

THE MATHEMATICAL ASSOCIATION OF AMERICA  
American Mathematics Competitions



22<sup>nd</sup> Annual

AMC 8

(American Mathematics Contest 8)

Tuesday, NOVEMBER 14, 2006

### INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids are permitted other than scratch paper, graph paper, rulers, erasers, and calculators that are accepted for use on the SAT. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

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The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it determines that the required security procedures were not followed.

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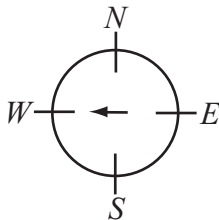
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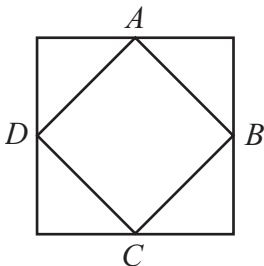
1. Mindy made three purchases for \$1.98, \$5.04 and \$9.89. What was her total, to the nearest dollar?  
 (A) \$10            (B) \$15            (C) \$16            (D) \$17            (E) \$18
2. On the AMC 8 contest Billy answers 13 questions correctly, answers 7 questions incorrectly and doesn't answer the last 5. What is his score?  
 (A) 1            (B) 6            (C) 13            (D) 19            (E) 26
3. Elisa swims laps in the pool. When she first started, she completed 10 laps in 25 minutes. Now she can finish 12 laps in 24 minutes. By how many minutes has she improved her lap time?  
 (A)  $\frac{1}{2}$             (B)  $\frac{3}{4}$             (C) 1            (D) 2            (E) 3



4. Initially, a spinner points west. Chenille moves it clockwise  $2\frac{1}{4}$  revolutions and then counterclockwise  $3\frac{3}{4}$  revolutions. In what direction does the spinner point after the two moves?  
 (A) north (B) east (C) south (D) west (E) northwest



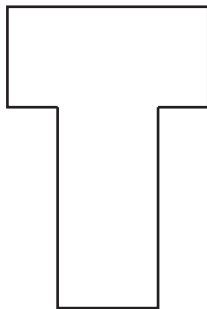
5. Points  $A$ ,  $B$ ,  $C$  and  $D$  are midpoints of the sides of the larger square. If the larger square has area 60, what is the area of the smaller square?



- (A) 15            (B) 20            (C) 24            (D) 30            (E) 40

6. The letter T is formed by placing two 2 inch  $\times$  4 inch rectangles next to each other, as shown. What is the perimeter of the T, in inches?

(A) 12                      (B) 16                      (C) 20                      (D) 22                      (E) 24

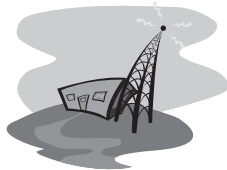


7. Circle  $X$  has a radius of  $\pi$ . Circle  $Y$  has a circumference of  $8\pi$ . Circle  $Z$  has an area of  $9\pi$ . List the circles in order from smallest to largest radius.

(A)  $X, Y, Z$               (B)  $Z, X, Y$               (C)  $Y, X, Z$               (D)  $Z, Y, X$               (E)  $X, Z, Y$

8. The table shows some of the results of a survey by radio station KAMC. What percentage of the males surveyed listen to the station?

	Listen	Don't Listen	Total
Male	?	26	?
Female	58	?	96
Total	136	64	200

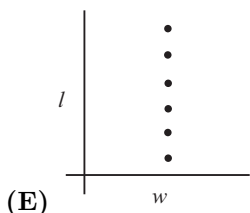
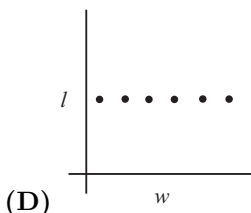
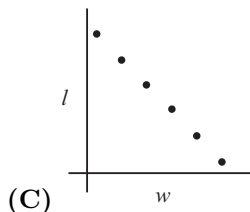
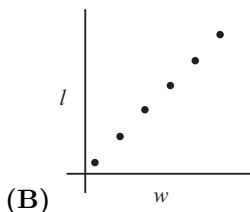
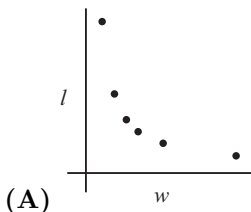


(A) 39              (B) 48              (C) 52              (D) 55              (E) 75

9. What is the product of  $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \dots \times \frac{2006}{2005}$ ?

(A) 1              (B) 1002              (C) 1003              (D) 2005              (E) 2006

10. Jorge's teacher asks him to plot all the ordered pairs  $(w, l)$  of positive integers for which  $w$  is the width and  $l$  is the length of a rectangle with area 12. What should his graph look like?



11. How many two-digit numbers have digits whose sum is a perfect square?  
 (A) 13                      (B) 16                      (C) 17                      (D) 18                      (E) 19
12. Antonette gets 70% on a 10-problem test, 80% on a 20-problem test and 90% on a 30-problem test. If the three tests are combined into one 60-problem test, which percent is closest to her overall score?  
 (A) 40                      (B) 77                      (C) 80                      (D) 83                      (E) 87
13. Cassie leaves Escanaba at 8:30 AM heading for Marquette on her bike. She bikes at a uniform rate of 12 miles per hour. Brian leaves Marquette at 9:00 AM heading for Escanaba on his bike. He bikes at a uniform rate of 16 miles per hour. They both bike on the same 62-mile route between Escanaba and Marquette. At what time in the morning do they meet?  
 (A) 10:00                      (B) 10:15                      (C) 10:30                      (D) 11:00                      (E) 11:30

Problems 14, 15 and 16 involve Mrs. Reed's English assignment.

#### A Novel Assignment

The students in Mrs. Reed's English class are reading the same 760-page novel. Three friends, Alice, Bob and Chandra, are in the class. Alice reads a page in 20 seconds, Bob reads a page in 45 seconds and Chandra reads a page in 30 seconds.

14. If Bob and Chandra both read the whole book, Bob will spend how many more seconds reading than Chandra?

(A) 7,600 (B) 11,400 (C) 12,500 (D) 15,200 (E) 22,800



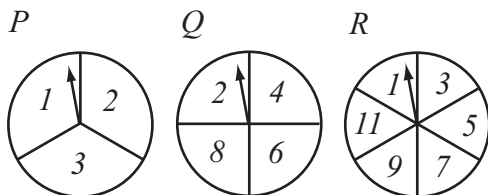
15. Chandra and Bob, who each have a copy of the book, decide that they can save time by “team reading” the novel. In this scheme, Chandra will read from page 1 to a certain page and Bob will read from the next page through page 760, finishing the book. When they are through they will tell each other about the part they read. What is the last page that Chandra should read so that she and Bob spend the same amount of time reading the novel?

(A) 425 (B) 444 (C) 456 (D) 484 (E) 506

16. Before Chandra and Bob start reading, Alice says she would like to team read with them. If they divide the book into three sections so that each reads for the same length of time, how many seconds will each have to read?

(A) 6400 (B) 6600 (C) 6800 (D) 7000 (E) 7200

17. Jeff rotates spinners P, Q and R and adds the resulting numbers. What is the probability that his sum is an odd number?

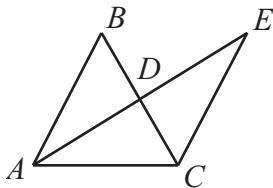


(A)  $\frac{1}{4}$  (B)  $\frac{1}{3}$  (C)  $\frac{1}{2}$  (D)  $\frac{2}{3}$  (E)  $\frac{3}{4}$

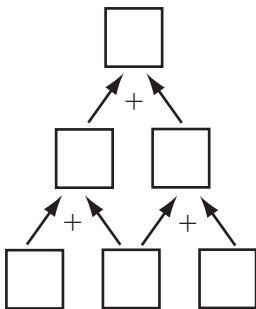
18. A cube with 3-inch edges is made using 27 cubes with 1-inch edges. Nineteen of the smaller cubes are white and eight are black. If the eight black cubes are placed at the corners of the larger cube, what fraction of the surface area of the larger cube is white?

(A)  $\frac{1}{9}$  (B)  $\frac{1}{4}$  (C)  $\frac{4}{9}$  (D)  $\frac{5}{9}$  (E)  $\frac{19}{27}$

19. Triangle  $ABC$  is an isosceles triangle with  $AB = BC$ . Point  $D$  is the midpoint of both  $\overline{BC}$  and  $\overline{AE}$ , and  $\overline{CE}$  is 11 units long. Triangle  $ABD$  is congruent to triangle  $ECD$ . What is the length of  $\overline{BD}$ ?



- (A) 4                      (B) 4.5                      (C) 5                      (D) 5.5                      (E) 6
20. A singles tournament had six players. Each player played every other player only once, with no ties. If Helen won 4 games, Ines won 3 games, Janet won 2 games, Kendra won 2 games and Lara won 2 games, how many games did Monica win?
- (A) 0                      (B) 1                      (C) 2                      (D) 3                      (E) 4
21. An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. The aquarium is filled with water to a depth of 37 cm. A rock with volume  $1000 \text{ cm}^3$  is then placed in the aquarium and completely submerged. By how many centimeters does the water level rise?
- (A) 0.25                      (B) 0.5                      (C) 1                      (D) 1.25                      (E) 2.5
22. Three different one-digit positive integers are placed in the bottom row of cells. Numbers in adjacent cells are added and the sum is placed in the cell above them. In the second row, continue the same process to obtain a number in the top cell. What is the difference between the largest and smallest numbers possible in the top cell?



- (A) 16                      (B) 24                      (C) 25                      (D) 26                      (E) 35

23. A box contains gold coins. If the coins are equally divided among six people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds the smallest number of coins that meets these two conditions, how many coins are left when equally divided among seven people?



(A) 0            (B) 1            (C) 2            (D) 3            (E) 5

24. In the multiplication problem below,  $A$ ,  $B$ ,  $C$  and  $D$  are different digits. What is  $A + B$ ?

$$\begin{array}{r} ABA \\ \times CD \\ \hline CDCD \end{array}$$

(A) 1            (B) 2            (C) 3            (D) 4            (E) 9

25. Barry wrote 6 different numbers, one on each side of 3 cards, and laid the cards on a table, as shown. The sums of the two numbers on each of the three cards are equal. The three numbers on the hidden sides are prime numbers. What is the average of the hidden prime numbers?



(A) 13            (B) 14            (C) 15            (D) 16            (E) 17

## **SOLUTIONS**

Your School Manager has been sent at least one copy of the 2006 AMC 8 Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

### **WRITE TO US**

*Comments about the problems and solutions for this AMC 8 should be addressed to:*

Ms. Bonnie Leitch, AMC 8 Chair / [bleitch@earthlink.net](mailto:bleitch@earthlink.net)  
548 Hill Avenue, New Braunfels, TX 78130

*Comments about administrative arrangements should be addressed to:*

MAA American Mathematics Competitions / [amcinfo@unl.edu](mailto:amcinfo@unl.edu)  
American Mathematics Competitions, University of Nebraska-Lincoln  
P.O. Box 880658, Lincoln, NE 68588-0658

### **AMC 10 & AMC 12**

The AMC 10 and AMC 12 are 25-question, 75-minute contests with 5 choices of answers for each problem (A through E). Schools with high scoring students on the AMC 8 will receive an Invitation Brochure for the 2007 AMC 10. The best way to prepare for these upper level contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

American Mathematics Competitions  
ATTN: Publications  
P.O. Box 81606  
Lincoln, NE 68501-1606

### **PUBLICATIONS**

A complete listing of current publications, with ordering instructions, is at our web site:  
[www.unl.edu/amc](http://www.unl.edu/amc).

2006

AMC 8

DO NOT OPEN UNTIL  
TUESDAY, NOVEMBER 14, 2006

**\*\*Administration On An Earlier Date Will Disqualify Your School's Results\*\***

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE NOVEMBER 14, 2006. Nothing is needed from inside this package until November 14.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed First Class to the AMC office no later than 24 hours following the exam.
4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.
5. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, e-mail, World Wide Web or media of any type is a violation of the competition rules.*

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