

THE MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions



24th Annual

AMC 8

(American Mathematics Contest 8)

Tuesday, November 18, 2008

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids are permitted other than scratch paper, graph paper, rulers, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

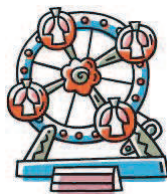
The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it determines that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the AMC 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules. After the contest period, permission to make copies of individual problems in paper or electronic form including posting on web-pages for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

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1. Susan had \$50 to spend at the carnival. She spent \$12 on food and twice as much on rides. How many dollars did she have left to spend?

(A) 12 (B) 14 (C) 26 (D) 38 (E) 50



2. The ten-letter code BEST OF LUCK represents the ten digits 0–9, in order. What 4-digit number is represented by the code word CLUE?

(A) 8671 (B) 8672 (C) 9781 (D) 9782 (E) 9872

3. If February is a month that contains Friday the 13th, what day of the week is February 1?

(A) Sunday (B) Monday (C) Wednesday (D) Thursday
(E) Saturday



4. In the figure, the outer equilateral triangle has area 16, the inner equilateral triangle has area 1, and the three trapezoids are congruent. What is the area of one of the trapezoids?



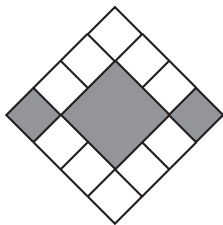
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

5. Barney Schwinn notices that the odometer on his bicycle reads 1441, a palindrome, because it reads the same forward and backward. After riding 4 more hours that day and 6 the next, he notices that the odometer shows another palindrome, 1661. What was his average speed in miles per hour?

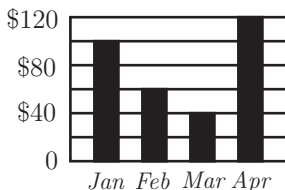
(A) 15 (B) 16 (C) 18 (D) 20 (E) 22



6. In the figure, what is the ratio of the area of the gray squares to the area of the white squares?



- (A) 3 : 10 (B) 3 : 8 (C) 3 : 7 (D) 3 : 5 (E) 1 : 1
7. If $\frac{3}{5} = \frac{M}{45} = \frac{60}{N}$, what is $M + N$?
- (A) 27 (B) 29 (C) 45 (D) 105 (E) 127
8. Candy sales of the Boosters Club for January through April are shown. What were the average sales per month in dollars?



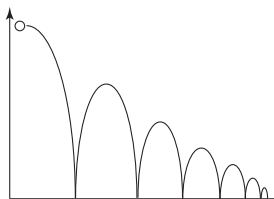
- (A) 60 (B) 70 (C) 75 (D) 80 (E) 85
9. In 2005 Tycoon Tammy invested \$100 for two years. During the first year her investment suffered a 15% loss, but during the second year the remaining investment showed a 20% gain. Over the two-year period, what was the change in Tammy's investment?
- (A) 5% loss (B) 2% loss (C) 1% gain (D) 2% gain (E) 5% gain
10. The average age of the 6 people in Room A is 40. The average age of the 4 people in Room B is 25. If the two groups are combined, what is the average age of all the people?
- (A) 32.5 (B) 33 (C) 33.5 (D) 34 (E) 35

11. Each of the 39 students in the eighth grade at Lincoln Middle School has one dog or one cat or both a dog and a cat. Twenty students have a dog and 26 students have a cat. How many students have both a dog and a cat?

(A) 7 (B) 13 (C) 19 (D) 39 (E) 46



12. A ball is dropped from a height of 3 meters. On its first bounce it rises to a height of 2 meters. It keeps falling and bouncing to $\frac{2}{3}$ of the height it reached in the previous bounce. On which bounce will it not rise to a height of 0.5 meters?



(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

13. Mr. Harman needs to know the combined weight in pounds of three boxes he wants to mail. However, the only available scale is not accurate for weights less than 100 pounds or more than 150 pounds. So the boxes are weighed in pairs in every possible way. The results are 122, 125 and 127 pounds. What is the combined weight in pounds of the three boxes?

(A) 160 (B) 170 (C) 187 (D) 195 (E) 354



14. Three As, three Bs and three Cs are placed in the nine spaces so that each row and column contain one of each letter. If A is placed in the upper left corner, how many arrangements are possible?

A		

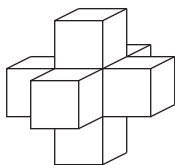
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

15. In Theresa's first 8 basketball games, she scored 7, 4, 3, 6, 8, 3, 1 and 5 points. In her ninth game, she scored fewer than 10 points and her points-per-game average for the nine games was an integer. Similarly in her tenth game, she scored fewer than 10 points and her points-per-game average for the 10 games was also an integer. What is the product of the number of points she scored in the ninth and tenth games?



(A) 35 (B) 40 (C) 48 (D) 56 (E) 72

16. A shape is created by joining seven unit cubes, as shown. What is the ratio of the volume in cubic units to the surface area in square units?

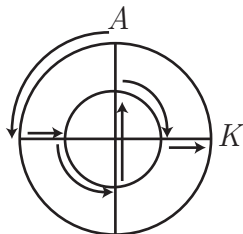


(A) 1 : 6 (B) 7 : 36 (C) 1 : 5 (D) 7 : 30 (E) 6 : 25

17. Ms. Osborne asks each student in her class to draw a rectangle with integer side lengths and a perimeter of 50 units. All of her students calculate the area of the rectangle they draw. What is the difference between the largest and smallest possible areas of the rectangles?

(A) 76 (B) 120 (C) 128 (D) 132 (E) 136

18. Two circles that share the same center have radii 10 meters and 20 meters. An aardvark runs along the path shown, starting at A and ending at K . How many meters does the aardvark run?

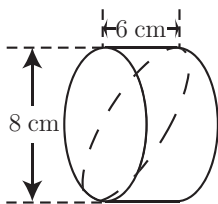


(A) $10\pi + 20$ (B) $10\pi + 30$ (C) $10\pi + 40$ (D) $20\pi + 20$ (E) $20\pi + 40$

19. Eight points are spaced at intervals of one unit around a 2×2 square, as shown. Two of the 8 points are chosen at random. What is the probability that the points are one unit apart?

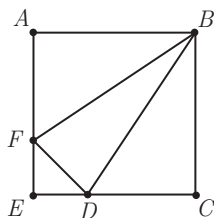


- (A) $\frac{1}{4}$ (B) $\frac{2}{7}$ (C) $\frac{4}{11}$ (D) $\frac{1}{2}$ (E) $\frac{4}{7}$
20. The students in Mr. Neatkin's class took a penmanship test. Two-thirds of the boys and $\frac{3}{4}$ of the girls passed the test, and an equal number of boys and girls passed the test. What is the minimum possible number of students in the class?
- (A) 12 (B) 17 (C) 24 (D) 27 (E) 36
21. Jerry cuts a wedge from a 6-cm cylinder of bologna as shown by the dashed curve. Which answer choice is closest to the volume of his wedge in cubic centimeters?



- (A) 48 (B) 75 (C) 151 (D) 192 (E) 603
22. For how many positive integer values of n are both $\frac{n}{3}$ and $3n$ three-digit whole numbers?
- (A) 12 (B) 21 (C) 27 (D) 33 (E) 34

23. In square $ABCE$, $AF = 2FE$ and $CD = 2DE$. What is the ratio of the area of $\triangle BFD$ to the area of square $ABCE$?



- (A) $\frac{1}{6}$ (B) $\frac{2}{9}$ (C) $\frac{5}{18}$ (D) $\frac{1}{3}$ (E) $\frac{7}{20}$

24. Ten tiles numbered 1 through 10 are turned face down. One tile is turned up at random, and a die is rolled. What is the probability that the product of the numbers on the tile and the die will be a square?

- (A) $\frac{1}{10}$ (B) $\frac{1}{6}$ (C) $\frac{11}{60}$ (D) $\frac{1}{5}$ (E) $\frac{7}{30}$

25. Margie's winning art design is shown. The smallest circle has radius 2 inches, with each successive circle's radius increasing by 2 inches. Approximately what percent of the design is black?



- (A) 42 (B) 44 (C) 45 (D) 46 (E) 48

2008

AMC 8

DO NOT OPEN UNTIL
TUESDAY, NOVEMBER 18, 2008

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE NOVEMBER 18, 2008. Nothing is needed from inside this package until November 18.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed First Class to the AMC office no later than 24 hours following the exam.
4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.
5. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, e-mail, World Wide Web or media of any type is a violation of the competition rules.*

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SOLUTIONS

Your School Manager has been sent at least one copy of the 2008 AMC 8 Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

WRITE TO US

Comments about the problems and solutions for this AMC 8 should be addressed to:

Ms. Bonnie Leitch, AMC 8 Chair / bleitch@earthlink.net
548 Hill Avenue, New Braunfels, TX 78130

Comments about administrative arrangements should be addressed to:

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American Mathematics Competitions, University of Nebraska-Lincoln
P.O. Box 880658, Lincoln, NE 68588-0658

AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute contests with 5 choices of answers for each problem (A through E). Schools with high scoring students on the AMC 8 will receive an Invitation Brochure for the 2009 AMC 10. The best way to prepare for these upper level contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

American Mathematics Competitions
ATTN: Publications
P.O. Box 81606
Lincoln, NE 68501-1606

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.unl.edu/amc.