pp. 300-301 (#4-36 evens)

- **4. a.** $\frac{1}{16}$
 - **b.** 64
- 6. a. $\frac{3}{2}$
- **b.** 48
- 8. a. $-\frac{7}{4}$
 - **b.** 6
- 10. exponential growth



12. exponential decay



14. exponential decay



16. exponential decay



18. exponential growth



- **20.** *b* = 5
- 22. a. exponential growth
 - **b.** 3% increase
 - c. about 6 years after the start of the decade
- **24. a.** $y = 325(0.71)^t$
 - **b.** about 3.4 h
- 26. Power of a Power Property; Evaluate power; Rewrite in form $y = a(1 r)^t$.
- **28.** about 56%
- **30.** $y = a(1 + 0.26)^{t}$; 26% growth
- **32.** $y = a(1 0.14)^t$; 14% decay
- **34.** $y = a(1 + 0.01)^t$; 1% growth
- **36.** $y = a(1 0.96)^t$; 96% decay

pp. 301-302 (#38-43 and #45-52)

- **38.** quarterly \approx \$432.11; monthly \approx \$433.29; daily \approx \$433.86
- **39.** The percent decrease needs to be subtracted from 1 to produce the decay factor;

$$y = {\operatorname{Initial}_{\operatorname{amount}}} {\operatorname{Decay}_{\operatorname{factor}}}^{t}; y = 500(1 - 0.02)^{t}; y = 500(0.98)^{t}$$

40. The percentage rate was not converted to a decimal;

$$A = 250 \left(1 + \frac{0.0125}{4}\right)^{4 \cdot 3}; A \approx \$259.54$$

- **41.** \$3982.92
- **42.** \$4014.98
- **43.** \$3906.18
- **45.** *a* represents the number of referrals it received at the start of the model. *b* represents the growth factor of the number of referrals each year; 50%; 1.50 can be rewritten as (1 + 0.50), showing the percent increase of 50%.
- 46. a. exponential decay
 - **b.** domain: all real numbers, range: y > 0; It is an exponential function and any real number can be used as an exponent. $f(x) \to 0$ as $x \to \infty$ and $f(x) \to \infty$ as $x \to -\infty$.
- **47.** no; $f(x) = 2^x$ eventually increases at a faster rate than $g(x) = x^2$, but not for all $x \ge 0$.
- **48.** *Sample answer:* $y = (1 b)^x$
- 49. about 221.5; The curve contains the points (0, 6850) and

(6, 8179.26) and
$$\frac{8179.26 - 6850}{6 - 0} \approx 221.5$$
.

50. a.
$$\frac{f(x+1)}{f(x)} = \frac{ab^{x+1}}{ab^x} = \frac{b^{x+1}}{b^x} = b^{(x+1)-x} = b^1 = b$$

- b. The equation shows that when a value of the function is divided by the previous value, the answer is the constant *b*. Dividing the *y*-values in the table by the previous value does not always produce the same number.
- **51. a.** The decay factor is 0.9978. The percent decrease is 0.22%.



- c. about 134 eggs per year
- **d.** Replace $\frac{w}{52}$ with y, where y represents the age of the chicken in years.
- **52.** $V = 1300(0.6782)^t$

pp. 307-308 (#2-42 evens)

- 2. exponential growth; A function of the form $y = ae^{rx}$ shows growth when a > 0 and r > 0. In this function, $a = \frac{1}{3}$ and r = 4.
- **4.** e^2
- **6.** 9*e*³
- 8. $\frac{64}{e^{6x}}$
- **10.** $2e^{4x}$
- 12. e^{2x+7}
- **14.** The exponent of the denominator was added, not subtracted, from the exponent of the numerator;

$$\frac{e^{5x}}{e^{-2x}} = e^{5x - (-2x)} = e^{5x + 2x} = e^{7x}$$

16. exponential decay



18. exponential growth



20. exponential decay



22. exponential growth



24. A; The graph shows decay and has a *y*-intercept of 1.

- 26. C; The graph shows growth and has a y-intercept of 0.75.
- **28.** $y = (1 0.528)^t$; 52.8% decay
- **30.** $y = 0.5(1 + 1.226)^{t}$; 122.6% growth



domain: all real numbers, range: y > 0



domain: all real numbers, range: y > -536. sodium-22; tritium **38.** Let $m = \frac{n}{r}$, so n = mr and $\frac{r}{n} = \frac{1}{m}$.

Substituting into $A = P(1 + \frac{r}{n})^{nt}$ gives $A = P(1 + \frac{1}{m})^{mrt}$

which can be written as $A = P\left[\left(1 + \frac{1}{m}\right)^m\right]^n$. By definition,

$$\left(1 + \frac{1}{m}\right)^m$$
 approaches *e* as *m* approaches $+\infty$. So, the

equation becomes $A = Pe^{rt}$.

- **40.** no; The value of f(x) at x = 1000 is too small for the calculator to display, so the calculator rounds the value to 0. The function $f(x) = e^{-x}$ has no *x*-intercept, but $f(x) \to 0$ as $x \to \infty$.
- **42.** a. ∞
 - **b.** −3

p. 314 (#2-34 evens)

- log base 3 of 9
 Evaluate 4²; 16; 2
 4¹ = 4
 7³ = 343
 3⁻¹ = ¹/₃
 log₁₂ 1 = 0
 log₅ ¹/₂₅ = -2
 log₄₉ 7 = ¹/₂
 log 2
 -3
 There is no power of 2 that gives you -1, and all powers of 1 give you 1.
 2.485
- **30.** -0.544
- **32.** 0.778
- **34. a.** 8
 - **b.** 3

pp. 315-316 (#36-66 evens, #67-68, #70-71)

36. 5*x*

38. 15

40. *x* + 1

42. 16 should also be raised to the power of *x*;

 $\log_4 64^x = \log_4(16^x \cdot 4^x) = \log_4((4^2)^x \cdot 4^x) = \log_4(4^{2x} \cdot 4^x)$ $= \log_4(4^{2x} + x) = \log_4(4^{3x}) = 3x$

- **44.** $y = \log_{11} x$
- **46.** $y = \left(\frac{1}{5}\right)^x$
- **48.** $y = \frac{1}{2}e^{x}$
- **50.** $y = \ln x + 4$
- **52.** $y = 10^{x 13}$
- **54. a.** 9
 - **b.** $E = 10^{3/2(M + 9.9)}$; The inverse gives the amount of energy released from an earthquake of magnitude *M*.







domain: x > 0, range: all real numbers, asymptote: x = 0**66.** b, c, a, d



- **b.** about 281 lb
- **c.** (3.4, 0); no; The *x*-intercept shows that an alligator with a weight of 3.4 pounds has no length. If an object has weight, it must have length.
- 68. a. As x → -∞, the exponential function approaches 0. The logarithmic function is not defined for x ≤ 0. As x → ∞, the exponential function approaches ∞ and the logarithmic function approaches ∞.
 - **b.** yes; They are symmetric in the line y = x.
 - **c.** The base of each function is 6; The logarithmic function passes through the point (6, 1). So, the equation for function g is $y = \log_6 b$. Graphing this function and $y = 6^x$ in your graphing calculator produces the same graphs as the ones shown.





71. a. $\frac{2}{3}$ **b.** $\frac{5}{3}$ **c.** $\frac{4}{3}$ **d.** $\frac{7}{2}$

pp. 322-323 (#4-30 evens, #31-34)

- 4. D; The graph of g is a translation 2 units left and 2 units up of the graph of the parent function $y = 2^x$.
- 6. B; The graph of k is a translation 2 units right and 2 units up of the graph of the parent function $y = 2^x$.
- 8. The graph of g is a translation 8 units down of the graph of f.



10. The graph of g is a translation 4 units up of the graph of f.



12. The graph of g is a translation 1 unit left of the graph of f.



14. The graph of g is a translation 9 units down of the graph of f.



16. The graph of g is a translation 2 units left and $\frac{2}{3}$ unit down of the graph of *f*.



18. The graph of g is a vertical stretch by a factor of $\frac{4}{3}$ of the graph of *f*.



20. The graph of g is a horizontal stretch by a factor of 2 followed by a translation 5 units right of the graph of f.



22. The graph of g is a horizontal shrink by a factor of $\frac{1}{5}$ followed by a translation 2 units up of the graph of f.



24. The graph of *g* is a reflection in the *x*-axis followed by a translation 7 units right and 1 unit up of the graph of *f*.



26. The graph of the parent function $f(x) = 3^x$ was reflected in the *x*-axis instead of the *y*-axis.



28. The graph of *g* is a reflection in the *y*-axis followed by a translation 6 units up of the graph of *f*.



30. The graph of g is a translation 2 units left and 3 units down of the graph of f.



- **31.** A; The graph of f has been translated 2 units right.
- **32.** D; The graph of f has been translated 2 units left.
- **33.** C; The graph of *f* has been stretched vertically by a factor of 2.
- **34.** B; The graph of *f* has been shrunk horizontally by a factor of $\frac{1}{2}$.

pp. 323-324 (#35-54) 35. $g(x) = 5^{-x} - 2$ 36. $g(x) = -6\left(\frac{2}{3}\right)^{x+4}$ 37. $g(x) = e^{2x} + 5$ 38. $g(x) = \frac{1}{3}e^{-x+4} - \frac{1}{3}$ 39. $g(x) = 6 \log_6 x - 5$ 40. $g(x) = -\log_5(x+9)$ 41. $g(x) = \log_{1/2}(-x+3) + 2$ 42. $g(x) = \ln\left(\frac{1}{8}x - 3\right) + 1$ 43. Multiply the output by -1; Substitute $\log_7 x$ for f(x). Subtract 6 from the output; Substitute $-\log_7 x$ for h(x). 44. Multiply output by 4; Substitute 8^x for f(x). Add 3 to the input and add 1 to the output; Replace x with x + 3 in h(x) and add 1 to the output.

- **45.** The graph of g is a translation 4 units up of the graph of f; y = 4
- **46.** The graph of g is a translation 9 units right of the graph of f; y = 0
- **47.** The graph of g is a translation 6 units left of the graph of f; x = -6
- **49.** The graph of *S* is a vertical shrink by a factor of 0.118 followed by a translation 0.159 unit up of the graph of *f*; For fine sand, the slope of the beach is about 0.05. For medium sand, the slope of the beach is about 0.09. For coarse sand, the slope of the beach is about 0.12. For very coarse sand, the slope of the beach is about 0.16.

- **49.** The graph of *S* is a vertical shrink by a factor of 0.118 followed by a translation 0.159 units up of the graph of *f*; For fine sand, the slope of the beach is about 0.05. For medium sand, the slope of the beach is about 0.09. For coarse sand, the slope of the beach is about 0.12. For very coarse sand, the slope of the beach is about 0.16.
- **50. a.** The graph of g is a reflection in the y-axis of the graph of f.
 - **b.** no; When $f(x) = b^x$ is reflected in the *y*-axis, it becomes b^{-x} , which is equal to $g(x) = \left(\frac{1}{b}\right)^x$ for any 0 < b < 1.
- **51.** yes; *Sample answer:* If the graph is reflected in the *y*-axis, the graphs will never intersect because there are no values of *x* where $\log x = \log(-x)$.
- **52.** yes; $y = \ln x$ and $y = e^x$ are inverses. So, the graph of $y = \ln x$ is a reflection in the line y = x of the graph of $y = e^x$.
- **53.** a. never; The asymptote of $f(x) = \log x$ is a vertical line and would not change by shifting the graph vertically.
 - **b.** always; The asymptote of $f(x) = e^x$ is a horizontal line and would be changed by shifting the graph vertically.
 - c. always; The domain of $f(x) = \log x$ is x > 0 and would not be changed by a horizontal shrink.
 - **d.** sometimes; The graph of the parent exponential function does not intersect the *x*-axis, but if it is shifted down, the graph would intersect the *x*-axis.

- **54.** a. domain: $t \ge 0$, range: $0 < P \le 100$
 - **b.** 69.77 g
 - **c.** The graph would be a vertical stretch by a factor of 5.5 of the graph of the parent function.
 - **d.** The translation does not affect the domain but it does affect the range; It changes the range to $0 < P \le 550$ because the initial amount is 550, but in both cases there can never be 0 grams remaining.

p. 326 (#1-26)

- 1. exponential growth; The base is greater than 1.
- 2. exponential decay; The base is greater than 0 and less than 1.
- 3. exponential growth; a = 1 and r = 0.6 are positive.
- 4. exponential decay; a = 5 is positive and r = -2 is negative.
- 5. e^{12}
- 6. $5e^2$
- 7. $125e^{12x}$
- **8.** 9
- **9.** 2*x*
- **10.** -8x
- **11.** $4^5 = 1024$

12.
$$\left(\frac{1}{3}\right)^{-3} = 27$$

- **13.** $\log_7 2401 = 4$
- 14. $\log_4 0.0625 = -2$
- **15.** 1.653
- **16.** 0.336
- 17. 518.





- **23.** $g(x) = -\log_{1/2} x$
- **24.** $y = 150(1.0215)^t$

25. a. first account: \$1592.68, second account: \$2323.23; first account: \$92.68, second account: \$323.23;

$$1500 \left(1 + \frac{0.02}{12}\right)^{3 \cdot 12} \approx 1592.68,$$
$$2000 \left(1 + \frac{0.03}{12}\right)^{5 \cdot 12} \approx 2323.23;$$

If you subtract the minimum balance required from the total balance at the end of the term, you will get the amount of interest earned.

b. The first account requires a lesser minimum balance, but has a lesser interest rate. The second account requires a greater minimum balance, but has a greater interest rate.



p. 331 (#2-32 evens)

- 2. The expression can be evaluated using the change-of-base formula with the common logarithm or with the natural logarithm.
- **4.** 1.989
- **6.** 2.136
- **8.** −0.565
- **10.** D; Power Property
- 12. C; Product Property
- 14. $\log_8 3 + \log_8 x$
- **16.** $\ln 3 + 4 \ln x$
- **18.** $\ln 6 + 2 \ln x 4 \ln y$
- **20.** $\frac{2}{3}\log_5 x + \frac{1}{3}\log_5 y$
- **22.** The 3 is with the wrong term; $\ln 8x^3 = \ln 8 + 3 \ln x$
- **24.** ln 3
- **26.** $\log 11x^2$
- **28.** $\ln \frac{64}{v^4}$
- **30.** $\log_3 x$
- **32.** B;

 $9 \log x - 2 \log y = \log x^9 - \log y^2$

Power Property

```
=\log\frac{x^9}{y^2}
```

Quotient Property

pp. pp. 338-339 (#6-46 evens)

- 6. x = 18. x = 1**10.** $x \approx 2.173$ 12. $x = \frac{1}{2}$ 14. $x \approx 0.253$ **16.** $x \approx 0.896$ **18.** about 6967 years **20.** about 20 min **22.** x = 1024. x = 5**26.** x = 4**28.** x = 123**30.** x = 9**32.** x = -6 and x = -3**34.** x = 9**36.** $x \approx 13.22$ **38.** x = -2 and x = -8**40.** $x \approx 2.72$ 42. The solutions were not checked to see if either was extraneous; $\log_4(x + 12) + \log_4 x = 3$
 - $log_{4}[(x + 12)(x)] = 3$ $4^{log_{4}[(x + 12)(x)]} = 4^{3}$ (x + 12)(x) = 64 $x^{2} + 12x 64 = 0$ (x + 16)(x 4) = 0 x = 4
- **44.** 100 mm
- 46. no; The solution can be negative. For example, log(9 x) = 1 has the solution x = -1.

pp. 339-340 (#48-62 evens, #63-64, #67-74)

- **48.** $x \le 2.585$ **50.** 0 < x < 256**52.** x > -0.534
 - **54.** $0 < x \le \frac{1}{25}$
- 56. after 5.22 years; after 35.85 years
- 58. from the time it was purchased until about 4.25 years later
- **60.** $x \approx 1.23$

62.
$$x \approx 0.54$$
 and $x \approx 3.69$

63. a.
$$a = -\frac{1}{0.09} \ln\left(\frac{45 - \ell}{25.7}\right)$$

- **b.** 36 cm footprint: 11.7 years old; 32 cm footprint:
 7.6 years old; 28 cm footprint: 4.6 years old;
 24 cm footprint: 2.2 years old
- 64. x > 2.1; The graph of $y = 4 \ln x + 6$ is above the graph of y = 9 for all values such that x > 2.1.
- **67.** $x \approx 0.89$
- **68.** $x \approx 2.88$
- **69.** $x \approx 10.61$
- **70.** x = 16
- **71.** x = 2 and x = 3
- **72.** x = 1
- **73.** To solve exponential equations with different bases, take a logarithm of each side. Then use the Power Property to move the exponent to the front of the logarithm, and solve for *x*. To solve logarithmic equations of different bases, find a common multiple of the bases, and exponentiate each side with this common multiple as the base. Rewrite the base as a power that will cancel out the given logarithm and solve the resulting equation.

- 74. a. 2.80 cm
 - **b.** 0.37 cm
 - **c.** 0.03 cm
 - **d.** A lead apron does not need to be as thick as aluminum or copper to result in the same intensity.

pp. 346-347 (#4-16 evens, #17-24)

- 4. exponential; The data have a common ratio of $\frac{1}{2}$.
- 6. linear; The first differences are constant.

8.
$$y = \frac{2}{3}(6)^x$$

10.
$$y = 3^x$$

12.
$$y = 10(4)^x$$

14.
$$y = 0.4(4)^x$$

16.
$$y = 0.4 \left(\frac{1}{3}\right)^x$$

- **17.** Data are linear when the first differences are constant; The outputs have a common ratio of 3, so the data represents an exponential function.
 - **18.** The *x*-values are not evenly spaced; The data are modeled by a quartic function that can be found using the *regression* feature of a graphing calculator.
- **19.** Sample answer: $y = 7.20(1.39)^x$
- **20.** Sample answer: $y = 12.26(1.79)^x$; about 13,266
- **21.** yes; *Sample answer:* $y = 8.88(1.21)^x$
- **22.** yes; Sample answer: $y = 11.12(1.77)^x$
- **23.** no; *Sample answer:* y = -0.8x + 66
- **24.** yes; *Sample answer:* $y = 11.08(0.96)^x$



The points $(x, \ln y)$ are not linear, so an exponential model does not fit the data.

32. $y = 153.07(0.93)^x$; about 48 beats/min

- **33.** $t = 12.59 2.55 \ln d$; 2.6 h
- **34.** $s = 0.000398 + 2.89 \ln f$; about 5 units of light
- 35. a.



Sample answer: $y = 0.50(1.47)^{x}$

- **b.** about 47%; The base is 1.47 which means that the function shows 47% growth.
- **36.** yes; The points $(x, \ln y)$ are linear.
- **37.** no; When *d* is the independent variable and *t* is the dependent variable, the data can be modeled with a logarithmic function. When the variables are switched, the data can be modeled with an exponential function.
- **38.** yes; The data have a common ratio of 2.
- **39. a.** 5.9 weeks



The asymptote is the line y = 256 and represents the maximum height of the sunflower.

pp. 350-352 (#2-38 evens and #39) 2. exponential growth; 400% increase



4. \$1725.39

6.
$$\frac{2}{e^3}$$

8. exponential growth



10. exponential decay





- 14. $g(x) = \log_8 x$
- **16.** $y = 10^x 9$
- **18.** The graph of g is a horizontal shrink by a factor of $\frac{1}{5}$ followed by a translation 8 units down of the graph of *f*.



- **20.** $g(x) = 3e^{x+6} + 3$
- **22.** $\log_8 3 + \log_8 x + \log_8 y$
- **24.** $\ln 3 + \ln y 5 \ln x$

26.
$$\log_2 \frac{12}{x^2}$$

- **28.** about 3.32
- **30.** about 1.19

32. x = 7 **34.** x > 1.39 **36.** $x \ge 1.19$ **38.** *Sample answer:* $y = 3.60(1.43)^x$ **39.** $s = 3.95 + 27.48 \ln t$; 53 pairs