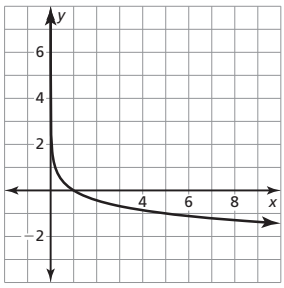
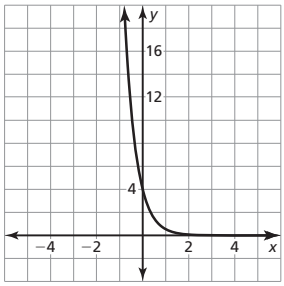
Chapter 6

2. 

The domain is $x > 0$, the range is all real numbers, and the asymptote is $x = 0$.

3. 

The domain is all real numbers, the range is $y > 0$, and the asymptote is $y = 0$.

4. The graph of g is a reflection in the x-axis followed by a translation 4 units right of the graph of f. So, $g(x) = -\log(x - 4)$.

5. The graph of g is a reflection in the y-axis followed by a translation 2 units up of the graph of f. So, $g(x) = e^{-x} + 2$.

6. The graph of g is a vertical stretch by a factor of 2 of the graph of f. So, $g(x) = 2\left(\frac{1}{4}\right)^x$.

7. $\log_3 52 = \log_3 4 + \log_3 13$
$\approx 1.262 + 2.335$
$= 3.597$

8. $\log_3 \frac{13}{9} = \log_3 13 - \log_3 9$
$= \log_3 13 - 2$
$\approx 2.335 - 2$
$= 0.335$

9. $\log_3 16 = 2 \log_3 4$
$\approx 2(1.262)$
$= 2.524$

10. $\log_3 8 + \log_3 \frac{1}{2} = \log_3 2 + \log_3 4 - \log_3 2$
$= \log_3 4$
≈ 1.262

11. To solve the first equation use a logarithm on both sides and to solve the second equation use exponentials on both sides.

$4^{5x-2} = 16$ $\qquad$ $\log_4(10x + 6) = 1$
$\log_4 4^{5x-2} = \log_4 16$ $\qquad$ $4^{\log_4(10x+6)} = 4^1$
$5x - 2 = 2$ $\qquad$ $10x + 6 = 4$
$5x = 4$ $\qquad$ $10x = -2$
$x = \frac{4}{5}$ $\qquad$ $x = -\frac{1}{5}$

12. All three are equivalent by the change-of-base formula.

13. One thousand wells will produce
$y = 12.263 \ln 1000 - 45.381 \approx 39$ billion barrels.

$y = 12.263 \ln x - 45.381$
$x = 12.263 \ln y - 45.381$
$x + 45.381 = 12.263 \ln y$
$\frac{x + 45.381}{12.263} = \ln y$
$e^{\frac{x+45.381}{12.263}} = y$
$e^{(x/12.263) + (45.381/12.263)} = y$
$(e^{x/12.263})(e^{45.381/12.263}) = y$
$(e^{1/12.263})^x (e^{45.381/12.263}) = y$
$40.473(1.085)^x \approx y$

The function $y \approx 40.473(1.085)^x$ represents the number of wells needed to produce a certain number of billions of barrels of oil.

14. a. The function is $L(x) = 100e^{-0.02x}$.

b. The function in part (a) represents exponential decay since the base, $e^{-0.02} \approx 0.98$, is greater than 0 and less than 1.

c. When the depth is 40 meters, the percent of surface light is $L(40) = 100\, e^{-0.02(40)} \approx 44.9$ or about 44.9%.

15. *Sample answer:* The three ways to find the exponential model are:

1. Use two points and the model $y = ab^x$ to determine the values of a and b.

2. Convert the pairs to $(x, \ln y)$, then solve the related linear equation for y.

3. Enter the points into a graphing calculator and perform exponential regression.

The model is $y = 4200\ (0.89)^x$ and the snowmobile is worth \$2500 in about 4.5 years.

Chapter 6 Standards Assessment (pp. 354–355)

1. The possible values for b are 0.94 and $e^{-1/2}$.

2. Your friend's claim is correct, interest compounded continuously produces the most interest when compound interest is used.

Chapter 6

3. a. $T(x) = (x)(4x)(2x) = 8x^3$

b. $C(x) = (x - 2)(4x - 2)(2x - 4)$
$= 8x^3 - 36x^2 + 48x - 16$

c. The relationship is $I(x) = T(x) - C(x)$.

d. $I(x) = T(x) - C(x)$
$= (8x^3) - (8x^3 - 36x^2 + 48x - 16)$
$= 36x^2 - 48x + 16$

The volume of the insulation is $I(8) = 36(8)^2 - 48(8) + 16 = 1936$ cubic inches when the width is 8 inches.

4. $-4 \log_2 x \geq -20$

$\log_2 x \leq 5$

$2^{\log_2 x} \leq 2^5$

$x \leq 32$

Because $\log_2 x$ is only defined when $x > 0$, the solution is $0 < x \leq 32$, which is choice C.

5. The graph of g is a reflection in the y-axis followed by a translation 2 units down of the graph of f.

6. $(f + g)(x) = f(x) + g(x) = -3x^4 - 4x^3 - 4x^2 + 10x + 4$

$(hg)(x) = h(x)\, g(x)$
$= -3x^6 - 9x^5 - 15x^4 + 44x^3 + 7x^2 - 9x - 35$

$(h - f)(x) = h(x) - f(x) = -2x^3 + 5x^2 - 7x - 6$

$(fh)(x) = f(x)\, h(x) = 2x^5 - 2x^4 - 10x^3 - 35x^2 - 57x + 7$

The order of the polynomials from least degree to greatest degree is C, A, D, B.

7. a. Substitute the coordinates of two points, such as (2, 20) and (3, 40), into the model $y = ab^x$.

$20 = ab^2$

$40 = ab^3$

Solve for a in the first equation to obtain $a = \frac{20}{b^2}$, then $b = 2$ and $a = 5$. So, the exponential model is $y = 5(2)^x$.

b. Substitute the coordinates of two points, such as (2, 4.5) and (3, 13.5), into the model $y = ab^x$.

$4.5 = ab^2$

$13.5 = ab^3$

Solve for a in the first equation to obtain $a = \frac{4.5}{b^2}$, then $b = 3$ and $a = \frac{1}{2}$. So, the exponential model is $y = \frac{1}{2}(3)^x$.

The equation in part (a) has a larger y-intercept and grows at a rate of 100%. The equation in part (b) has a smaller y-intercept but grows at a faster rate of 200% and will become larger than the equation in part (a) for $x > 5.7$.

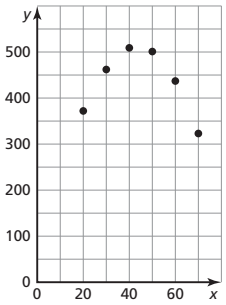
8. a. *Sample answer:* Use the Quadratic Formula because the polynomial $x^2 + 4x - 10$ does not factor.

b. *Sample answer:* Use the Square Root method because there is only one term with a variable.

c. *Sample answer:* Use the Quadratic Formula because the resulting polynomial does not factor.

d. *Sample answer:* Use the factoring method because $x^2 - 3x - 18$ factors.

9.



The scatter plot shows a quadratic relationship. Enter the data into a graphing calculator and perform quadratic regression. The model is $y = -0.261x^2 + 22.591x + 23.029$. When $y = 500$, the model estimates x to be about 50.0° or 36.5°.