pp. 466-467 (#2-40 evens)

- 2. The angle of elevation and angle of depression are always equal.
 - 4. What is the ratio of the side opposite θ to the hypotenuse?; $\frac{4}{6}$ or $\frac{2}{3}$; $\frac{6}{4}$ or $\frac{3}{2}$

6.
$$\sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5}, \tan \theta = \frac{3}{4}, \csc \theta = \frac{5}{3}, \sec \theta = \frac{5}{4}, \cot \theta = \frac{4}{3}$$

8.
$$\sin \theta = \frac{1}{3}, \cos \theta = \frac{2\sqrt{2}}{3}, \tan \theta = \frac{\sqrt{2}}{4}, \csc \theta = 3,$$

sec
$$\theta = \frac{3\sqrt{2}}{4}$$
, cot $\theta = 2\sqrt{2}$

10.
$$\sin \theta = \frac{2\sqrt{30}}{13}, \cos \theta = \frac{7}{13}, \tan \theta = \frac{2\sqrt{30}}{7}, \csc \theta = \frac{13\sqrt{30}}{60},$$

 $\sec \theta = \frac{13}{7}, \cot \theta = \frac{7\sqrt{30}}{60}$

12.
$$\sin(90^{\circ} - \theta) = \cos \theta, \cos(90^{\circ} - \theta) = \sin \theta,$$
$$\tan(90^{\circ} - \theta) = \cot \theta$$
$$\csc(90^{\circ} - \theta) = \sec \theta, \sec(90^{\circ} - \theta) = \csc \theta,$$
$$\cot(90^{\circ} - \theta) = \tan \theta$$

14.
$$\sin \theta = \frac{\sqrt{119}}{12}, \tan \theta = \frac{\sqrt{119}}{5}, \csc \theta = \frac{12\sqrt{119}}{119}, \sec \theta = \frac{12}{5},$$

$$\cot \theta = \frac{5\sqrt{119}}{119}$$

16.
$$\sin \theta = \frac{8}{15}, \cos \theta = \frac{\sqrt{161}}{15}, \tan \theta = \frac{8\sqrt{161}}{161},$$

 $\sec \theta = \frac{15\sqrt{161}}{161}, \cot \theta = \frac{\sqrt{161}}{8}$

18. $\sin \theta = \frac{11\sqrt{377}}{377}, \cos \theta = \frac{16\sqrt{377}}{377}, \tan \theta = \frac{11}{16},$ $\csc \theta = \frac{\sqrt{377}}{11}$, $\sec \theta = \frac{\sqrt{377}}{16}$ **20.** The reciprocal of csc θ is $\frac{1}{\sin \theta}$; $\csc \ \theta = \frac{1}{\sin \ \theta} = \frac{1}{\frac{6\sqrt{2}}{11}} = \frac{11}{6\sqrt{2}} = \frac{11\sqrt{2}}{12}$ 22. x = 324. x = 6.5**26.** x = 7**28.** 0.6009 **30.** 0.3907 **32.** 1.0187 **34.** $B = 63^{\circ}, a \approx 4.59, c \approx 10.10$ **36.** $A = 74^{\circ}, a \approx 48.82, c \approx 50.79$ **38.** $A = 59^{\circ}, b \approx 13.82, c \approx 26.83$ **40.** $B = 26^{\circ}, b \approx 3.61, c \approx 8.23$

pp. 467-468 (#41-53)

- **41.** $w \approx 514 \text{ m}$
- 42. about 586 ft
- **43.** about 427 m
- **44.** about 400 ft
- **45. a.** about 451 ft
 - **b.** about 5731 ft
- **46.** Answers will vary.
- **47. a.** about 22,818 mi
 - **b.** about 7263 mi
- **48. a.** x
 - **b.** *y*

c. yes;
$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{h}$$
 and $\sin(90^\circ - \theta) = \frac{\text{opp}}{\text{hyp}} = \frac{x}{h}$

- **49. a.** about 59,155 ft
 - **b.** about 53,613 ft
 - c. about 39,688 ft; Use the tangent function to find the horizontal distance, x + y, from the airplane to the second town to be about 93,301 ft. Subtract 53,613 ft to find the distance between the two towns.
- 50. about 59 m
- **51.** yes; The triangle must be a 45-45-90 triangle because both acute angles would be the same and have the same cosine value.

52.
$$\sin \theta = \frac{1}{2}, \cos \theta = \frac{\sqrt{3}}{2}, \tan \theta = \frac{\sqrt{3}}{3}, \csc \theta = 2, \sec \theta = \frac{2\sqrt{3}}{3},$$

cot $\theta = \sqrt{3}$; The triangle on the right is equilateral, so all of its angles must be 60°. Using geometry, it can be shown that $\theta = 30^{\circ}$ and that the two triangles form a larger 30-60-90 triangle. Because the large triangle is a right triangle, the 6 trig ratios can be found. The legs of the triangle are 1 and $\sqrt{3}$ and the hypotenuse is 2.

- **53. a.** x = 0.5; 6 units
 - **b.** Sample answer: Each side is part of two right triangles, with opposing angles $\left(\frac{180^{\circ}}{n}\right)$. So, each side length is $2 \sin\left(\frac{180^{\circ}}{n}\right)$, and there are *n* sides. **c.** $n \cdot \sin\left(\frac{180^{\circ}}{n}\right)$; about 3.14

pp. 474-476 (#4-22 evens, #23-28, #30-40, #42-47)

4. -90° ; It has a different terminal side than the other three angles.



22.
$$\frac{15\pi}{4}$$
; $-\frac{\pi}{4}$; Sample answer: 315° is equivalent to $\frac{7\pi}{4}$ radians,
and $\frac{7\pi}{4} + 2\pi = \frac{15\pi}{4}$ and $\frac{7\pi}{4} - 2\pi = -\frac{\pi}{4}$.
23. B
24. D
25. A
26. C
27. about 15.7 yd, about 78.5 yd²
28. a. about 13.3 m
b. about 146 m²

30. The angle was not converted to radians;

$$40^{\circ} = 40 \text{ degrees}\left(\frac{\pi \text{ radians}}{180 \text{ degrees}}\right) = \frac{2\pi}{9} \text{ radians}$$
$$A = \frac{1}{2}(6)^2 \left(\frac{2\pi}{9}\right) \approx 12.57 \text{ cm}^2$$
31. 72,000°, 400 π **32.**

240°, $\frac{4\pi}{3}$; *Sample answer:* The minute hand would generate

an angle of 2880° or 16π .

- **33.** -0.5
- **34.** 0.383

- **35.** 3.549 **36.** -1.376 **37.** -0.138 **38.** 0.960 **39.** 528 in.² **40. a.** $\frac{\pi}{2}$
 - **b.** about 45.6 ft
- **42.** $\pi 1$; *Sample answer:* Using $s = r(\pi \theta)$, the arc length of the small sector can be found to be 1. Therefore, $\pi \theta = 1$ and $\theta = \pi 1$.
- **43.** about 6.89 in.², about 0.76 in.², about 0.46 in.²
- 44. Sample answer: This continued fraction (which is irrational) gives rise to a sequence of rational approximations for π . When the next fraction is added, the value gets closer to the value of $\pi = 3.1415926535...$, as shown.

$$3 = 3$$

$$3 + \frac{1}{7} = \frac{22}{7} = 3.142857143...$$

$$3 + \frac{1}{7 + \frac{1}{15}} = \frac{333}{106} = 3.141509434...$$

$$3 + \frac{1}{7 + \frac{1}{15}} = \frac{355}{113} = 3.141592920...$$

45. yes; When the arc length is equal to the radius, the equation $s = r\theta$ shows that $\theta = 1$ and $A = \frac{1}{2}r^2\theta$ is equivalent to $A = \frac{s^2}{2}$ for r = s and $\theta = 1$.

46. a. about 16.49 in.

- **b.** $\frac{15\pi}{8}$
- **c.** about 5195.4 in.²
- **47. a.** 70°33′

b.
$$110.76^{\circ}$$
; $110 + \frac{45}{60} + \frac{30}{3600} \approx 110.76^{\circ}$

pp. 482-484 (#2-38 evens, #40-41, #43-47)

- 2. After finding the reference angle θ' , evaluate $-\cos \theta'$ (cosine is negative in Quadrant III).
- 4. $\sin \theta = -\frac{12}{13}, \cos \theta = \frac{5}{13}, \tan \theta = -\frac{12}{5}, \csc \theta = -\frac{13}{12},$ $\sec \theta = \frac{13}{5}, \cot \theta = -\frac{5}{12}$ 6. $\sin \theta = \frac{\sqrt{10}}{10}, \cos \theta = \frac{3\sqrt{10}}{10}, \tan \theta = \frac{1}{3}, \csc \theta = \sqrt{10},$ $\sec \theta = \frac{\sqrt{10}}{3}, \cot \theta = 3$ 8. $\sin \theta = -\frac{2\sqrt{5}}{5}, \cos \theta = \frac{\sqrt{5}}{5}, \tan \theta = -2, \csc \theta = -\frac{\sqrt{5}}{2},$ $\sec \theta = \sqrt{5}, \cot \theta = -\frac{1}{2}$ 10. $\sin \theta = 0, \cos \theta = -1, \tan \theta = 0, \csc \theta = \text{undefined},$ $\sec \theta = -1, \cot \theta = \text{undefined}$
- 12. $\sin \theta = -1$, $\cos \theta = 0$, $\tan \theta =$ undefined, $\csc \theta = -1$, $\sec \theta =$ undefined, $\cot \theta = 0$
- 14. $\sin \theta = 0$, $\cos \theta = 1$, $\tan \theta = 0$, $\csc \theta =$ undefined, sec $\theta = 1$, $\cot \theta =$ undefined

16. y ; 30° 30° 150° x



24. The angle found is the angle between the terminal side and the y-axis instead of the x-axis; θ is coterminal with 290°, whose terminal side lies in Quadrant IV. So, $\theta' = 360^\circ - 290^\circ = 70^\circ$

- 26. $\sqrt{3}$ 28. $-\frac{2\sqrt{3}}{3}$ 30. $\frac{\sqrt{3}}{3}$ 32. $\frac{2\sqrt{3}}{3}$ 34. about 6.1 ft
- **36.** yes; $\frac{71^2}{32}$ sin(2 40) ≈ 155.1
- **38.** about 104 ft; no. *Sample answer:* The initial height that the Ferris wheel is above the ground is not doubled so the entire height is not doubled.
- **40.** 125 ft
- 41.



43. $\tan \theta = \frac{\sin \theta}{\cos \theta}$; $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$, so $\tan 90^\circ$ is

undefined because you cannot divide by 0, but

$$\cot 90^\circ = \frac{0}{1} = 0.$$

- **44.** Sine and cosecant are negative because the *y*-coordinate is negative in Quadrant IV. Cosine and secant are positive because the *x*-coordinate is positive in Quadrant IV. Tangent and cotangent are negative because the *y*-coordinate is negative and the *x*-coordinate is positive.
- 45. $m = \tan \theta$
- **46.** no; $\theta = 240^{\circ}$ is also a solution; any angles coterminal with 60° and 240° are also solutions.
- **47. a.** (-58.1, 114)
 - **b.** about 218 pm

pp. 491-492 (#2-40 evens)

- 2. The amplitude of the first function is $\frac{1}{2}$ and the amplitude of the second function is 3. The period of the first function is 2π and the period of the second function is π .
- **4.** y = -2
- 6. yes; 2π
- 8. no
- **10.** $\frac{1}{2}$, 2
- **12.** 3, 8π
- 14. 2, 2π ; The graph of g is a vertical stretch by a factor of 2 of the graph of $f(x) = \sin x$.



16. 1, $\frac{\pi}{2}$; The graph of g is a horizontal shrink by a factor of $\frac{1}{4}$ of

the graph of $f(x) = \cos x$.



18. 3, π ; The graph of *g* is a horizontal shrink by a factor of $\frac{1}{2}$ and a vertical stretch by a factor of 3 of the graph of $f(x) = \sin x$.



20. $\frac{1}{2}$, $\frac{1}{2}$; The graph of g is a horizontal shrink by a factor of $\frac{1}{4\pi}$

and a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f(x) = \cos x$.



22. a. $y = \sin \frac{2\pi}{5}x$

- **b.** $y = 10 \sin \frac{\pi}{2} x$
- **c.** $y = 2 \sin x$ **d.** $y = \frac{1}{2} \sin \frac{2}{3}x$

24. The period is 6 and represents the amount of time, in seconds, that it takes for the buoy to bob up and down and return to the same position. The amplitude is 1.75 and represents the maximum distance, in feet, the buoy will be from its midline.





36. $\frac{\pi}{2}$ should be added to the *x*-coordinate;

Maximum:
$$\left(\left(\frac{1}{4} \cdot 2\pi\right) + \frac{\pi}{2}, 2\right) = \left(\frac{\pi}{2} + \frac{\pi}{2}, 2\right)$$
$$= (\pi, 2)$$

- **38.** The graph of g is a vertical stretch by a factor of 3 followed by a translation $\frac{\pi}{4}$ units left and 2 units down of the graph of f.
- **40.** The graph of g is a horizontal shrink by a factor of $\frac{1}{6}$ followed by a translation π units right and 9 units up of the graph of f.

pp. 492-494 (#42-48 evens, #49-57, #59-67)



- 50. a. B; The graph of sine has been translated 3 units up.
 - **b.** C; The graph of cosine has been translated 3 units down.
 - **c.** A; The graph of sine has been shrunk horizontally by a

factor of $\frac{1}{2}$ then translated $\frac{\pi}{2}$ units right.

- d. D; The graph of cosine has been shrunk horizontally by a factor of $\frac{1}{2}$ then translated $\frac{\pi}{2}$ units right. **51.** $g(x) = 3\sin(x - \pi) + 2$ 52. $g(x) = \cos 2\pi (x+3) - 4$ 53. $g(x) = -\frac{1}{3}\cos \pi x - 1$ 54. $g(x) = -\frac{1}{2}\sin 6(x-1) - \frac{3}{2}$ h 8 55. : 4.3 ft 7 Height (feet) 6 5 4 3 2 1 0 0 10 20 30 40 50 60 70 80 90 θ Angle (degrees) **56.** a. $\frac{45}{11.5}, \frac{27.5}{18}, \frac{10}{11.5}, \frac{27.5}{5}$
 - **b.** When the number of lynx is at the midline and increasing, the number of hares decreases. Once the number of lynx reaches a maximum and begins to decrease, the number of hares decreases to its minimum. At this point, the number of hares begins to increase to its midline and the number of lynx starts to decrease until it reaches its minimum. The number of lynx then starts to increase as the number of hares reaches its maximum.

- **57.** days 205 and 328; When the function is graphed with the line y = 10, the two points of intersection are (205.5, 10) and (328.7, 10).
- **59. a.** about -1.27
 - **b.** about 0.64
 - **c.** about 0.64
- 60. a.

x	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0
$y = \sin(-x)$	0	-1	0	1	0
$y = \cos(-x)$	1	0	-1	0	1
x	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	
$y = \sin(-x)$	-1	0	1	0	
$y = \cos(-x)$	0	-1	0	1	

b.



c. Both graphs are reflections across the y-axis. Because cosine is an even function, it is symmetrical across the y-axis and the graph of y = cos(-x) is the same as the graph of y = cos(x).



- **b.** 4.5
- **c.** 175 ft, 5 ft

62. a.
$$f(x) = a \cos bx$$
; The y-coordinate is 5 when $x = 0$.

b. maximum value = 5, minimum value = -5; period = π , amplitude = 5

63. The *x*-intercepts occur when
$$x = \pm \frac{\pi}{4}, \pm \frac{3\pi}{4}, \pm \frac{5\pi}{4}, \ldots$$

Sample answer: The x-intercepts can be represented by the

expression $(2n + 1)\frac{\pi}{4}$, where *n* is an integer.

- **64.** no; The value of *a* indicates a vertical stretch or a vertical shrink and changes the amplitude of the graph. It does not affect the *x*-intercepts of the function. The value of *b* indicates a horizontal stretch or a horizontal shrink and changes the period of the graph which is the horizontal length of each cycle. So, only the value of *b* affect the *x*-intercepts of the function.
- 65. The graph of $g(x) = \cos x$ is a translation $\frac{\pi}{2}$ units to the right of the graph of $f(x) = \sin x$.
- $66. \quad y = \sin x + \cos 2x$
- 67. 80 beats per minute

p. 496 (#1-17)





14. 3, 2π ; The graph of g is a vertical stretch by a factor of 3 of the graph of f;



15. 1, $\frac{2}{5}$; The graph of g is a horizontal shrink by a factor of $\frac{1}{5\pi}$

followed by a translation 3 units up of the graph of f;



- 16. a. About 380 ft
 - **b.** About 170 ft
- **17.** Seated at a window table, you will have traveled about 569 feet in that time. A person at a table 5 feet from the window will have traveled about 509 feet in that time.

p. 502 (#2-28 evens, #29-34)

- 2. cosecant; cotangent
- 4. To graph $y = a \sec bx$, first graph $y = a \cos bx$. Use the asymptotes and several points of $y = a \sec bx$ to graph the function.



The graph of g is a vertical stretch by a factor of 3 of the graph of $f(x) = \tan x$.



The graph of g is a horizontal shrink by a factor of $\frac{1}{2}$ of the graph of $f(x) = \cot x$.



The graph of g is a horizontal stretch by a factor of 2 and a vertical stretch by a factor of 4 of the graph of $f(x) = \cot x$.



The graph of g is a horizontal shrink by a factor of $\frac{1}{2\pi}$ and a

vertical shrink by a factor of $\frac{1}{3}$ of the graph of $f(x) = \tan x$.

- 14. The horizontal and vertical shrink factors are switched; A vertical stretch by a factor of 2 and a horizontal shrink by a factor of $\frac{1}{5}$.
- **16.** A



The graph of g is a vertical stretch by a factor of 2 of the graph of $f(x) = \csc x$.

20.



The graph of g is a horizontal shrink by a factor of $\frac{1}{3}$ of the graph of $f(x) = \sec x$.



The graph of g is a horizontal stretch by a factor of $\frac{4}{\pi}$ of the graph of $f(x) = \csc x$.

- **26.** $y = \frac{1}{2} \tan x$
- **28.** $y = 5 \tan 2x$
- **29.** B; The parent function is the tangent function and the graph has an asymptote at $x = \frac{\pi}{2}$.

30. C; The parent function is the cotangent function and the graph has an asymptote at x = 0.

- **31.** D; The parent function is the cosecant function and the graph has an asymptote at x = 1.
- 32. F; The parent function is the secant function and the graph has an asymptote at $x = -\frac{1}{2}$.
- **33.** A; The parent function is the secant function and the graph

has an asymptote at $x = \frac{\pi}{4}$.

34. E; The parent function is the cosecant function and the graph has an asymptote at $x = \frac{\pi}{2}$.

pp. 502-504 (#36-40 evens, #41-50)

36. a.



The graph of *g* is a translation 3 units up of the graph of $f(x) = \sec x$.



The graph of g is a translation 2 units down of the graph of $f(x) = \csc x$.



The graph of g is a translation π units right of the graph of $f(x) = \cot x$.



The graph of g is a reflection across the x-axis of the graph of $f(x) = \tan x$.

- **38.** $g(x) = 2 \tan(3x \pi)$
- **40.** $g(x) = -8 \csc x$
- **41.** Function B has a local maximum value of -5 so Function A's local maximum value of $-\frac{1}{4}$ is greater. Function A has a local minimum of $\frac{1}{4}$ so Function B's local minimum value of 5 is greater.



As d increases, θ increases because, as the car gets farther away, the angle required to see the car gets larger.



0 < t < 15 and 0 < h < 320; The Statue of Liberty is approximately 305 feet tall so it would take almost 15 seconds to span the statue.



The graph shows a negative correlation meaning that as the angle gets larger, the distance from your friend to the top of the building gets smaller. As the angle gets smaller, the distance from your friend to the top of the building gets larger.

46. a. $d = -300 \tan \theta + 200$



c. About 18.4°



- **48. a.** 4
 - **b.** y > 2 and y < -2
 - **c.** $y = a \csc bx$; The cosecant function has an asymptote at x = 0.

$$49. \quad a \sec bx = \frac{a}{\cos bx}$$

Because the cosine function is at most 1, $y = a \cos bx$ will produce a maximum when $\cos bx = 1$ and $y = a \sec bx$ will produce a minimum. When $\cos bx = -1$, $y = a \cos bx$ will produce a minimum and $y = a \sec bx$ will produce a maximum.

50.
$$\csc x = \frac{1}{2} \left(\tan \frac{x}{2} + \cot \frac{x}{2} \right)$$
; Graphing the function produces

the same graph as the cosecant function with asymptotes at 0, $\pm \pi, \pm 2\pi, \ldots$

pp. 510-511 (#3-20)





- **13.** $y = 3 \sin 2x$
- **14.** $y = 5 \cos 4x$

15.
$$y = -2\cos\frac{\pi}{2}(x+4)$$

- **16.** $y = -\sin \pi x 2$
- 17. To find the amplitude, take half of the difference between the maximum and the minimum; $\frac{10 (-6)}{2} = 8$
- 18. To find the vertical shift, use the y-coordinates of the points; $\frac{-2 + (-8)}{2} = -5$
- **19.** $h = -2.5 \cos \pi t + 6.5$ **20.** $h = -36.25 \cos \frac{\pi}{12}t + 34.25$

pp. 511-512 (#21-31)

- **21.** $D = 19.81 \sin(0.549t 2.40) + 79.8$; The period of the graph represents the amount of time it takes for the weather to repeat its cycle, which is about 11.4 months.
- 22. $D = 7.38 \sin(0.498t 2.05) + 78.6$; The period of the graph represents the amount of time it takes for the weather to repeat its cycle, which is about 12.6 months.
- 23. $V = 100 \sin 4\pi t$
- 24. Louisville; The graph of the average daily temperature for Louisville is always higher than the one for Lexington.



The slope of the graph of $y = \sin x$ is given by the function $y = \cos x$.

- 27. a. and b. A cosine function because it does not require determining a horizontal shift.
 - **c.** A sine function because it does not require determining a horizontal shift.
- **28.** 2; The graph completes 2 full cycles in 1 unit of x.

29.
$$y = 2.5 \sin 4\left(x - \frac{\pi}{8}\right) + 5.5, y = -2.5 \cos 4x + 5.5$$

30. no; The period is the reciprocal of the frequency. The reciprocal of $\frac{1}{2}$ is greater than the reciprocal of 2.

31. a.
$$d = -6.5 \cos \frac{\pi}{6}t + 10$$

- **b.** low tide: 12:00 A.M., 12:00 P.M., high tide: 6:00 A.M., 6:00 P.M.
- **c.** It is a horizontal shift to the left by 3.
p. 517 (#2-10 evens, #11-22)

2. The reciprocal identity for secant can be used to write the expression in terms of cosine. The negative angle identity can be used to simplify the expression and then the reciprocal identity can again be used to write the expression in terms of cosine.

4.
$$\cos \theta = -\frac{\sqrt{51}}{10}, \tan \theta = \frac{7\sqrt{51}}{51}, \csc \theta = -\frac{10}{7},$$

 $\sec \theta = -\frac{10\sqrt{51}}{51}, \cot \theta = \frac{\sqrt{51}}{7}$
6. $\sin \theta = \frac{5\sqrt{29}}{29}, \cos \theta = -\frac{2\sqrt{29}}{29}, \tan \theta = -\frac{5}{2}, \csc \theta = \frac{\sqrt{29}}{5},$
 $\sec \theta = -\frac{\sqrt{29}}{2}$
8. $\sin \theta = -\frac{\sqrt{65}}{9}, \cos \theta = \frac{4}{9}, \tan \theta = -\frac{\sqrt{65}}{4}, \csc \theta = -\frac{9\sqrt{65}}{65},$
 $\cot \theta = -\frac{4\sqrt{65}}{65}$
10. $\sin \theta = -\frac{3}{5}, \cos \theta = -\frac{4}{5}, \tan \theta = \frac{3}{4}, \sec \theta = -\frac{5}{4}, \cot \theta = \frac{4}{3}$
11. $\cos x$
12. $\sec \theta$
13. $-\tan \theta$
14. $\sin^2 x$
15. $\sin^2 x$
16. 1
17. $-\sec x$
18. -1
19. 1
20. $\sin x$

21.
$$\sin^2 \theta = 1 - \cos^2 \theta;$$

$$1 - \sin^2 \theta = 1 - (1 - \cos^2 \theta) = 1 - 1 + \cos^2 \theta = \cos^2 \theta$$

22.
$$\tan x = \frac{\sin x}{\cos x};$$

$$\tan x \csc x = \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$$

$$=\frac{1}{\cos x}$$

 $= \sec x$

pp. 517-518 (#23-40)

23.
$$\sin x \csc x = \sin x \cdot \frac{1}{\sin x}$$

= 1
24. $\tan \theta \csc \theta \cos \theta = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta} \cdot \cos \theta$
= 1
25. $\cos(\frac{\pi}{2} - x) \cot x = \sin x \cdot \frac{\cos x}{\sin x}$
= $\cos x$
26. $\sin(\frac{\pi}{2} - x) \tan x = \cos x \cdot \frac{\sin x}{\cos x}$
= $\sin x$
27. $\frac{\cos(\frac{\pi}{2} - \theta) + 1}{1 - \sin(-\theta)} = \frac{\sin \theta + 1}{1 - \sin(-\theta)}$
= $\frac{\sin \theta + 1}{1 - (-\sin \theta)}$
= $\frac{\sin \theta + 1}{1 + \sin \theta}$
= 1
28. $\frac{\sin^2(-x)}{\tan^2 x} = \frac{(-\sin x)^2}{\tan^2 x} = \frac{\sin^2 x}{\tan^2 x}$
= $\sin^2 x \cdot \cot^2 x$
= $\sin^2 x \cdot \cot^2 x$

29.
$$\frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} = \frac{1 + \cos x}{\sin x} + \frac{\sin x(1 - \cos x)}{(1 + \cos x)(1 - \cos x)}$$
$$= \frac{1 + \cos x}{\sin x} + \frac{\sin x(1 - \cos x)}{1 - \cos^2 x}$$
$$= \frac{1 + \cos x}{\sin x} + \frac{\sin x(1 - \cos x)}{\sin^2 x}$$
$$= \frac{\sin x(1 + \cos x)}{\sin^2 x} + \frac{\sin x(1 - \cos x)}{\sin^2 x}$$
$$= \frac{\sin x(1 + \cos x) + \sin x(1 - \cos x)}{\sin^2 x}$$
$$= \frac{\sin x(1 + \cos x) + \sin x(1 - \cos x)}{\sin^2 x}$$
$$= \frac{\sin x(1 + \cos x + 1 - \cos x)}{\sin^2 x}$$
$$= \frac{\sin x(2)}{\sin^2 x}$$
$$= \frac{2}{\sin x}$$
$$= \frac{2}{\sin x}$$
$$= \frac{2}{\sin x}$$
$$= \frac{2}{\sin x}$$
$$= \frac{\sin x}{1 - \cos x}$$
$$= \frac{\sin x}{1 - \cos x}$$
$$= \frac{\sin x(1 + \cos x)}{1 - \cos^2 x}$$
$$= \frac{\sin x(1 + \cos x)}{1 - \cos^2 x}$$
$$= \frac{\sin x(1 + \cos x)}{1 - \cos^2 x}$$
$$= \frac{\sin x(1 + \cos x)}{\sin^2 x}$$
$$= \frac{1 + \cos x}{\sin^2 x}$$
$$= \frac{1 + \cos x}{\sin x}$$
$$= \frac{1 + \cos x}{\sin x}$$
$$= \frac{1 + \cos x}{\sin x}$$
$$= \cos x + \cot x$$



32. Just as $\frac{1}{x}$ decreases as x increases, sec $\theta = \frac{1}{\cos \theta}$ decreases as

 $\cos \theta$ increases; this happens on the intervals

$$\pi + 2n\pi < \theta < \frac{3\pi}{2} + 2n\pi$$
 and $\frac{3\pi}{2} + 2n\pi < \theta < 2\pi + 2n\pi$

where *n* is an integer.

33. yes; sec
$$x \tan x - \sin x = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} - \sin x$$

$$= \frac{\sin x}{\cos^2 x} - \sin x$$

$$= \sec^2 x \sin x - \sin x$$

$$= \sin x (\sec^2 x - 1)$$

$$= \sin x \tan^2 x$$

- **34.** a. sin θ is positive, cos θ is negative and tan θ is negative.
 - b. Quadrant III
 - c. $\sin(-\theta)$ is negative, $\cos(-\theta)$ is negative and $\tan(-\theta)$ is positive.

35.
$$s = \frac{h \sin(90^\circ - \theta)}{\sin \theta}$$
$$s = \frac{h \cos \theta}{\sin \theta}$$
$$s = h \cot \theta$$

36. Sample answer: Multiply both sides by $\frac{1}{\cos x}$, so $\frac{\sin x}{\cos x} = 1$.

Then $\tan x = 1$ because $\frac{\sin x}{\cos x} = \tan x$.

- **37. a.** $u = \tan \theta$
 - **b.** *u* increases from 0 to ∞ .

38. a.
$$\frac{n_1}{\sqrt{\cot^2 \theta_1 + 1}} = \frac{n_2}{\sqrt{\cot^2 \theta_2 + 1}}$$
$$\frac{n_1}{\sqrt{\csc^2 \theta_1}} = \frac{n_2}{\sqrt{\csc^2 \theta_2}}$$
$$\frac{n_1}{\csc \theta_1} = \frac{n_2}{\csc \theta_2}$$
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
b. showt 1.4

- **b.** about 1.4
- **c.** $n_1 = n_2$; This situation could occur when the mediums have the same composition.
- **39.** You can obtain the graph of $y = \cos x$ by reflecting the graph of $f(x) = \sin x$ in the y-axis and translating it $\frac{\pi}{2}$ units right.

40. a.
$$\ln|\sec \theta| = \ln \frac{1}{|\cos \theta|} = \ln|(\cos \theta)^{-1}| = -\ln|\cos \theta|$$

b. $\ln|\tan \theta| = \ln \frac{|\sin \theta|}{|\cos \theta|} = \ln|\sin \theta| - \ln|\cos \theta|$

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- **1.** cos 170°
- 2. First break 75° into the sum or difference of two angles whose tangent values are known such as $45^{\circ} + 30^{\circ}$. Rewrite the expression using the corresponding sum or difference formula and evaluate.

	und evaluate.
3.	$\sqrt{3} - 2$
4.	$2 - \sqrt{3}$
5.	$\frac{\sqrt{2}-\sqrt{6}}{4}$
6.	$\frac{\sqrt{2}-\sqrt{6}}{4}$
7.	$\frac{\sqrt{2}-\sqrt{6}}{4}$
8.	$\frac{-\sqrt{2}-\sqrt{6}}{4}$
9.	$\sqrt{3} + 2$
10.	$\frac{-\sqrt{2}-\sqrt{6}}{4}$
	4
11.	$-\frac{36}{85}$
11. 12.	36
	$-\frac{36}{85}$ 84
12.	$-\frac{36}{85}$ $\frac{84}{85}$ 13
12. 13.	$-\frac{36}{85}$ $\frac{84}{85}$ $-\frac{13}{85}$ 77
12. 13. 14.	$-\frac{36}{85} \\ \frac{84}{85} \\ -\frac{13}{85} \\ \frac{77}{85} \\ -\frac{36}{85}$
12. 13. 14. 15.	$-\frac{36}{85} \\ \frac{84}{85} \\ -\frac{13}{85} \\ \frac{77}{85} \\ -\frac{36}{77} \\ -\frac{84}{13}$
12. 13. 14. 15. 16.	$-\frac{36}{85} \\ \frac{84}{85} \\ -\frac{13}{85} \\ \frac{77}{85} \\ -\frac{36}{77} \\ -\frac{84}{13} \\ \tan x$
12. 13. 14. 15. 16. 17.	$-\frac{36}{85} \\ \frac{84}{85} \\ -\frac{13}{85} \\ \frac{77}{85} \\ -\frac{36}{77} \\ -\frac{84}{13} \\ \tan x$
12. 13. 14. 15. 16. 17. 18.	$ -\frac{36}{85} \\ \frac{84}{85} \\ -\frac{13}{85} \\ \frac{77}{85} \\ -\frac{36}{77} \\ -\frac{36}{77} \\ -\frac{84}{13} \\ tan x \\ sin x \\ cos x $

- **22.** $-\cot x$
- **23.** The sign in the denominator should be negative when using the sum formula;

$$\frac{\tan x + \tan \frac{\pi}{4}}{1 - \tan x \tan \frac{\pi}{4}} = \frac{\tan x + 1}{1 - \tan x}$$

24. The *a* and *b* were reversed when the difference formula was used; (--)

$$\sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4} = \sin x \left(\frac{\sqrt{2}}{2}\right) - \cos x \left(\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{2}}{2} (\sin x - \cos x)$$

25. B, D

26. B, D

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27. $x = \frac{\pi}{3}, \frac{5\pi}{3}$ 28. $x = \frac{\pi}{4}, \frac{5\pi}{4}$ 29. $x = \frac{3\pi}{2}$ 30. $x = 0, \pi$ 31. $x = 0, \pi$ 32. $x = \frac{3\pi}{4}, \frac{7\pi}{4}$ 33. $\sin(\frac{\pi}{2} - \theta) = \sin\frac{\pi}{2}\cos\theta - \cos\frac{\pi}{2}\sin\theta$ $= (1)\cos\theta - (0)\sin\theta$ $= \cos\theta$

34. no; The difference formula for $tan\left(\frac{\pi}{2} - \theta\right)$ would require

finding $\tan \frac{\pi}{2}$, which is undefined.

35.
$$\frac{35 \tan(\theta - 45^\circ) + 35 \tan 45^\circ}{h \tan \theta}$$
$$= \frac{35 \left(\frac{\tan \theta - \tan 45^\circ}{1 + \tan \theta \tan 45^\circ}\right) + 35 \tan 45^\circ}{h \tan \theta}$$
$$= \frac{35 \left(\frac{\tan \theta - 1}{1 + \tan \theta}\right) + 35}{h \tan \theta}$$
$$= \frac{35 \left(\frac{\tan \theta - 1}{1 + \tan \theta}\right) + 35(1 + \tan \theta)}{h \tan \theta(1 + \tan \theta)}$$
$$= \frac{35 \tan \theta - 35 + 35 + 35 \tan \theta}{h \tan \theta(1 + \tan \theta)}$$
$$= \frac{70 \tan \theta}{h \tan \theta(1 + \tan \theta)}$$
$$= \frac{70 \tan \theta}{h \tan \theta(1 + \tan \theta)}$$

36.
$$A \cos\left(\frac{2\pi}{3} - \frac{2\pi x}{5}\right) + A \cos\left(\frac{2\pi}{3} + \frac{2\pi x}{5}\right)$$

$$= A \cos\left(\frac{2\pi}{3} \cos\left(\frac{2\pi x}{5}\right) + A \sin\left(\frac{2\pi}{3} \sin\left(\frac{2\pi x}{5}\right) + A \cos\left(\frac{2\pi}{3} \cos\left(\frac{2\pi x}{5}\right)\right)\right)$$

$$= A \cos\left(\frac{2\pi}{3} \cos\left(\frac{2\pi x}{5}\right) + A \cos\left(\frac{2\pi}{3} \cos\left(\frac{2\pi x}{5}\right)\right)$$

$$= A \cos\left(\frac{2\pi}{3} \cos\left(\frac{2\pi x}{5}\right)\right)$$

$$= 2A \cos\left(\frac{2\pi x}{5}\right)$$

$$= 2A \left(\cos\left(\frac{2\pi x}{5}\right)\right)$$

$$= -A \cos\left(\frac{2\pi x}{5}\right)$$

$$= -A \cos\left(\frac{2\pi x}{5}\right)$$

$$= -A \cos\left(\frac{2\pi x}{5}\right)$$

$$= -A \cos\left(\frac{2\pi x}{5}\right)$$

$$= \cos(1100\pi t - 140\pi t) + \cos(1100\pi t + 140\pi t)\right)$$

$$= \cos(1100\pi t \cos 140\pi t + \sin 1100\pi t \sin 140\pi t)$$

$$+ \cos 1100\pi t \cos 140\pi t - \sin 1100\pi t \sin 140\pi t$$

$$= \cos 1100\pi t \cos 140\pi t + \cos 1100\pi t \cos 140\pi t$$

$$= 2 \cos 1100\pi t \cos 140\pi t + \cos 1100\pi t \cos 140\pi t$$

$$= 2 \cos 1100\pi t \cos 140\pi t$$
38. Any point where the two graphs intersect is a solution because if $f(x) = g(x)$ then $f(x) - g(x) = 0$.
39. **a.** $\tan(\theta_2 - \theta_1) = \frac{m_2 - m_1}{1 + m_2 m_1}$
b. 60°
40. **a.** $3 \sin x - 4 \sin^3 x$
b. $4 \cos^3 x - 3 \cos x$
c. $\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

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about 1497 m²

10.
$$\sin \theta = -\frac{7}{25}, \cos \theta = \frac{24}{25}, \tan \theta = -\frac{7}{24}, \csc \theta = -\frac{25}{7}, \sec \theta = \frac{25}{24}, \cot \theta = -\frac{24}{7}$$

12. $-\frac{\sqrt{3}}{3}$

- 14. $\frac{1}{2}$
- 16. 8, 2π ; The graph of g is a vertical stretch by a factor of 8 of the graph of $f(x) = \cos x$;



18. $\frac{1}{4}, \frac{\pi}{2}$; The graph of g is a horizontal shrink by a factor of $\frac{1}{4}$

and a vertical shrink by a factor of $\frac{1}{4}$ of the graph of $f(x) = \cos x$;



The graph of g is a horizontal stretch by a factor of 2 of the graph of $f(x) = \tan x$.



vertical stretch by a factor of 4 of the graph of $f(x) = \tan x$.



30. Sample answer: $y = \cos \pi x - 2$

32. $P = 1.08 \sin(0.585t - 2.33) + 1.5$; The period represents the amount of time it takes for the precipitation level to complete one cycle, which is about 10.7 months.

34.
$$\tan x$$

36. $\frac{\cos x \sec x}{1 + \tan^2 x} = \frac{\cos x \sec x}{\sec^2 x}$
 $= \frac{\cos x}{\sec^2 x}$
 $= \cos x \cos x$
 $= \cos^2 x$
38. $\frac{\sqrt{2} + \sqrt{6}}{4}$
40. $\frac{\sqrt{6} + \sqrt{2}}{4}$
42. $x = \frac{3\pi}{4}, \frac{5\pi}{4}$