20th Annual

AMC 8

(American Mathematics Contest 8)
Tuesday, NOVEMBER 16, 2004

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids are permitted other than scratch paper, graph paper, rulers, erasers, and calculators that are accepted for use on the SAT. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

1. On a map, a 12-centimeter length represents 72 kilometers. How many kilometers does a 17-centimeter length represent?

   (A) 6 **(B)** 102 **(C)** 204 **(D)** 864 **(E)** 1224

2. How many different four-digit numbers can be formed by rearranging the four digits in 2004?

   (A) 4 **(B)** 6 **(C)** 16 **(D)** 24 **(E)** 81

3. Twelve friends met for dinner at Oscar's Overstuffed Oyster House, and each ordered one meal. The portions were so large, there was enough food for 18 people. If they share, how many meals should they have ordered to have just enough food for the 12 of them?

   (A) 8 **(B)** 9 **(C)** 10 **(D)** 15 **(E)** 18

   The following information is needed to solve problems 4, 5 and 6.

   Ms. Hamilton's eighth-grade class wants to participate in the annual three-person-team basketball tournament.

4. Lance, Sally, Joy and Fred are chosen for the team. In how many ways can the three starters be chosen?

   (A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

5. The losing team of each game is eliminated from the tournament. If sixteen teams compete, how many games will be played to determine the winner?

   (A) 4 **(B)** 7 **(C)** 8 **(D)** 15 **(E)** 16

6. After Sally takes 20 shots, she has made 55% of her shots. After she takes 5 more shots, she raises her percentage to 56%. How many of the last 5 shots did she make?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

7. An athlete's target heart rate, in beats per minute, is 80% of the theoretical maximum heart rate. The maximum heart rate is found by subtracting the athlete's age, in years, from 220. To the nearest whole number, what is the target heart rate of an athlete who is 26 years old?

   (A) 134 **(B)** 155 **(C)** 176 **(D)** 194 **(E)** 243

8. Find the number of two-digit positive integers whose digits total 7.

   (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

9. The average of the five numbers in a list is 54. The average of the first two numbers is 48. What is the average of the last three numbers?

   (A) 55 **(B)** 56 **(C)** 57 **(D)** 58 **(E)** 59

10. Handy Aaron helped a neighbor $1\frac{1}{4}$ hours on Monday, 50 minutes on Tuesday, from 8:20 to 10:45 on Wednesday morning, and a half-hour on Friday. He is paid $3 per hour. How much did he earn for the week?

(A) $8   (B) $9   (C) $10   (D) $12   (E) $15

11. The numbers -2, 4, 6, 9 and 12 are rearranged according to these rules:

    1. The largest isn't first, but it is in one of the first three places.

    2. The smallest isn't last, but it is in one of the last three places.

    3. The median isn't first or last.

What is the average of the first and last numbers?

(A) 3.5   (B) 5   (C) 6.5   (D) 7.5   (E) 8

12. Niki usually leaves her cell phone on. If her cell phone is on but she is not actually using it, the battery will last for 24 hours. If she is using it constantly, the battery will last for only 3 hours. Since the last recharge, her phone has been on 9 hours, and during that time she has used it for 60 minutes. If she doesn't talk any more but leaves the phone on, how many more hours will the battery last?
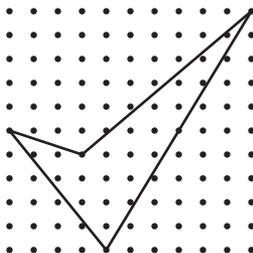
(A) 7   (B) 8   (C) 11   (D) 14   (E) 15

13. Amy, Bill and Celine are friends with different ages. Exactly one of the following statements is true.

    I. Bill is the oldest.

    II. Amy is not the oldest.
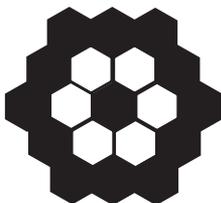
    III. Celine is not the youngest.

Rank the friends from the oldest to the youngest.

(A) Bill, Amy, Celine   (B) Amy, Bill, Celine   (C) Celine, Amy, Bill
    (D) Celine, Bill, Amy   (E) Amy, Celine, Bill

14. What is the area enclosed by the geoboard quadrilateral below?

(A) 15   (B) $18\frac{1}{2}$ (C) $22\frac{1}{2}$ (D) 27   (E) 41

15. Thirteen black and six white hexagonal tiles were used to create the figure below. If a new figure is created by attaching a border of white tiles with the same size and shape as the others, what will be the difference between the total number of white tiles and the total number of black tiles in the new figure?

(A) 5   (B) 7   (C) 11   (D) 12   (E) 18

16. Two 600 ml pitchers contain orange juice. One pitcher is $\frac{1}{3}$ full and the other pitcher is $\frac{2}{5}$ full. Water is added to fill each pitcher completely, then both pitchers are poured into one large container. What fraction of the mixture in the large container is orange juice?

(A) $\dfrac{1}{8}$ (B) $\dfrac{3}{16}$ (C) $\dfrac{11}{30}$ (D) $\dfrac{11}{19}$ (E) $\dfrac{11}{15}$

17. Three friends have a total of 6 identical pencils, and each one has at least one pencil. In how many ways can this happen?

(A) 1   (B) 3   (C) 6   (D) 10   (E) 12

18. Five friends compete in a dart-throwing contest. Each one has two darts to throw at the same circular target, and each individual's score is the sum of the scores in the target regions that are hit. The scores for the target regions are the whole numbers 1 through 10. Each throw hits the target in a region with a different value. The scores are: Alice 16 points, Ben 4 points, Cindy 7 points, Dave 11 points, and Ellen 17 points. Who hits the region worth 6 points?
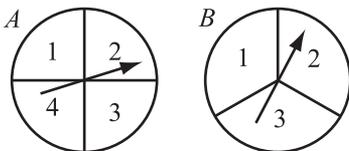
(A) Alice   (B) Ben   (C) Cindy   (D) Dave   (E) Ellen

19. A whole number larger than 2 leaves a remainder of 2 when divided by each of the numbers 3, 4, 5 and 6. The smallest such number lies between which two numbers?

(A) 40 and 49   (B) 60 and 79   (C) 100 and 129   (D) 210 and 249
(E) 320 and 369

20. Two-thirds of the people in a room are seated in three-fourths of the chairs. The rest of the people are standing. If there are 6 empty chairs, how many people are in the room?

(A) 12   (B) 18   (C) 24   (D) 27   (E) 36

21. Spinners A and B are spun. On each spinner, the arrow is equally likely to land on each number. What is the probability that the product of the two spinners' numbers is even?
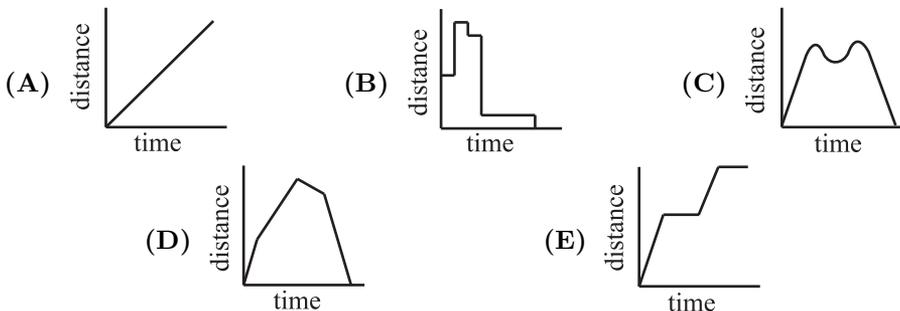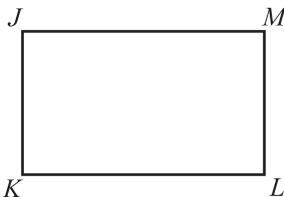

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{2}{3}$ (E) $\dfrac{3}{4}$

22. At a party there are only single women and married men with their wives. The probability that a randomly selected woman is single is $\frac{2}{5}$. What fraction of the people in the room are married men?
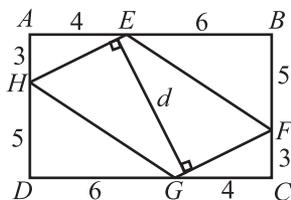
(A) $\dfrac{1}{3}$ (B) $\dfrac{3}{8}$ (C) $\dfrac{2}{5}$ (D) $\dfrac{5}{12}$ (E) $\dfrac{3}{5}$
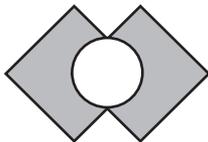
23. Tess runs counterclockwise around rectangular block $JKLM$. She lives at corner J. Which graph could represent her straight-line distance from home?

24. In the figure, $ABCD$ is a rectangle and $EFGH$ is a parallelogram. Using the measurements given in the figure, what is the length d of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$?

(**A**) 6.8   (**B**) 7.1   (**C**) 7.6   (**D**) 7.8   (**E**) 8.1

25. Two 4×4 squares intersect at right angles, bisecting their intersecting sides, as shown. The circle's diameter is the segment between the two points of intersection. What is the area of the shaded region created by removing the circle from the squares?

(**A**) $16 - 4\pi$ (**B**) $16 - 2\pi$ (**C**) $28 - 4\pi$ (**D**) $28 - 2\pi$ (**E**) $32 - 2\pi$

SOLUTIONS

Your School Manager will be sent at least one copy of the 2004 AMC 8 Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

WRITE TO US

Comments about the problems and solutions for this AMC 8 should be addressed to:
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548 Hill Avenue, New Braunfels, TX 78130

Comments about administrative arrangements should be addressed to:
MAA American Mathematics Competitions / amcinfo@unl.edu
American Mathematics Competitions, University of Nebraska-Lincoln
P.O. Box 880658, Lincoln, NE 68588-0658

AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute contests with 5 choices of answers for each problem (A through E). Schools with high scoring students on the AMC 8 will receive an Invitation Brochure for the 2005 AMC 10. The best way to prepare for these upper level contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

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Each price is for an exam and its solutions for one year. Specify the years you want and how many copies of each exam. All prices effective to September 1, 2005.

AMC 8 (Junior High/Middle School contests), 1986-2003, $1.00 per copy per year.
AMC 10 (9th and 10th grade High School contests) 2000-2004. $1.00 per copy per year.
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2004

AMC 8

DO NOT OPEN UNTIL
TUESDAY, NOVEMBER 16, 2004

Administration On An Earlier Date Will Disqualify Your School's Results

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE NOVEMBER 16, 2004. Nothing is needed from inside this package until November 16.

2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.

3. The Answer Forms must be mailed First Class to the AMC office no later than 24 hours following the exam.

4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.

5. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Duplication at any time via copier, telephone, e-mail, World Wide Web or media of any type is a violation of the competition rules.*