AMERICAN MATHEMATICS COMPETITIONS

14th ANNUAL

AMERICAN JUNIOR HIGH SCHOOL MATHEMATICS EXAMINATION

(AJHSME)

TUESDAY, NOVEMBER 17, 1998

Sponsored by

Mathematical Association of America

Society of Actuaries Mu Alpha Theta

National Council of Teachers of Mathematics

Casualty Actuarial Society American Statistical Association

American Mathematical Association of Two-Year Colleges

American Mathematical Society

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Consortium for Mathematics and its Applications

Pi Mu Epsilon National Association of Mathematicians

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AJHSME ANSWER FORM with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids are permitted other than scratch paper, graph paper, ruler, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working the problems. You will have **40 MINUTES** working time for the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. For $x = 7$, which of the following is smallest?

   (A) $\frac{6}{x}$ **(B)** $\frac{6}{x+1}$ **(C)** $\frac{6}{x-1}$ **(D)** $\frac{x}{6}$ **(E)** $\frac{x+1}{6}$

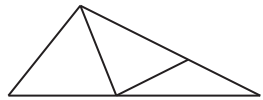
2. If $\frac{a|b}{c|d} = a \cdot d - b \cdot c$, what is the value of $\frac{3|4}{1|2}$?

   (A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

3. $\frac{\frac{3}{8} + \frac{7}{8}}{\frac{4}{5}} =$

   (A) 1 **(B)** $\frac{25}{16}$ **(C)** 2 **(D)** $\frac{43}{20}$ **(E)** $\frac{47}{16}$

4. How many triangles are in this figure?
   (Some triangles may overlap other triangles.)

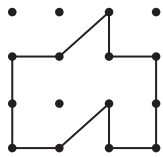
   

   (A) 9 **(B)** 8 **(C)** 7 **(D)** 6 **(E)** 5

5. Which of the following numbers is largest?

   (A) 9.12344 **(B)** $9.123\overline{4}$ **(C)** $9.12\overline{34}$ **(D)** $9.1\overline{234}$ **(E)** $9.\overline{1234}$

6. Dots are spaced one unit part, horizontally and vertically.
   The number of square units enclosed by the polygon is

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

7. $100 \times 19.98 \times 1.998 \times 1000 =$

   (A) $(1.998)^2$ **(B)** $(19.98)^2$ **(C)** $(199.8)^2$ **(D)** $(1998)^2$ **(E)** $(19980)^2$

8. A child's wading pool contains 200 gallons of water. If water evaporates at the rate of 0.5 gallons per day and no other water is added or removed, how many gallons of water will be in the pool after 30 days?

(A) 140 **(B)** 170 **(C)** 185 **(D)** 198.5 **(E)** 199.85

9. For a sale, a store owner reduces the price of a \$10 scarf by 20%. Later the price is lowered again, this time by one-half the reduced price. The price is now

(A) \$2.00 **(B)** \$3.75 **(C)** \$4.00 **(D)** \$4.90 **(E)** \$6.40

10. Each of the letters W, X, Y, and Z represents a different integer in the set $\{1, 2, 3, 4\}$, but not necessarily in that order. If $\frac{W}{X} - \frac{Y}{Z} = 1$, then the sum of W and Y is

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

11. Harry has 3 sisters and 5 brothers. His sister Harriet has S sisters and B brothers. What is the product of S and B?

(A) 8 **(B)** 10 **(C)** 12 **(D)** 15 **(E)** 18

12. $2(1 - \frac{1}{2}) + 3(1 - \frac{1}{3}) + 4(1 - \frac{1}{4}) + \ldots + 10(1 - \frac{1}{10}) =$

(A) 45 **(B)** 49 **(C)** 50 **(D)** 54 **(E)** 55

13. What is the ratio of the area of the shaded square to the area of the large square? (The figure is drawn to scale.)

(A) $\frac{1}{6}$ **(B)** $\frac{1}{7}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{12}$ **(E)** $\frac{1}{16}$

14. At Annville Junior High School, 30% of the students in the Math Club are in the Science Club, and 80% of the students in the Science Club are in the Math Club. There are 15 students in the Science Club. How many students are in the Math Club?

(A) 12 **(B)** 15 **(C)** 30 **(D)** 36 **(E)** 40

Problems 15, 16, 17 all refer to the following:

Don't Crowd The Isles

In the very center of the Irenic Sea lie the beautiful Nisos Isles. In 1998 the number of people on these islands is only 200, but the population triples every 25 years. Queen Irene has decreed that there must be at least 1.5 square miles for every person living in the Isles. The total area of the Nisos Isles is 24,900 square miles.

15. Estimate the population of Nisos in the year 2050.

(A) 600 **(B)** 800 **(C)** 1000 **(D)** 2000 **(E)** 3000

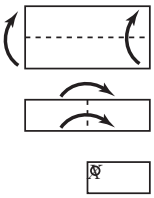
16. Estimate the year in which the population of Nisos will be approximately 6,000.

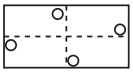
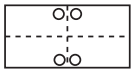
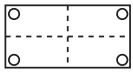
(A) 2050 **(B)** 2075 **(C)** 2100 **(D)** 2125 **(E)** 2150

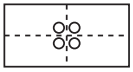
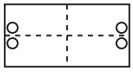
17. In how many years, approximately, from 1998 will the population of Nisos be as much as Queen Irene has proclaimed that the islands can support?

(A) 50 yrs. **(B)** 75 yrs. **(C)** 100 yrs. **(D)** 125 yrs. **(E)** 150 yrs.

18. As indicated by the diagram at the right, a rectangular piece of paper is folded bottom to top, then left to right, and finally, a hole is punched at X. What does the paper look like when unfolded?



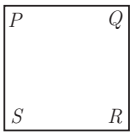
(A)  **(B)**  **(C)** 

(D)  **(E)** 

19. Tamika selects two different numbers at random from the set $\{8, 9, 10\}$ and adds them. Carlos takes two different numbers at random from the set $\{3, 5, 6\}$ and multiplies them. What is the probability that Tamika's result is greater than Carlos' result?

(A) $\frac{4}{9}$ **(B)** $\frac{5}{9}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{2}{3}$

20. Let $PQRS$ be a square piece of paper. P is folded onto R and then Q is folded onto S. The area of the resulting figure is 9 square inches. Find the perimeter of square $PQRS$.



(A) 9 **(B)** 16 **(C)** 18 **(D)** 24 **(E)** 36

21. A $4 \times 4 \times 4$ cubical box contains 64 identical small cubes that exactly fill the box. How many of these small cubes touch a side or the bottom of the box?

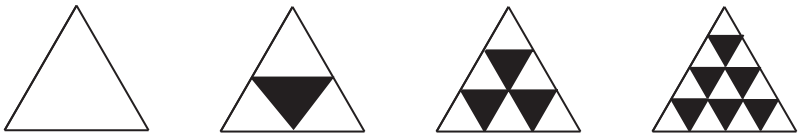
(A) 48 **(B)** 52 **(C)** 60 **(D)** 64 **(E)** 80

22. Terri produces a sequence of positive integers by following three rules. She starts with a positive integer, then applies the appropriate rule to the result, and continues in this fashion.
Rule 1: If the integer is less than 10, multiply it by 9.
Rule 2: If the integer is even and greater than 9, divide it by 2.
Rule 3: If the integer is odd and greater than 9, subtract 5 from it.

A sample sequence: 23, 18, 9, 81, 76, ...
Find the 98^{th} term of the sequence that begins 98, 49, ...

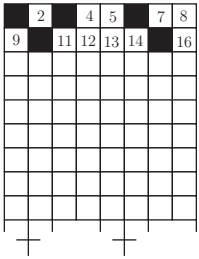
(A) 6 **(B)** 11 **(C)** 22 **(D)** 27 **(E)** 54

23. If the pattern in the diagram continues, what fraction of the interior would be shaded in the eighth triangle?



(A) $\frac{3}{8}$ **(B)** $\frac{5}{27}$ **(C)** $\frac{7}{16}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{11}{45}$

24. A rectangular board of 8 columns has squares numbered beginning in the upper left corner and moving left to right so row one is numbered 1 through 8, row two is 9 through 16, and so on. A student shades square 1, then skips one square and shades square 3, skip two squares and shades square 6, ships 3 squares and shades square 10, and continues in this way until there is at least one shaded square in each column. What is the number of the shaded square that first achieves this result?



(A) 36 **(B)** 64 **(C)** 78 **(D)** 91 **(E)** 120

25. Three generous friends, each with some cash, redistribute their money as follows: Ami gives enough money to Jan and Toy to double the amount that each has. Jan then gives enough to Ami and Toy to double their amounts. Finally, Toy gives Ami and Jan enough to double their amounts. If Toy has \$36 when they begin and \$36 when they end, what is the total amount that all three friends have?

(A) \$108 **(B)** \$180 **(C)** \$216 **(D)** \$252 **(E)** \$288

SOLUTIONS

Your School Examination Manager will be sent at least one copy of the 1998 AJHSME Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

WRITE TO US!

Correspondence about the problems and solutions for this AJHSME should be addressed to:

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1999 AHSME

The American High School Mathematics Examination [AHSME] is a 30-question, 90-minute, multiple choice examination. Schools with high-scoring students on the AJHSME will receive a 1999 AHSME Invitation Brochure containing information about the AHSME and the registration procedure. The best way to prepare for the AHSME is to study the exams from previous years. Orders for all publications listed below should be addressed to:

Dr. Walter E. Mientka, AMC Executive Director
American Mathematics Competitions
University of Nebraska-Lincoln
P.O. Box 81606
Lincoln, NE 68501-1606

PUBLICATIONS

MINIMUM ORDER: $5 (before handling fee), US FUNDS ONLY. Canada and US orders must be prepaid. Orders are mailed 4th class, unless you specify 1st class, in which case add 20% of the total order, with a minimum of $3 and a maximum of $15 (USA) and a $4 mimimum and $20 maximum(Canada). **Please note that if the correct 1st class cost is not included, the order will be sent 4th class.** Make checks payable to the American Mathematics Competitions; or give Visa or MasterCard number, expiration date and cardholders home address.

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Examinations: Each price is for an examination **and** its solutions for one year. Specify the years you want and how many copies of each. All prices effective to September 1, 1999.

- **AJHSME** (Junior High Exam), 1985-1998, $1 per copy per year.
- **AHSME** (High School Exam) 1980-98, $1 per copy per year.
- **AJHSME Summary** of Results and Awards, 1985-98, $5 per copy per year.
- **AHSME Summary** of Results and Awards, 1980-98, $10 per copy per year.

Books (Exams and Solutions):

- Problem Book I, AHSMEs 1950-60, $9.00
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- Problem Book III, AHSMEs 1966-72, $13.00
- Problem Book IV, AHSMEs 1973-82, $13.00
- Problem Book V, AHSMEs and AIMEs 1983-88, $30.00

1998

American Junior High School Mathematics Examination

(AJHSME)

DO NOT OPEN UNTIL TUESDAY, NOVEMBER 17, 1998

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer the AJHSME is contained in the AJHSME TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE NOVEMBER 17.** Nothing is needed from inside this package until November 17.

2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AJHSME CERTIFICATION Form that all rules associated with the conduct of the examination were followed.

3. The Answer Forms must be mailed by First Class Mail to Dr. Mientka no late than 24 hours following the Examination.

4. THE AJHSME IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAMINATION MAY BE GIVEN DURING THE REGULAR MATHEMATICS CLASS PERIOD OF THE STUDENTS IF IT IS NOT POSSIBLE TO ADMINISTER THE EXAMINATION TO ALL STUDENTS DURING ONE 40 MINUTE PERIOD.

5. *The publication, reproduction or communications of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Duplication at any time via copier, telephone, eMail, World Wide Web or media of any type is a violation of the copyright law.*