Chapter 5

12. $(f+g)(x) = f(x) + g(x) = 6x^{3/5} - x^{3/5} = 5x^{3/5}$

The functions f and g each have the same domain: all real numbers. So, the domain of $f + g$ is all real numbers. When $x = 32$, the value of the sum is
$(f+g)(32) = 5(32)^{3/5} = 5(8) = 40.$

$(f-g)(x) = f(x) - g(x) = 6x^{3/5} + x^{3/5} = 7x^{3/5}$

The functions f and g each have the same domain: all real numbers. So, the domain of $f - g$ is all real numbers. When $x = 32$, the value of the difference is
$(f-g)(32) = 7(32)^{3/5} = 7(8) = 56.$

13. $(fg)(x) = f(x)g(x) = \left(\frac{1}{2}x^{3/4}\right)(8x) = 4x^{7/4}$

The domain of f is $x \geq 0$ and the domain of g is all real numbers. So, the domain of fg is $x \geq 0$. When $x = 16$, the value of the product is

$(fg)(16) = 4(16)^{7/4} = 4(128) = 512.$

$\left(\frac{f}{g}\right)(x) = \frac{\frac{1}{2}x^{3/4}}{8x} = \frac{1}{16x^{1/4}}$

The domain of f is $x \geq 0$ and the domain of g is all real numbers. So, the domain of $\frac{f}{g}$ consists of $x > 0$. When $x = 16$, the value of the quotient is

$\left(\frac{f}{g}\right)(16) = \frac{1}{16(16)^{1/4}} = \frac{1}{16(2)} = \frac{1}{32}.$

14.
$$\begin{aligned} h &= \tfrac{1}{64}s^2 \\ 64h &= s^2 \\ \sqrt{64h} &= s \\ 8\sqrt{h} &= s \end{aligned}$$

The initial speed of the player is about $8\sqrt{3} \approx 13.9$ feet per second.

Step 1 Show that $h(s(h)) = h$.
$$\begin{aligned} h(s(h)) &= h\left(8\sqrt{h}\right) \\ &= \tfrac{1}{64}\left(8\sqrt{h}\right)^2 \\ &= \tfrac{1}{64}(64h) \\ &= h \ \checkmark \end{aligned}$$

Step 2 Show that $s(h(s)) = s$.
$$\begin{aligned} s(h(s)) &= s\left(\tfrac{1}{64}s^2\right) \\ &= 8\sqrt{\tfrac{1}{64}s^2} \\ &= 8 \cdot \tfrac{1}{8}s \\ &= s \ \checkmark \end{aligned}$$

Chapter 5 Standards Assessment (pp. 290–291)

1. The pairs of equivalent expressions are

1. a and $\sqrt[n]{a^n}$ because $\sqrt[n]{a^n} = a^{n/n} = a$.
2. $a^{1/n}$ and $\sqrt[n]{a}$ because $a^{1/n} = \sqrt[n]{a}$.
3. $(\sqrt{a})^n$ and $\sqrt{a^n}$ because $(\sqrt{a})^n = \sqrt{a^n}$.

2. The parent function is $y = x^2$. The graph has been translated 3 units left and 2 units up. So, the function is $f(x) = (x - (-3))^2 + 2$.

3. a. $n = 2$: $s = 4.62\sqrt[9]{2} \approx 5.0$ m/sec
$n = 4$: $s = 4.62\sqrt[9]{4} \approx 5.4$ m/sec
$n = 8$: $s = 4.62\sqrt[9]{8} \approx 5.8$ m/sec

b. no; the boat speed does not double when the number of people are doubled because of the ninth root, so it increases by a factor of $2^{1/9}$.

c. $n = 2$: $\frac{2000}{4.99} \approx 400.8 \text{ sec} = 400.8 \text{ sec} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \approx 6.7 \text{ min}$

$n = 4$: $\frac{2000}{5.39} \approx 371.1 \text{ sec} = 371.1 \text{ sec} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \approx 6.2 \text{ min}$

$n = 8$: $\frac{2000}{5.82} \approx 343.6 \text{ sec} = 343.6 \text{ sec} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \approx 5.7 \text{ min}$

4.

28 2 −6 −2 8 18
26 8 −4 −10 −10
18 12 6 0
6 6 6

The third differences are constant, so the degree of the polynomial is 3. Work backwards to find the missing values in the table.

−2 8 18 22 14
−10 −10 −4 8
0 −6 −12
6 6

5. 18 minus what number is −4?
6. 22 minus what number is 8?
3. −10 minus what number is −6?
4. −4 minus what number is −12?
1. 0 minus what number is 6?
2. −6 minus what number is 6?

The missing values are 22 and 14.

5.
$$\begin{aligned} 42 &= \tfrac{1}{2}(x+8)x \\ 84 &= (x+8)x \\ 84 &= x^2 + 8x \\ 0 &= x^2 + 8x - 84 \\ 0 &= (x-6)(x+14) \end{aligned}$$
$x - 6 = 0$ or $x + 14 = 0$
$x = 6$ or $x = -14$

Reject the negative solution because a negative length does not make sense. So, $x = 6$.

Chapter 5

6.

Equation	Parabola	Function
$y = (x + 3)^2$	✓	✓
$x = 4y^2 - 2$	✓	
$y = (x - 1)^{1/2} + 6$		✓
$y^2 = 10 - x^2$		

Parabolas are of the form $y = x^2$ or $x = y^2$. There are two equations in the table that are parabolas. Two of the equations represent functions because they pass the Vertical Line Test but the other two do not.

7. C; $2\sqrt{x+3} - 1 < 3$

$$2\sqrt{x+3} < 4$$
$$\sqrt{x+3} < 2$$
$$x + 3 < 4$$
$$x < 1$$

Consider the radicand.

$$x + 3 \geq 0$$
$$x \geq -3$$

So, the solution is $-3 \leq x < 1$.

8. C; The graph is a horizontal shrink by a factor of $\frac{1}{2}$ and a translation 1 unit down of the parent absolute value function.

9.

$$d = \sqrt{2500 + h^2}$$
$$d^2 = 2500 + h^2$$
$$d^2 - 2500 = h^2$$
$$\sqrt{d^2 - 2500} = h$$

When $d = 100$: $h = \sqrt{100^2 - 2500} = \sqrt{7500} \approx 87$

So, the height of the balloon is about 87 feet.

10. They are not inverse functions because they are not reflections of each other in the line $y = x$.