =$ Divide, then multiply. $8 \cdot 4 \div 2 =$ Add, then subtract.	Yes No Same?
f.	$8 \times 4 - 2 =$ Multiply, then divide. $8 \cdot 4 \div 2 =$	$8 \times 4 - 2 =$ Divide, then multiply. $8 \cdot 4 \div 2 =$	Yes No Same? Yes No
f.	$8 \times 4 - 2 =$ Multiply, then divide. $8 \cdot 4 \div 2 =$ Subtract, then add. 13 - 4 + 6 =	$8 \times 4 - 2 =$ Divide, then multiply. $8 \cdot 4 \div 2 =$ Add, then subtract. 13 - 4 + 6 =	Yes No Same? Yes No Same? Yes No
f. g.	$8 \times 4 - 2 =$ Multiply, then divide. $8 \cdot 4 \div 2 =$ Subtract, then add.	$8 \times 4 - 2 =$ Divide, then multiply. $8 \cdot 4 \div 2 =$ Add, then subtract.	Yes No Same? Yes No Same?



Numerical Expressions In this lesson, you will

 evaluate numerical expressions with whole-number exponents.
 Learning Standard
 6.EE.1

ACTIVITY: Using Parentheses

Work with a partner. Use all the symbols and numbers to write an expression that has the given value.

	Symbols and Numbers	Value	Expression
a.	(), +, ÷, 3, 4, 5	3	
b.	(), -, ×, 2, 5, 8	11	
c.	(), ×, ÷, 4, 4, 16	16	
d.	(), -, ÷, 3, 8, 11	1	
e.	(), +, ×, 2, 5, 10	70	

3 ACTIVITY: Reviewing Fractions and Decimals

Work with a partner. Evaluate the expression.



What Is Your Answer?

- **4.** In an expression with two or more operations, why is it necessary to agree on an order of operations? Give examples to support your explanation.
- **5. IN YOUR OWN WORDS** What is the effect of inserting parentheses into a numerical expression?



Use what you learned about the order of operations to complete Exercises 3–5 on page 20.



1.3 Lesson



Key Vocabulary numerical expression, p. 18 evaluate, p. 18 order of operations, p. 18

A **numerical expression** is an expression that contains only numbers and operations. To evaluate, or find the value of, a numerical expression, use a set of rules called the **order of operations**.

Key Idea

Order of Operations

- 1. Perform operations in Parentheses.
- 2. Evaluate numbers with Exponents.
- **3.** Multiply or **D**ivide from left to right.
- 4. Add or Subtract from left to right.

Using Order of Operations ฦ **EXAMPLE**

b.

a. Evaluate $12 - 2 \times 4$.

$12 - 2 \times 4 = 12 - 12$	8	Multiply 2 and 4.
=4		Subtract 8 from 12.
Evaluate 7 + 60 \div	(3 × 5).	
$7 + 60 \div (3 \times 5) =$	$7+60 \div 15$	Perform operation in parentheses.
=	7 + 4	Divide 60 by 15.
=	11	Add 7 and 4.

EXAMPLE 2 Using Order of Operations with Exponents



Remember to multiply and divide from left to right. In Example 2, you should divide before multiplying because the division symbol comes first when reading from left to right.

Evaluate $30 \div (7 + 2^3) \times 6$.

Evaluate the power in parentheses first.

$$30 \div (7 + 2^3) \times 6 = 30 \div (7 + 8) \times 6$$
 Evaluate 2³.
= 30 ÷ 15 × 6 Perform operation in parentheses.
= 2 × 6 Divide 30 by 15.
= 12 Multiply 2 and 6.

On Your Own

1. 4.

Now You're Ready Exercises 6–14

Evaluate the expression.

$7 \cdot 5 + 3$	2.	$(28 - 20) \div 4$	3.	$6\times 15-10\div 2$
$6+2^4-1$	5.	$4 \cdot 3^2 + 18 - 9$	6.	$16 + (5^2 - 7) \div 3$

Multi-Language Glossary at BigIdeasMath com

The symbols \times and \cdot are used to indicate multiplication. You can also use parentheses to indicate multiplication. For example, 3(2 + 7) is the same as $3 \times (2 + 7)$.

EXAMPLE 3 Using Order of Operations

a. Evaluate 9 + 7(5 - 2).

$$9 + 7(5 - 2) = 9 + 7(3)$$

= 9 + 21

= 30

- b. Evaluate $15 4(6 + 1) \div 2^2$.
 - $15 4(6 + 1) \div 2^2 = 15 4(7) \div 2^2$ $= 15 4(7) \div 4$ $= 15 28 \div 4$ = 15 7= 8

Perform operation in parentheses. Multiply 7 and 3. Add 9 and 21.

Perform operation in parentheses.
Evaluate 2 ² .
Multiply 4 and 7.
Divide 28 by 4.
Subtract 7 from 15.

EXAMPLE

4 Real-Life Application



You buy foam spheres, paint bottles, and wooden rods to construct a model of our solar system. What is your total cost?

Item	Quantity	Cost per Item
Spheres	9	\$2
Paint	6	\$3
Rods	8	\$1

Use a verbal model to solve the problem.

cost of 9 spheres+ cost of 6 paint bottles+ cost of 8 rods $9 \cdot 2$ + $6 \cdot 3$ + $8 \cdot 1$ $9 \cdot 2 + 6 \cdot 3 + 8 \cdot 1 = 18 + 18 + 8$ Multiply.= 44Add.

• Your total cost is \$44.



Evaluate the expression.

On Your Own

- **7.** $50 + 6(12 \div 4) 8^2$ **8.** $5^2 5(10 5)$ **9.** $\frac{8(3 + 4)}{7}$
- **10. WHAT IF?** In Example 4, you add the dwarf planet Pluto to your model. Use a verbal model to find your total cost assuming you do not need more paint. Explain.



Vocabulary and Concept Check

- **1. WRITING** Why does $12 8 \div 2 = 8$, but $(12 8) \div 2 = 2$?
- **2. REASONING** Describe the steps in evaluating the expression $8 \div (6 4) + 3^2$.

Practice and Problem Solving

Find the value of the expression.

3. $(4 \times 15) - 3$ **4.** 10 - (7 + 1) **5.** $18 \div (6 + 3)$

Evaluate the expression.

1 2 6. $5 + 18 \div 6$	7. $(11-3) \div 2 + 1$	8. 45 ÷ 9 × 12
9. $6^2 - 3 \cdot 4$	10. $42 \div (15 - 2^3)$	11. $4^2 \cdot 2 + 8 \cdot 7$
12. $3^2 + 12 \div (6 - 3) \times 8$	13. (10 + 4) ÷ (26 - 19)	14. $(5^2 - 4) \cdot 2 - 18$

ERROR ANALYSIS Describe and correct the error in evaluating the expression.



17. POETRY You need to read 20 poems in 5 days for an English project. Each poem is 2 pages long. Evaluate the expression $20 \times 2 \div 5$ to find how many pages you need to read each day.

Evaluate the expression.

- **B 18.** $9^2 8(6+2)$ **19.** $(3-1)^3 + 7(6) - 5^2$ **20.** $8\left(1\frac{1}{6} + \frac{5}{6}\right) \div 4$ **21.** $7^2 - 2\left(\frac{11}{8} - \frac{3}{8}\right)$ **22.** $8(7.3 + 3.7) - 14 \div 2$ **23.** $2^4(5.2 - 3.2) \div 4$
 - **24. MONEY** You have four \$10 bills and eighteen \$5 bills in your piggy bank. How much money do you have?
 - **25. THEATER** Before a show, there are 8 people in a theater. Five groups of 4 people enter, and then three groups of 2 people leave. Evaluate the expression 8 + 5(4) 3(2) to find how many people are in the theater.



Evaluate the expression.

26. $\frac{6(3+5)}{4}$

27.
$$\frac{12^2 - 4(6) + 1}{11^2}$$

28.
$$\frac{26 \div 2 + 5}{3^2 - 3}$$

- **29. FIELD TRIP** Eighty students are going on a field trip to a history museum. The total cost includes
 - 2 bus rentals and
 - \$10 per student for lunch.

What is the total cost per student?

30. OPEN-ENDED Use all four operations without parentheses to write an expression that has a value of 100.





- **31. SHOPPING** You buy 6 notebooks, 10 folders, 1 pack of pencils, and 1 lunch box for school. After using a \$10 gift card, how much do you owe? Explain how you solved the problem.
- **32. LITTER CLEANUP** Two groups collect litter along the side of a road. It takes each group 5 minutes to clean up a 200-yard section. How long does it take to clean up 2 *miles*? Explain how you solved the problem.

33. Sense Copy each statement. Insert +, -, \times , or \div symbols to make each statement true.

a.	27	3	5	2 = 19	b.	9 ²	11	8	4	1 = 60
c.	5	6	15	9 = 24	d.	14	2	7	3	9 = 10

R		Fair Gan	ne Review wh	nat you learned in previous	s grades & lessons
1	Add	or subtract.	(Skills Review Han	ndbook)	
	34.	5.2 + 0.5	35. 8 – 1.9	36. 12.6 – 3	37. 0.7 + 0.2
	38.	MULTIPLE CH	OICE You are makir	ng two recipes. One recip	e calls for $2\frac{1}{3}$ cups of
		flour. The otl	her recipe calls for 1	$\frac{1}{4}$ cups of flour. How much	ch flour do you need
		to make both	n recipes? <i>(Skills Re</i>	• view Handbook)	
		(A) $1\frac{1}{12}$ cup	$B = 3\frac{1}{12}$	cups $\bigcirc 3\frac{2}{7}c$	ups (D) $3\frac{7}{12}$ cups

Study Help



You can use an **information frame** to help you organize and remember concepts. Here is an example of an information frame for powers.



On Your Own

Make information frames to help you study these topics.

- **1.** adding whole numbers
- 2. subtracting whole numbers
- 3. multiplying whole numbers
- 4. dividing whole numbers
- 5. order of operations

After you complete this chapter, make information frames for the following topics.

- **6.** prime factorization
- 7. greatest common factor (GCF)
- 8. least common multiple (LCM)
- **9.** least common denominator (LCD)



"Dear Mom, I am sending you an information frame card for Mother's Day!"



- **15. DUATHLON** In an 18-mile duathlon, you run, then bike 12 miles, and then run again. The two runs are the same distance. Find the distance of each run. *(Section 1.3)*
- **16. AMUSEMENT PARK** Tickets for an amusement park cost \$10 for adults and \$6 for children. Find the total cost for 2 adults and 3 children. *(Section 1.3)*



Essential Question Without dividing, how can you tell when a number is divisible by another number?

ACTIVITY: Finding Divisibility Rules for 2, 3, 5, and 10

Work with a partner. Copy the set of numbers (1-50) as shown.

0	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	1 <i>8</i>	19	20
	21	22	23	24	25	26	27	28	29	30
	31	32	33	34	35	36	37	38	39	40
							· ·			
	41	42	43	44	45	46	47	48	49	50

- **a.** Highlight all the numbers that are divisible by 2.
- **b.** Put a box around the numbers that are divisible by 3.
- **c.** Underline the numbers that are divisible by 5.
- **d.** Circle the numbers that are divisible by 10.
- e. **STRUCTURE** In parts (a)–(d), what patterns do you notice? Write four rules to determine when a number is divisible by 2, 3, 5, and 10.



Common Factors

• use divisibility rules to

Preparing for Standard

find prime factorizations

and Multiples In this lesson, you will

of numbers.

6.NS.4

2 ACTIVITY: Finding Divisibility Rules for 6 and 9

Work with a partner.

- **a.** List ten numbers that are divisible by 6. Write a rule to determine when a number is divisible by 6. Use a calculator to check your rule with large numbers.
- **b.** List ten numbers that are divisible by 9. Write a rule to determine when a number is divisible by 9. Use a calculator to check your rule with large numbers.



ACTIVITY: Rewriting a Number Using 2s, 3s, and 5s

Work with three other students. Use the following rules and only the prime factors 2, 3, and 5 to write each number below as a product.



- Your group should have four sets of cards: a set with all 2s, a set with all 3s, a set with all 5s, and a set of blank cards. Each person gets one set of cards.
- Begin by choosing two cards to represent the given number as a product of two factors. The person with the blank cards writes any factors that are not 2, 3, or 5.
- Use the cards again to represent any number written on a blank card as a product of two factors. Continue until you have represented each handwritten card as a product of two prime factors.
- You may use only one blank card for each step.
- **Sample:** 108 a.



Compare your results with those of other groups. Are your steps the same e. for each number? Is your final answer the same for each number?

What Is Your Answer?

- 4. IN YOUR OWN WORDS Without dividing, how can you tell when a number is divisible by another number? Give examples to support your explanation.
- 5. Explain how you can use your divisibility rules from Activities 1 and 2 to help with Activity 3.



Use what you learned about divisibility rules to complete Exercises 4-7 on page 28.



1.4 Lesson



Because 2 is factor of 10 and $2 \cdot 5 = 10, 5$ is also a factor of 10. The pair 2, 5 is called a **factor pair** of 10.

EXAMPLE

Key Vocabulary ∎ factor pair, *p. 26* prime factorization, *p. 26* factor tree, *p. 26*

Study Tip 🖌

When making an organized list of factor pairs, stop finding pairs when the factors begin to repeat.



Remember 🗕

A prime number is a whole number greater than 1 with exactly two factors, 1 and itself. A composite number is a whole number greater than 1 with factors other than 1 and itself.

1 Finding Factor Pairs

The brass section of a marching band has 30 members. The band director arranges the brass section in rows. Each row has the same number of members. How many possible arrangements are there?

Use the factor pairs of 30 to find the number of arrangements.

$30 = 1 \cdot 30$	There could be 1 row of 30 or 30 rows of 1.
$30 = 2 \cdot 15$	There could be 2 rows of 15 or 15 rows of 2.
$30 = 3 \cdot 10$	There could be 3 rows of 10 or 10 rows of 3.
$30=5 \cdot 6$	There could be 5 rows of 6 or 6 rows of 5.
$30=6 \bullet 5$	The factors 5 and 6 are already listed.

There are 8 possible arrangements: 1 row of 30, 30 rows of 1, 2 rows of 15, 15 rows of 2, 3 rows of 10, 10 rows of 3, 5 rows of 6, or 6 rows of 5.

On Your Own

List the factor pairs of the number.

- **1.** 18 **2.** 24 **3.** 51
- **4. WHAT IF?** The woodwinds section of the marching band has 38 members. Which has more possible arrangements, the brass section or the woodwinds section? Explain.

🔎 Key Idea

Prime Factorization

The **prime factorization** of a composite number is the number written as a product of its prime factors.

You can use factor pairs and a **factor tree** to help find the prime factorization of a number. The factor tree is complete when only prime factors appear in the product. A factor tree for 60 is shown.

$$60$$
(2) • 30
(2) • 15
(3) • 5

$$60 = 2 • 2 • 3 • 5, \text{ or } 2^2 • 3 • 5$$

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Writing a Prime Factorization EXAMPLE 2

Write the prime factorization of 48.

Choose any factor pair of 48 to begin the factor tree.



The prime factorization of 48 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$, or $2^4 \cdot 3$. ÷.

Using a Prime Factorization **EXAMPLE** 3

What is the greatest perfect square that is a factor of 1575?

Because 1575 has many factors, it is not efficient to list all of its factors and check for perfect squares. Use the prime factorization of 1575 to find any perfect squares that are factors.

$$\begin{array}{r}
 1575 \\
 25 \cdot 63 \\
 \overline{5} \cdot \overline{5} \cdot \overline{7} \cdot 9 \\
 \overline{3} \cdot \overline{3}
 \end{array}$$

 $1575 = 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7$

The prime factorization shows that 1575 has three factors other than 1 that are perfect squares.

 $3 \cdot 3 = 9$ $5 \cdot 5 = 25$ $(3 \cdot 5) \cdot (3 \cdot 5) = 15 \cdot 15 = 225$

-So, the greatest perfect square that is a factor of 1575 is 225.

On Your Own

Now You're Ready
Exercises 16–23
and 29–32

Study Tip

Notice that beginning

pairs results in the same

with different factor

prime factorization. Every composite

number has only one

prime factorization.

Write the prime factorization of the number. **5**. 20 7. 90 **6.** 88 8. 462

9. What is the greatest perfect square that is a factor of 396? Explain.

1.4 Exercises

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Vocabulary and Concept Check

- 1. VOCABULARY What is the prime factorization of a number?
- **2. VOCABULARY** How can you use a factor tree to help you write the prime factorization of a number?
- **3.** WHICH ONE DOESN'T BELONG? Which factor pair does not belong with the other three? Explain your reasoning.



Practice and Problem Solving

Use divisibility rules to determine whether the number is divisible by 2, 3, 5, 6, 9, and 10. Use a calculator to check your answer.

	4.	1044	5.	1485	6.	1620	7.	1709
	List	the factor pairs of	the	number.				
1	8.	15	9.	22	10.	34	11.	39
	12.	45	13.	54	14.	59	15.	61
	Writ	e the prime factor	izati	ion of the number.				
2	16.	16	17.	25	18.	30	19.	26
	20.	84	21.	54	22.	65	23.	77

The prime factorization of

$$72$$

 2
 36
 $72 = 2 \cdot 2 \cdot 2 \cdot 9$
 $= 2^3 \cdot 9.$

- **24. ERROR ANALYSIS** Describe and correct the error in writing the prime factorization.
- **25. FACTOR RAINBOW** You can use a factor rainbow to check whether a list of factors is correct. To create a factor rainbow, list the factors of a number in order from least to greatest. Then draw arches that link the factor pairs. For perfect squares, there is no connecting arch in the middle. So, just circle the middle number. A factor rainbow for 12 is shown. Create factor rainbows for 6, 24, 36, and 48.



Find the number represented by the prime factorization.

27. $3^2 \cdot 5^2 \cdot 7$ **26.** $2^2 \cdot 3^2 \cdot 5$

28. $2^3 \cdot 11^2 \cdot 13$

Find the greatest perfect square that is a factor of the number.

- 3 29. 244 **30.** 650 **31.** 756 **32.** 1290
 - **33.** CRITICAL THINKING Is 2 the only even prime number? Explain.
 - 34. **BASEBALL** The coach of a baseball team separates the players into groups for drills. Each group has the same number of players. Is the total number of players on the baseball team prime or composite? Explain.
 - **35. SCAVENGER HUNT** A teacher divides 36 students into equal groups for a scavenger hunt. Each group should have at least 4 students but no more than 8 students. What are the possible group sizes?
 - **36. PERFECT NUMBERS** A *perfect number* is a number that equals the sum of its factors, not including itself. For example, the factors of 28 are 1, 2, 4, 7, 14, and 28. Because 1 + 2 + 4 + 7 + 14 = 28, 28 is a perfect number. What are the perfect numbers between 1 and 28?
 - 37. BAKE SALE One table at a bake sale has 75 oatmeal cookies. Another table has 60 lemon cupcakes. Which table allows for more rectangular arrangements when all the cookies and cupcakes are displayed? Explain.
 - **38. MODELING** The stage manager of a school play creates a rectangular acting area of 42 square yards. String lights will outline the acting area. To the nearest whole number, how many yards of string lights does the manager need to enclose this area?



39. Volume The volume of a rectangular prism can be found using the formula $volume = length \times width \times height$. Using only whole number dimensions, how many different prisms are possible? Explain.

Volume = 40 cubic inches

Fair Game Review What you learned in previous grades & lessons **Find the difference.** (*Skills Review Handbook*) **40.** 192 - 47 **41.** 451 – 94 **42.** 3210 - 815 **43.** 4752 - 3504 44. MULTIPLE CHOICE You buy 168 pears. There are 28 pears in each bag. How many bags of pears do you buy? (*Skills Review Handbook*) **B** 6 (\mathbf{A}) 5 **(C)** 7 (\mathbf{D}) 28

1.5 **Greatest Common Factor**

Essential Question How can you find the greatest common factor

of two numbers?

A Venn diagram uses circles to describe relationships between two or more sets. The Venn diagram shows the names of students enrolled in two activities. Students enrolled in both activities are represented by the overlap of the two circles.



ACTIVITY: Identifying Common Factors

Work with a partner. Copy and complete the Venn diagram. Identify the common factors of the two numbers.



Look at the Venn diagrams in parts (a)–(d). Explain how to identify the greatest e. common factor of each pair of numbers. Then circle it in each diagram.

COMMON CORE

Common Factors

- In this lesson, you will
- use diagrams to identify common factors.
- find greatest common factors.

Learning Standards 6.NS.4 6.EE.2b

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ACTIVITY: Interpreting a Venn Diagram of Prime Factors

Work with a partner. The Venn diagram represents the prime factorization of two numbers. Identify the two numbers. Explain your reasoning.



3 ACTIVITY: Identifying Common Prime Factors

Math Practice

Solution What does the diagram of the resulting prime factorization mean?

Work with a partner.

a. Write the prime factorizations of 36 and 48. Use the results to complete the Venn diagram.



- **b.** Repeat part (a) for the remaining number pairs in Activity 1.
- **c. STRUCTURE** Compare the numbers in the overlap of the Venn diagrams to your results in Activity 1. What conjecture can you make about the relationship between these numbers and your results in Activity 1?

-What Is Your Answer?

- **4. IN YOUR OWN WORDS** How can you find the greatest common factor of two numbers? Give examples to support your explanation.
- **5.** Can you think of another way to find the greatest common factor of two numbers? Explain.



Use what you learned about greatest common factors to complete Exercises 4–6 on page 34.

1.5 Lesson



Factors that are shared by two or more numbers are called **common factors**. The greatest of the common factors is called the **greatest common factor** (GCF). One way to find the GCF of two or more numbers is by listing factors.

EXAMPLE

Key Vocabulary

Venn diagram, p. 30

common factors,

greatest common

factor, p. 32

p. 32

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Find the GCF of 24 and 40.

List the factors of each number.

Factors of 24:(1),(2), 3,(4), 6,(8), 12, 24

Finding the GCF Using Lists of Factors

Circle the common factors.

Factors of 40:1,2,4, 5,8, 10, 20, 40

The common factors of 24 and 40 are 1, 2, 4, and 8. The greatest of these common factors is 8.

So, the GCF of 24 and 40 is 8.

Another way to find the GCF of two or more numbers is by using prime factors. The GCF is the product of the common prime factors of the numbers.

EXAMPLE 2 Finding the GCF Using Prime Factorizations

Find the GCF of 12 and 56.

Make a factor tree for each number.





Write the prime factorization of each number.

• So, the GCF of 12 and 56 is 4.

On Your Own

Now You're Ready	
Exercises 7–18	

Find the GCF of the numbers using lists of factors.						
1.	8, 36	2.	18, 72	3.	14, 28, 49	
Find the GCF of the numbers using prime factorizations.						
4.	20, 45	5.	32,90	6.	45, 75, 120	

Study Tip

Examples 1 and 2 show two different methods for finding the GCF. After solving with one method, you can use the other method to check your answer.

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EXAMPLE

Finding Two Numbers with a Given GCF

Which pair of numbers has a GCF of 15?

(A) 10, 15 (B) 30, 60 (C) 21, 45 (D) 45, 75

The number 15 cannot be a factor of the lesser number 10. So, you can eliminate Statement A.

The number 15 cannot be a factor of a number that does not have a 0 or 5 in the ones place. So, you can eliminate Statement C.

List the factors for Statements B and D. Then identify the GCF for each.

Choice B: Factors of 30:(1),(2),(3),(5),(6),(10),(15),(30)

Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

The GCF of 30 and 60 is 30.

Choice D: Factors of 45:(1),(3),(5), 9,(15), 45

Factors of 75:1,3,5,15, 25, 75

The GCF of 45 and 75 is 15.

• The correct answer is \bigcirc .

Real-Life Application

EXAMPLE

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You are filling piñatas for your sister's birthday party. The list shows the gifts you are putting into the piñatas. You want identical groups of gifts in each piñata with no gifts left over. What is the greatest number of piñatas you can make?

The GCF of the numbers of gifts represents the greatest number of identical groups of gifts you can make with no gifts left over. So, to find the number of piñatas, find the GCF.

$$18 = 2 \cdot 3 \cdot 3$$
$$24 = 2 \cdot 3 \cdot 2 \cdot 2$$
$$42 = 2 \cdot 3 \cdot 7$$

 $2 \cdot 3 = 6$ Find the product of the common prime factors.

The GCF of 18, 24, and 42 is 6.

• So, you can make at most 6 piñatas.

On Your Own

- 7. Write a pair of numbers whose greatest common factor is 10.
- **8. WHAT IF?** In Example 4, you add 6 more pairs of earrings. Does this change your answer? Explain your reasoning.

1.5 Exercises



Vocabulary and Concept Check

- 1. VOCABULARY What is the greatest common factor (GCF) of two numbers?
- 2. WRITING Describe how to find the GCF of two numbers by using prime factorization.
- 3. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

What is the greatest common factor of 24 and 32?

What is the greatest common divisor of 24 and 32?

What is the greatest prime factor of 24 and 32?

What is the product of the common prime factors of 24 and 32?

> Practice and Problem Solving

Use a Venn diagram to find the greatest common factor of the numbers.

4. 12, 30	5. 32, 54	6. 24, 108
Find the GCF of the n	numbers using lists of factors.	
1 7. 6, 15	8. 14, 84	9. 45, 76
10. 39, 65	11. 51, 85	12. 40, 63
Find the GCF of the n	umbers using prime factorization	18.
2 13. 45, 60	14. 27, 63	15. 36, 81
16. 72, 84	17. 61, 73	18. 189, 200

ERROR ANALYSIS Describe and correct the error in finding the GCF.

19. $42 = 2 \cdot 3 \cdot 7$ $154 = 2 \cdot 7 \cdot 11$ The GCF is 7.

 $36 = 2^{2} \cdot 3^{2}$ $60 = 2^{2} \cdot 3 \cdot 5$ The GCF is 2 \cdot 3 = 6.

- **21. CLASSROOM** A teacher is making identical activity packets using 92 crayons and 23 sheets of paper. What is the greatest number of packets the teacher can make with no items left over?
- **22. BALLOONS** You are making balloon arrangements for a birthday party. There are 16 white balloons and 24 red balloons. Each arrangement must be identical. What is the greatest number of arrangements you can make using every balloon?

20.

Find the GCF of the numbers.

4 23. 35, 56, 63

24. 30, 60, 78

25. 42, 70, 84

- **26. OPEN-ENDED** Write a set of three numbers that have a GCF of 16. What procedure did you use to find your answer?
- **27. REASONING** You need to find the GCF of 256 and 400. Would you rather list their factors or use their prime factorizations? Explain.

CRITICAL THINKING Tell whether the statement is always, sometimes, or never true.

- **28.** The GCF of two even numbers is 2.
- **29.** The GCF of two prime numbers is 1.
- **30.** When one number is a multiple of another, the GCF of the numbers is the greater of the numbers.
- **31. BOUQUETS** A florist is making identical bouquets using 72 red roses, 60 pink roses, and 48 yellow roses. What is the greatest number of bouquets that the florist can make if no roses are left over? How many of each color are in each bouquet?
- **32.** VENN DIAGRAM Consider the numbers 252, 270, and 300.
 - **a.** Create a Venn diagram using the prime factors of the numbers.
 - **b.** Use the Venn diagram to find the GCF of 252, 270, and 300.
 - **c.** What is the GCF of 252 and 270? 252 and 300? Explain how you found your answer.
- **33. FRUIT BASKETS** You are making fruit baskets using 54 apples, 36 oranges, and 73 bananas.
 - a. Explain why you cannot make identical fruit baskets without leftover fruit.
 - **b.** What is the greatest number of identical fruit baskets you can make with the least amount of fruit left over? Explain how you found your answer.
- **34.** Two rectangular, adjacent rooms share a wall. One-foot-by-one-foot tiles cover the floor of each room. Describe how the greatest possible length of the adjoining wall is related to the total number of tiles in each room. Draw a diagram that represents one possibility.



àdjoining wall

Fair Game Review What you learned in previous grades & lessons

Tell which property is being illustrated. (Skills Review Handbook)

35. 13 + (29 + 7) = 13 + (7 + 29)**36.** 13 + (7 + 29) = (13 + 7) + 29**37.** $(6 \times 37) \times 5 = (37 \times 6) \times 5$ **38.** $(37 \times 6) \times 5 = 37 \times (6 \times 5)$ **39.** MULTIPLE CHOICE In what order should you perform the operations in the

expression $4 \times 3 - 12 \div 2 + 5$? (Section 1.3) (A) $\times, -, \div, +$ (B) $\times, \div, -, +$ (C) $\times, \div, +, -$ (D) $\times, +, -, \div$


Essential Question How can you find the least common multiple

of two numbers?

ACTIVITY: Identifying Common Multiples

Work with a partner. Using the first several multiples of each number, copy and complete the Venn diagram. Identify any common multiples of the two numbers.



e. Look at the Venn diagrams in parts (a)–(d). Explain how to identify the *least* common multiple of each pair of numbers. Then circle it in each diagram.

COMMON CORE **Common Multiples**

In this lesson, you will

• find least common multiples. Learning Standard

6.NS.4

common multiples.

ACTIVITY: Interpreting a Venn Diagram of Prime Factors

Work with a partner.

- **a.** Write the prime factorizations of 8 and 12. Use the results to complete the Venn diagram.
- **b.** Repeat part (a) for the remaining number pairs in Activity 1.
- **c. STRUCTURE** Compare the numbers from each section of the Venn



diagrams to your results in Activity 1. What conjecture can you make about the relationship between these numbers and your results in Activity 1?

What Is Your Answer?

- **3. IN YOUR OWN WORDS** How can you find the least common multiple of two numbers? Give examples to support your explanation.
- 4. The Venn diagram shows the prime factors of two numbers.



Use the diagram to do the following tasks.

- **a.** Identify the two numbers.
- **b.** Find the greatest common factor.
- **c.** Find the least common multiple.
- **5.** A student writes the prime factorizations of 8 and 12 in a table as shown. She claims she can use the table to find the greatest common factor and the least common multiple of 8 and 12. How is this possible?

9 -	2	2	2		
0 - 17 =	2	2	2	3	
16 -	-	-			

6. Can you think of another way to find the least common multiple of two or more numbers? Explain.



Use what you learned about least common multiples to complete Exercises 3–5 on page 40.



Construct Arguments

How can you use diagrams to support your explanation?

1.6 Lesson



Multiples that are shared by two or more numbers are called **common** multiples. The least of the common multiples is called the least common **multiple** (LCM). You can find the LCM of two or more numbers by listing multiples or using prime factors.

EXAMPLE



Finding the LCM Using Lists of Multiples

Find the LCM of 4 and 6.

List the multiples of each number.

Multiples of 4: 4, 8, **(2)**, 16, 20, **24**, 28, 32, **36**, ... Circle the common multiples. **Multiples of 6:** 6, (12), 18, (24), 30, (36), ...

Some common multiples of 4 and 6 are 12, 24, and 36. The least of these common multiples is 12.

So, the LCM of 4 and 6 is 12. -

On Your Own



eady	Find the LCM of	the numbers using lists of	nultiples.
1	1. 3, 8	2. 9, 12	3. 6, 10



Find the LCM of 16 and 20.

Make a factor tree for each number.





Write the prime factorization of each number. Circle each different factor where it appears the greater number of times.

2 appears more often here, so circle all 2s.

$20 = 2 \cdot 2 \cdot 5$	5 appears once. Do not circle the 2s again.
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 = 80$	Find the product of the circled factors.

Find the product of the circled factors.

So, the LCM of 16 and 20 is 80.

On Your Own



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EXAMPLE 3 Finding the LCM of Three Numbers

Find the LCM of 4, 15, and 18.

Write the prime factorization of each number. Circle each different factor where it appears the greatest number of times.

$4 = 2 \cdot 2$	2 appears most often here, so circle both 2s.
$15 = 3 \cdot 5$	5 appears here only, so circle 5.
$18 = 2 \cdot 3 \cdot 3$	3 appears most often here, so circle both 3s.
$2 \cdot 2 \cdot 5 \cdot 3 \cdot 3 = 180$	Find the product of the circled factors.

So, the LCM of 4, 15, and 18 is 180.

On Your Own



Find the LCM of the numbers.

- **7.** 2, 5, 8
- **8.** 6, 10, 12
- 9. Write a set of numbers whose least common multiple is 100.

EXAMPLE 4 Real-Life Application



A traffic light changes every 30 seconds. Another traffic light changes every 40 seconds. Both lights just changed. After how many minutes will both lights change at the same time again?

Find the LCM of 30 and 40 by listing multiples of each number. Circle the least common multiple.

Multiples of 30: 30, 60, 90, 120, . . .

Multiples of 40: 40, 80, 120, 160, ...

The LCM is 120. So, both lights will change again after 120 seconds.

Because there are 60 seconds in 1 minute, there are $120 \div 60 = 2$ minutes in 120 seconds.

Both lights will change at the same time again after 2 minutes.

👂 On Your Own

10. WHAT IF? In Example 4, the traffic light that changes every 40 seconds is adjusted to change every 45 seconds. Both lights just changed. After how many minutes will both lights change at the same time again?







Vocabulary and Concept Check

- 1. VOCABULARY What is the least common multiple (LCM) of two numbers?
- **2. WRITING** Describe how to find the LCM of two numbers by using prime factorization.



Use a Venn diagram to find the least common multiple of the numbers.

3. 3, 7	4. 6, 8	5. 12, 15
Find the LCM of the n	numbers using lists of multiples.	
1 6. 2, 9	7. 3, 4	8. 8, 9
9. 5, 8	10. 15, 20	11. 12, 18
Find the LCM of the n	numbers using prime factorization	IS.
2 12. 9, 21	13. 12, 27	14. 18, 45
15. 22, 33	16. 36, 60	17. 35, 50

 $6 \times 9 = 54$ The LCM of 6 and 9 is 54.

- **18. ERROR ANALYSIS** Describe and correct the error in finding the LCM.
- **19. AQUATICS** You have diving lessons every fifth day and swimming lessons every third day. Today you have both lessons. In how many days will you have both lessons on the same day again?
- **20. HOT DOGS** Hot dogs come in packs of 10, while buns come in packs of eight. What are the least numbers of packs you should buy in order to have the same numbers of hot dogs and buns?



21. MODELING Which model represents an LCM that is different from the other three? Explain your reasoning.



Find the LCM of the numbers.

1

3 22.	2, 3, 7	23. 3, 5, 11	24. 4, 9, 12
25.	6, 8, 15	26. 7, 18, 21	27. 9, 10, 28

28. REASONING You need to find the LCM of 13 and 14. Would you rather list their multiples or use their prime factorizations? Explain.

CRITICAL THINKING Tell whether the statement is *always*, sometimes, or never true.

- **29.** The LCM of two different prime numbers is their product.
- **30.** The LCM of a set of numbers is equal to one of the numbers in the set.
- **31.** The GCF of two different numbers is the LCM of the numbers.
- **32. SUBWAY** At Union Station, you notice that three subway lines just arrived at the same time. The table shows their arrival schedule. How long must you wait until all three lines arrive at Union Station at the same time again?

Subway Line	Arrival Time
А	every 10 min
В	every 12 min
С	every 15 min

- **33. RADIO CONTEST** A radio station gives away \$15 to every 15th caller, \$25 to every 25th caller, and free concert tickets to every 100th caller. When will the station first give away *all* three prizes to one caller?
- **34. TREADMILL** You and a friend are running on treadmills. You run 0.5 mile every 3 minutes, and your friend runs 2 miles every 14 minutes. You both start and stop running at the same time and run a whole number of miles. What is the least possible number of miles you and your friend can run?



36.

- **35. VENN DIAGRAM** Refer to the Venn diagram.
 - **a.** Copy and complete the Venn diagram.
 - **b.** What is the LCM of 16, 24, and 40?
 - **c.** What is the LCM of 16 and 40? 24 and 40?

Sense When is the LCM of two numbers equal to their product?

Fair Game	Review What you learned in	previous grades & le	essons
Write the product a	s a power. (Section 1.2)		
37. 3 × 3	38. 5 • 5 • 5 • 5	39. 17 2	$\times 17 \times 17 \times 17 \times 17$
40. MULTIPLE CHOI (A) 1^3 and 3^1	CE Which two powers have the sa (\mathbf{B}) 2 ⁴ and 4 ² (\mathbf{G})		D 4^3 and 3^4



Key Vocabulary least common denominator, p. 42 Recall that you can add and subtract fractions with unlike denominators by writing equivalent fractions with a common denominator. One way to do this is by multiplying the numerator and the denominator of each fraction by the denominator of the other fraction.

EXAMPLE

Adding Fractions Using a Common Denominator 1

Find
$$\frac{5}{8} + \frac{1}{6}$$
.

Rewrite the fractions with a common denominator. Use the product of the denominators as the common denominator.

$\frac{5}{8} + \frac{1}{6} = \frac{5 \cdot 6}{8 \cdot 6} + \frac{1 \cdot 8}{6 \cdot 8}$	Rewrite the fractions using a common denominator of $8 \cdot 6 = 48$.
$=\frac{30}{48}+\frac{8}{48}$	Multiply.
$=\frac{38}{48}$	Add the numerators.
$=\frac{1}{2}\frac{\cancel{2}\cdot19}{\cancel{2}\cdot24}$	Divide out the common factor 2.
$=\frac{19}{24}$	Simplify.

The **least common denominator** (LCD) of two or more fractions is the least common multiple (LCM) of the denominators. The LCD provides another method for adding and subtracting fractions with unlike denominators.

Adding Fractions Using the LCD EXAMPLE 2

Find
$$\frac{5}{8} + \frac{1}{6}$$
.

Find the LCM of the denominators.

Multiples of 8: 8, 16, 24, 32, 40, 48, ...

The LCM of 8 and 6 is 24. So, the LCD is 24.

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, ...

COMMON CORE

In this extension, you will use least common multiples to add and subtract fractions. **Applying Standard**

Common Multiples

6.NS.4



Rewrite the fractions using the LCD, 24.

Multiply.

Add the numerators.



A fraction is in simplest form when the numerator and the denominator have no common factors other than 1.

To add or subtract mixed numbers, first rewrite the numbers as improper fractions. Then find the common denominator.

EXAMPLE 3 Subtracting Mixed Numbers

Find
$$4\frac{3}{4} - 2\frac{3}{10}$$
.

Write the difference using improper fractions.

$$4\frac{3}{4} - 2\frac{3}{10} = \frac{19}{4} - \frac{23}{10}$$

Method 1: Use the product of the denominators as the common denominator.

$\frac{19}{4} - \frac{23}{10} = \frac{19 \cdot 10}{4 \cdot 10} - \frac{23 \cdot 4}{10 \cdot 4}$	Rewrite the fractions using a common denominator of $4 \cdot 10 = 40$.
$=\frac{190}{40}-\frac{92}{40}$	Multiply.
$=\frac{98}{40}$	Subtract the numerators.
$=\frac{49}{20}$, or $2\frac{9}{20}$	Simplify.

Method 2: Use the LCD. The LCM of 4 and 10 is 20.

$$\frac{19}{4} - \frac{23}{10} = \frac{19 \cdot 5}{4 \cdot 5} - \frac{23 \cdot 2}{10 \cdot 2}$$
Rewrite the fractions
using the LCD, 20.
$$= \frac{95}{20} - \frac{46}{20}$$
Multiply.
$$= \frac{49}{20}, \text{ or } 2\frac{9}{20}$$
Simplify.

Practice

Use the LCD to rewrite the fractions with the same denominator.

1. $\frac{1}{6}, \frac{3}{8}$ 2. $\frac{7}{7}, \frac{3}{10}$ 3. $\frac{3}{12}, \frac{2}{9}$ 4. $\frac{3}{4}, \frac{3}{8}, \frac{3}{8}$
--

Copy and complete	e the statement using <	<i>z</i> , >, or =.		
5. $\frac{4}{5}$ $\frac{5}{6}$	6. $\frac{5}{14}$ $\frac{3}{8}$	7. $2\frac{2}{5}$ $\frac{24}{10}$	8. $4\frac{9}{25}$ $4\frac{7}{20}$	
Add or subtract. Write the answer in simplest form.				
0 0	C 1	7 -	10 5	

9. $\frac{2}{3} + \frac{3}{4}$	10. $\frac{6}{7} + \frac{1}{2}$	11. $\frac{7}{10} - \frac{5}{12}$	12. $\frac{13}{18} - \frac{5}{8}$
13. $2\frac{1}{6} + 3\frac{4}{9}$	14. $4\frac{3}{16} + 1\frac{1}{10}$	15. $1\frac{5}{6} - \frac{3}{4}$	16. $3\frac{2}{3} - 2\frac{4}{11}$

17. COMPARING METHODS List some advantages and disadvantages of each method shown in the examples. Which method do you prefer? Why?



in Example 1. You can generalize the procedure using

 $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}.$

the rule

1.4–1.6 Quiz



List the factor pairs of the number. (Section	1.4) B		
1. 48	2. 56		
Write the prime factorization of the number.	(Section 1.4)		
3. 60	4. 72		
Find the GCF of the numbers using lists of fac	ctors. (Section 1.5)		
5. 18, 42	6. 24, 44, 52		
Find the GCF of the numbers using prime fac	torizations. (Section 1.5)		
7. 38, 68	8. 68, 76, 92		
Find the LCM of the numbers using lists of multiples. (Section 1.6)			
9. 8, 14	10. 3, 6, 16		
Find the LCM of the numbers using prime factorizations. (Section 1.6)			
11. 18, 30	12. 6, 24, 32		

Add or subtract. Write the answer in simplest form. (Section 1.6)

13. $\frac{3}{-} + \frac{2}{-}$	$\frac{3}{2} + \frac{2}{2}$	14. $\frac{7}{-}-\frac{3}{-}$			
	5 3	• ••	8	4	



- **15. PICNIC BASKETS** You are creating identical picnic baskets using 30 sandwiches and 42 apples. What is the greatest number of baskets that you can fill using all of the food? (Section 1.5)
- **16. RIBBON** You have 52 inches of yellow ribbon and 64 inches of red ribbon. You want to cut the ribbons into pieces of equal length with no leftovers. What is the greatest length of the pieces that you can make? (Section 1.5)
- 17. MUSIC LESSONS You have piano lessons every fourth day and guitar lessons every sixth day. Today you have both lessons. In how many days will you have both lessons on the same day again? Explain. (Section 1.6)
- **18. HAMBURGERS** Hamburgers come in packs of 20, while buns come in packs of 12. What is the least number of packs you should buy in order to have the same numbers of hamburgers and buns? (Section 1.6)



Review Key Vocabulary

power, p. 12 base, p. 12 exponent, p. 12 perfect square, p. 13 numerical expression, p. 18 evaluate, p. 18 order of operations, p. 18 factor pair, p. 26 prime factorization, p. 26 factor tree, p. 26 Venn diagram, p. 30 common factors, p. 32 greatest common factor (GCF), p. 32



common multiples, *p. 38* least common multiple (LCM), *p. 38* least common denominator (LCD), *p. 42*

Review Examples and Exercises

1.1

Whole Number Operations (pp. 2–9)

Use the tens place because 203 is less than 508.

2 203 5081 Divide 508 by 203: There are two groups of 203 in 508. - 406 Multiply 2 and 203. Subtract 406 from 508.

Next, bring down the 1 and divide the ones.

6

	25	R
203	5081	
_	406	
	1021	
—	1015	
	6	

Divide 1021 by 203: There are five groups of 203 in 1021.

Multiply 5 and 203. Subtract 1015 from 1021.

 \therefore The quotient of 5081 and 203 is $25\frac{6}{203}$

Exercises

Find the value of the expression. Use estimation to check your answer.

1. 4382 + 2899	2. 8724 - 3568

3. 192 × 38 **4.** 216 ÷ 31

1.2 **Powers and Exponents** (pp. 10–15)

Evaluate 6².

 $6^2 = 6 \cdot 6 = 36$

Write as repeated multiplication and simplify.

Exercises

Find the value of the power.

5. 7³

7. 4⁴

1.3 **Order of Operations** (pp. 16–21)

Evaluate $4^3 - 15 \div 5$. $4^3 - 15 \div 5 = 64 - 15 \div 5$ Evaluate 4^3 . = 64 - 3= 61

Divide 15 by 5. Subtract 3 from 64.

Exercises

Evaluate the expression.

8. $3 \times 6 - 12 \div 6$

9. $20 \times (3^2 - 4) \div 50$ **10.** $5 + (4^2 + 2) \div 6$

Prime Factorization (pp. 24–29) 14



Exercises

List the factor pair	s of the number.			
11. 28	12. 44	13. 63		
Write the prime factorization of the number.				
14. 42	15. 50	16. 66		

1.5 **Greatest Common Factor** (pp. 30–35)

a. Find the GCF of 32 and 76.

Factors of 32: (1), (2), (4), 8, 16, 32 Factors of 76: 1,2,4, 19, 38, 76

The greatest of the common factors is 4.

So, the GCF of 32 and 76 is 4.



Exercises

Find the GCF of the numbers using lists of factors.			
17. 27, 45	18. 30, 48	19. 28, 48, 64	
Find the GCF of the numbers using prime factorizations.			
20. 24, 90	21. 52, 68	22. 32, 56, 96	

1.6 Least Common Multiple (pp. 36–43)

a. Find the LCM of 8 and 12.

Make a factor tree for each number.



Write the prime factorization of each number. Circle each different factor where it appears the greater number of times.

$8 = (2) \cdot (2) \cdot (2)$
$12 = 2 \cdot 2 \cdot 3$
$2 \cdot 2 \cdot 2 \cdot 3 = 24$

2 appears more often here, so circle all 2s.3 appears once. Do not circle the 2s again.

Find the product of the circled factors.

So, the LCM of 8 and 12 is 24.

b. Find
$$\frac{1}{2} + \frac{1}{3}$$
.

The LCM of 2 and 3 is 6. So, the LCD is 6.

 $\frac{1}{2} + \frac{1}{3} = \frac{1 \cdot 3}{2 \cdot 3} + \frac{1 \cdot 2}{3 \cdot 2} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

Exercises

Find the LCM of the numbers using lists of multiples.				
23. 4, 14	24. 6, 20	25. 12, 28		
Find the LCM of the numb	ers using prime factorization	ons.		
26. 6, 45	27. 10, 12	28. 18, 27		
Add or subtract. Write the answer in simplest form.				
29. $\frac{2}{7} + \frac{1}{4}$	30. $\frac{5}{9} + \frac{3}{8}$	31. $3\frac{5}{6} - 2\frac{7}{15}$		
32. WATER PITCHER A water pitcher contains $\frac{2}{3}$ gallon of water. You add $\frac{5}{7}$ gallon				
of water to the pitcher. How much water does the pitcher contain?				

Find the value of the expres your answer.	sion. Use estimation to check	Test Practice BigIdeasMath
1. 3963 + 2379	2. 6184 – 2348	8
3. 184 × 26	4. 207 ÷ 23	
Find the value of the power		
5. 2 ³	6. 15 ²	7. 5 ⁴
Evaluate the expression.		
8. $11 \times 8 - 6 \div 2$	9. $5+2^3 \div 4-2$	10. $6 + 4(11 - 2) \div 3^2$
List the factor pairs of the n	umber.	
11. 52	12. 66	
Write the prime factorization	on of the number.	
13. 46	14. 28	
Find the GCF of the number	rs using lists of factors.	
15. 24, 54	16. 16, 32, 72	
Find the GCF of the number	rs using prime factorizations.	
17. 52, 65	18. 18, 45, 63	
Find the LCM of the numbe	rs using lists of multiples.	
19. 14, 21	20. 9, 24	
Find the LCM of the numbe	rs using prime factorizations.	
21. 26, 39	22. 6, 12, 14	

- **23. BRACELETS** You have 16 yellow beads, 20 red beads, and 24 orange beads to make identical bracelets. What is the greatest number of bracelets that you can make using all the beads?
- 24. MARBLES A bag contains equal numbers of green and blue marbles. You can divide all the green marbles into groups of 12 and all the blue marbles into groups of 16. What is the least number of each color of marble that can be in the bag?
- **25.** SCALE You place a $3\frac{3}{8}$ -pound weight on the left side of a balance scale and a $1\frac{1}{5}$ -pound weight on the right side. How much weight do you need to add to the right side to balance the scale?



Check It Out

Standards Assessment

- 1. You are making identical bagel platters using 40 plain bagels, 30 raisin bagels, and 24 blueberry bagels. What is the greatest number of platters that you can make if there are no leftover bagels? (6.NS.4)
 - **A.** 2 **C.** 8
 - **B.** 6 **D.** 10
- **2.** The top of an end table is a square with a side length of 16 inches. What is the area of the tabletop? *(6.EE.1)*





- F. 16 in.²
 H. 64 in.²

 G. 32 in.²
 I. 256 in.²
- **3.** Which number is equivalent to the expression below? (6.EE.1)

$$3 \cdot 2^3 - 8 \div 4$$

- **A.** 0 **C.** 22
- **B.** 4 **D.** 214
- 4. What is the least common multiple of 14 and 49? (6.NS.4)



- **5.** Which number is equivalent to the expression $7059 \div 301$? (6.NS.2)
 - F. 23
 H. $23\frac{136}{301}$

 G. $23\frac{136}{7059}$ I. 136
- **6.** You are building identical displays for the school fair using 65 blue boxes and 91 yellow boxes. What is the greatest number of displays you can build using all the boxes? (6.NS.4)
 - A. 13B. 35C. 91D. 156
- 7. You hang the two strands of decorative lights shown below.



Strand 1: changes between red and blue every 15 seconds



Strand 2: changes between green and gold every 18 seconds

H. 90 seconds

I. 270 seconds

Both strands just changed color. After how many seconds will the strands change color at the same time again? (6.NS.4)

- F. 3 seconds
- G. 30 seconds

8. Which expression is equivalent to $\frac{29}{63}$? (6.NS.4) A. $\frac{28}{60} + \frac{1}{3}$ C. $\frac{5}{21} + \frac{2}{9}$ B. $\frac{4}{27} + \frac{25}{36}$ D. $\frac{22}{47} + \frac{7}{16}$

- **9.** Which expression is *not* equivalent to 32? (6.EE.1)
 - F. $6^2 8 \div 2$ H. $30 + 4^2 \div (2 + 6)$ G. $30 \div 2 + 5^2 8$ I. $8^2 \div 4 2$
- **10.** Which number is equivalent to the expression 148×27 ? (6.NS.2)

Α.	3696	C.	3946
В.	3896	D.	3996



11. You have 60 nickels, 48 dimes, and 42 quarters. You want to divide the coins into identical groups with no coins left over. What is the greatest number of groups that you can make? (6.NS.4)

12. Erica was evaluating the expression in the box below.

$$56 \div (2^{3} - 1) \times 4 = 56 \div (8 - 1) \times 4$$
$$= 56 \div 7 \times 4$$
$$= 56 \div 28$$
$$= 2$$

What should Erica do to correct the error that she made? (6.EE.1)

- **F.** Divide 56 by 8 because operations are performed left to right.
- **G.** Multiply 1 by 4 because multiplication is done before subtraction.
- H. Divide 56 by 7 because operations are performed left to right.
- **I.** Divide 56 by 8 and multiply 1 by 4 because division and multiplication are performed before subtraction.
- **13.** Find the greatest common factor for each pair of numbers.

Think Solve	10 and 15	10 and 21	15 and 21
Explain	What can you conclude about the greatest common factor of 10, 15, and 21? Explain your reasoning. <i>(6.NS.4)</i>		
14.	14. Which number is <i>not</i> a perfect square? (6.EE.1)		
	A. 64		C. 96
	B. 81		D. 100
15.	Which number pair ha F. 4, 12 G. 6, 8	as a least common	multiple of 48? <i>(6.NS.4)</i> H. 8, 24 I. 16, 24
16. Which number is equivalent to the expression below? (6.EE.1) $\frac{3(6+2^2)+2}{8}$			
	A. 3		C. 7
	B. 4		D. $24\frac{1}{4}$