12.4

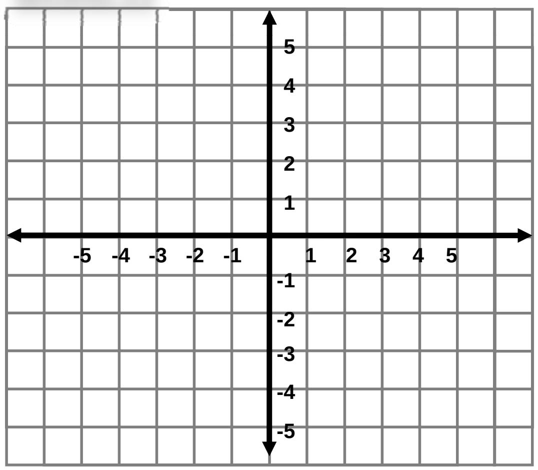
Graphs of Quadratic Equations: The Discriminant

Solve using the quadratic formula

1) $x^2 - 2x - 3 = 0$

GRAPH

$$f(x) = x^2 - 2x - 3$$



x	$f(x)$

Observations…

Example 1

Equation: $x^2 + 4x + 1 = 0$

Solution:

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm 2\sqrt{3}}{2}$$

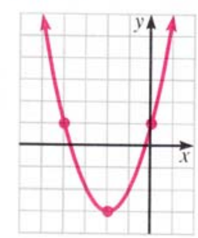
$$= -2 \pm \sqrt{3}$$

$\therefore$ the solution set is $\{-2 + \sqrt{3}, -2 - \sqrt{3}\}$.

Number of roots: two real-number roots

Related equation: $y = x^2 + 4x + 1$

Graph:



Number of x-intercepts: two

Observations...

Example 2

Equation: $x^2 + 2x + 1 = 0$

Related equation: $y = x^2 + 2x + 1$

Solution:

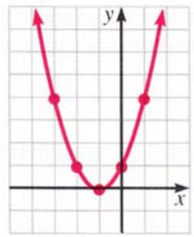
$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{0}}{2}$$

$$= -1$$

$\therefore$ the solution set is $\{-1\}$.

Graph:



Number of roots: one real-number root

Number of x-intercepts: one

Observations...

Example 3

Equation: $x^2 - 4x + 7 = 0$

Related equation: $y = x^2 - 4x + 7$

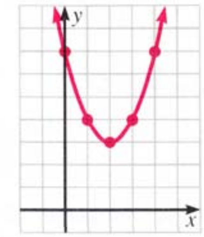
Solution:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-12}}{2}$$

There is no real-number root since $\sqrt{-12}$ does not represent a real number.

Graph:



Number of roots: no real-number roots

Number of x-intercepts: none

Key to Finding the Number of Roots

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b^2 - 4ac$$

Discriminant

$b^2 - 4ac$	# of Roots (x-intercepts)

Use the discriminant to find the number of roots

1) $x^2 - 3x + 4 = 0$

2) $a^2 + 4a - 5 = 0$

3) $4b^2 - 3b - 6 = 0$

Use the discriminant to find the number of roots

4) $25x^2 + 20x + 4 = 0$

5) $0.5x^2 + 1.2x + 3 = 0$

Find the indicated information:

a) the number x-intercepts

b) whether it's vertex lies above, below, or on the x-axis (without finding the vertex)

6) $-x^2 + 5x + 6$